Observation of Universal Thermopower Fluctuations

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We have measured the thermopower of small n^+-type GaAs wires using a novel electron heating technique. The results show very large fluctuations which are in good agreement with existing theories. We believe this to be the first observation of universal thermopower fluctuations.

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In the usual semiclassical theory of electron conduction the interference between scattered waves is neglected since it is assumed that such effects will be averaged away. For conductors of size comparable with the electron phase-coherence length such averaging breaks down and these interference effects become intrinsically nonself-averaging, giving rise to universal conductance fluctuations as a function of magnetic field[1,2] or Fermi energy.[3] Other transport coefficients are also expected to show aperiodic fluctuations of which the thermopower is predicted to show the most dramatic behavior.[4]

A simple estimate of the diffusion thermopower fluctuations for a degenerate conductor can be obtained from the Mott formula:

$$S=-\frac{\pi^2}{3}\frac{k}{|e|}\frac{kT}{G}\frac{dG}{dE}\bigg|_{E_F}, \tag{1}$$

where G is the electrical conductance, E_F is the Fermi energy, and k is Boltzmann's constant. Since quantum interference effects lead to a conductance which is a rapidly fluctuating function of E_F, (1) immediately implies that the associated thermopower will fluctuate about zero with a large amplitude. We now replace the derivative by $\delta G/\Delta E$, where δG is the rms amplitude of the conductance fluctuations. ΔE, their average energy scale, will be the larger of kT, hD/L_i^2, or hD/L^2, where L is the specimen length in the direction of the temperature gradient, L_i is the inelastic-scattering length, and D is the diffusion coefficient. In our specimens in the temperature range studied, kT is the largest of these so the rms amplitude of the thermopower fluctuations will be

$$\delta S\simeq\beta\frac{\pi^2}{3}\frac{k}{|e|}\frac{\delta G}{G},\quad \beta\simeq 1. \tag{2}$$

This is only a crude estimate since the Mott formula assumes that G is slowly varying on energy scales kT which is untrue in this case. However, detailed theoretical calculations[4,5] show that (2) will still be correct provided that for a phase-coherent subunit $\delta G/G$ is small, as it is in our specimens. Furthermore, for the case when $\Delta E=kT$ the prefactor β can be easily calculated. In the Appendix we show that $\beta=0.17\pm0.01$. The ratio of δS to $\delta G/G$ for $\Delta E=kT$ is thus a simple universal value, independent of temperature, size of specimen, or the degree of disorder. This remarkable result is implicit in Refs. 4 and 5 but is not given the emphasis it deserves. The key result is that the temperature and size dependences of δS, which can be complicated, arise solely from those of $\delta G/G$.

The specimens we have studied are n^+-type GaAs wires fabricated by electron-beam lithography and dry etching from molecular-beam-epitaxy grown layers. They are of physical width 500 μm, thickness 50 nm, and length 190 μm with many side arms. The electron concentration derived from Hall and Shubnikov-de Haas measurements is 1.09×10^{24} m^{-3}. The electron-transport mean free path (MFP) is about 50 nm and the scattering MFP will be smaller than this so a diffusive-transport model should still be valid.

The two-terminal magnetoresistance of a 9-μm section of wire (Fig. 1) shows that the relatively large conductance fluctuations persist to the highest fields and coexist

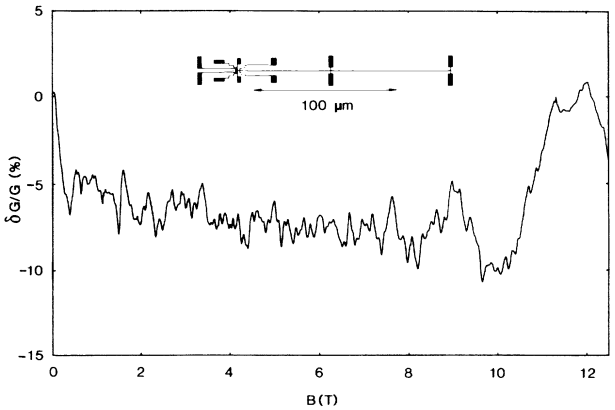

FIG. 1. Two-terminal resistance fluctuations in a 9-μm section of the wire. The wire resistance at zero field is 22 kΩ. Inset: Wire geometry.

with the high-field Shubnikov–de Haas oscillations. The underlying negative magnetoresistance is due to weak localization and electron-electron interaction effects.

The fundamental problem in measuring thermopower is how to establish a significant temperature difference along such micron-scale devices. In practice, we find that the largest gradients in the lattice temperature which we can establish will only give thermoelectric voltages of a few nanovolts. This is of the same order as the Johnson noise. This has led us to use gradients in the electron temperature produced by electron heating. Such gradients, which are easily established, will give the same result for the diffusion thermopower as the conventional approach, since electron-phonon scattering is unimportant, and will also eliminate the phonon-drag contribution.

In our experiment an electric field is applied along the wire and the thermoelectric voltage, V_{AB}, is measured between the ends of the side arms (see Fig. 2). The electrons at C will be at a temperature $T_{E\max}$ and those at A and B close to the lattice temperature T_L since the energy relaxation length is ~ 2 μm and the distance $AC=CB=4.5$ μm. The local electron temperature will be well defined since the electron-electron scattering time ($\tau_e \sim 6\times 10^{-12}$ s) is far shorter than the electron-phonon scatter time ($\tau_e \sim 6\times 10^{-10}$ s) in these specimens.[2] In fact, $T_{E\max}$ will itself show fluctuations, but this is not important as they will only be $\sim 1\%$. For a homogeneous conductor the voltages V_{AC} and V_{CB} would cancel, but the thermopower fluctuations in each arm are uncorrelated and so simply add in an rms fashion. Another unique consequence of this technique is that the symmetry of the temperature distributions allows an absolute rather than a relative thermopower to be determined. This can be seen from Fig. 2. Points A and B are the junctions between the sample and the external circuitry and these will be the same temperature. Point C is at a higher temperature but is internal to the sample; the normal circuit of a hot junction and a cold junction between differing materials does not exist. In practice, the field along the wire is alternating at low frequency so any contribution to V_{AB} from the Hall effect is eliminated. The calibration of $T_E(V_{ac})$ is obtained from $G(V_{ac})$ and $G(T_L)$ which are measured to high accuracy with a six-decade inductive ratio bridge. This "hot-electron thermopower" technique is an extension of our previous work on the thermopower of 2D electron systems.[6]

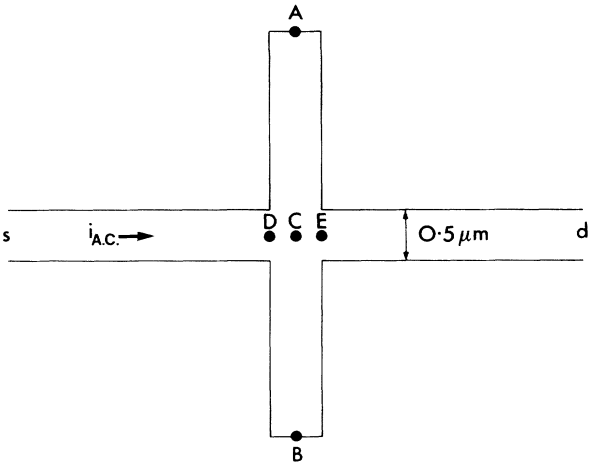

FIG. 2. Detail of the wire geometry. Length AB is 9 μm.

The measurements are made by recording the dc voltage (V_{AB}) between points A and B as the magnetic field is slowly swept (typically 0.5 T/min). Figure 3 shows a series of measured thermoelectric voltages. One can see that they show the expected large oscillations about zero. At higher fields we find no contribution from the Shubnikov–de Haas oscillation as this will be the same in each arm and thus eliminated. Such traces are totally reproducible over weeks so long as the wire is kept in the dark at helium temperatures. In Fig. 4 the ratio of $\delta V/(\Delta G/G)$ is plotted against $T_{E\max}-T_L$, the value of $\Delta G/G$ being taken at $T_{E\max}$ since, as we will show later, the dominant contribution to δV comes from the hotter regions. One sees that the data for different T_L all fall close to the same line as expected and agree well with that predicted by Eq. (2), except that the results are a factor of about 3 larger than the theory predicts. However, this comparison implicitly assumes that each phase-coherent subunit contributes equally to the fluctuations in conductivity and thermoelectric voltage, i.e., that the temperature gradient is linear, which will not be true.

The full form of $T_E(x)$ along the wire may be obtained by numerical solution of the diffusion equation provided that the energy relaxation length $L_E(T_E,T_L)$ is known.[7] This we obtain from our calibration of $T_E(V_{ac})$

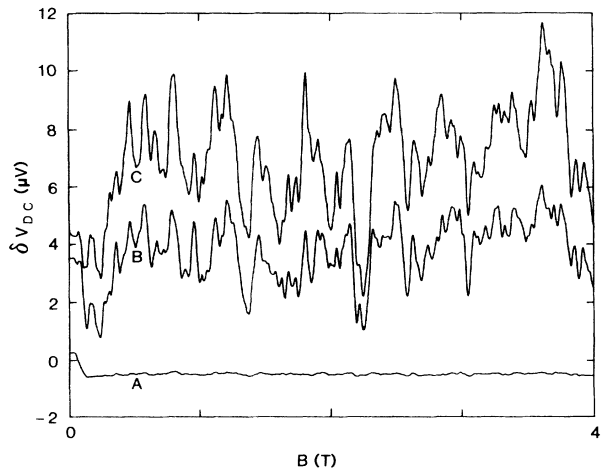

FIG. 3. dc thermoelectric voltage fluctuations due to the electron temperature gradient. Heating voltage (ac) across DE is 0, 17, and 45 μV for traces A, B, and C. Trace B has been offset by 4 μV and C by 7 μV.

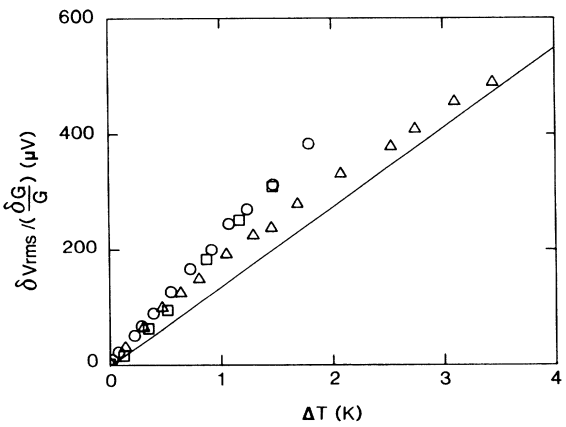

FIG. 4. rms dc thermoelectric voltage fluctuations as a function of $\Delta T = T_{E\max} - T_L$ compared with the theoretical prediction assuming a linear temperature gradient. The theoretical values have been increased by a factor of 2. Data points: $T_L = 1.08$ K (△), 2.00 K (○), and 4.00 K (□).

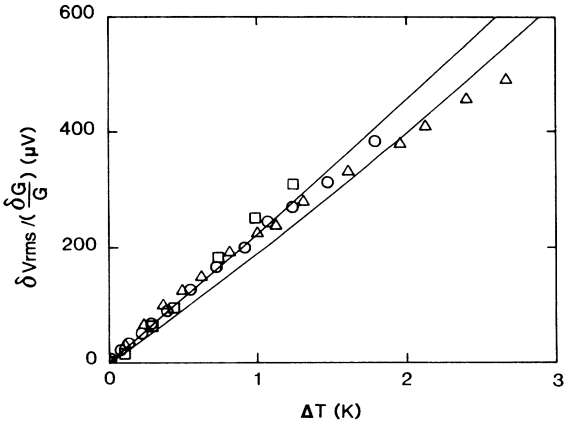

FIG. 5. rms dc thermoelectric voltage fluctuations as a function of $\Delta T = T_{E\max} - T_{E\min}$ compared with the theoretical predictions for the true temperature gradient. Upper line prediction for $T_L = 4$ K; lower line for $T_L = 1$ K. The predicted values have been increased by a factor of 2. Data points: $T_L = 1.08$ K (△), 2.00 K (○), and 4.00 K (□).

by use of the energy-balance equation.[7] The phase breaking length, L_ϕ, should be given by $(hD/kT)^{1/2}$. To check this L_ϕ is calculated by assuming that for each phase-coherence subunit $\delta G = e^2/h$ and adding these in an rms fashion. We obtain good agreement in magnitude and temperature with the expected form. This then allows us to calculate the temperature difference across each subunit and to directly calculate the expected thermoelectric voltage for a given $T_{E\max}$ and T_L. The predictions of this full theory are compared with the experimental data in Fig. 5. In this case the values of $\delta G/G$ are taken at the appropriate average temperature. The true temperature difference along the wire is also used since the calculated $T_{E\min}$ can be slightly larger than T_L. One sees that the good agreement with the predicted form is retained and that now the results are a factor of 2 greater than the prediction. These calculated thermopowers are close to the prediction of the simple theory. There are two reasons for this. First, we choose the distances AC and CB to be approximately $2L_E$ so that $T_{E\min}$ is not very much larger than T_L and the departure from linearity not too dramatic. Second, we find that the ratio L_E/L_ϕ is almost constant over the temperature range studied which leads to the temperature difference across each subunit being a weak function of distance along the wire.

We feel confident that these voltages are indeed thermoelectric. It is difficult to see what other effects could mimic the observed behavior. The EM electronics N1A nanovoltmeter used to measure the dc voltages has an input bias current of typically < 5 pA with an upper bound of 20 pA, and an effective input impedance of 1 G Ω. Therefore a fluctuating voltage of larger than 4 nV could not be produced due to the input bias current. Further, such currents would result in traces looking like Fig. 1 including the Shubnikov-de Haas oscillations at high fields. The structure observed was also totally unaffected by sweep rate or direction so induced voltages are of no importance. A fluctuating Hall-like voltage which *did not* reverse sign with current is perhaps conceivable but this would require ~10% of the voltage dropped between D and E (Fig. 2) to appear perpendicular to the current direction. More importantly, such a contribution would surely be linear in V_{ac} and this is very far from the case. In fact, we find the linear dependence upon the temperature difference most convincing because this difference is both strongly nonlinear in V_{ac} and strongly dependent upon T_L. We have also studied the rectification of alternating-voltage signals due to the non-Ohmic conduction[8] in these systems and can discount this as a possible cause of the voltage we measure.

In conclusion, we have employed a novel hot-electron technique to measure the thermopower of a mescoscopic conductor. The thermoelectric voltages obtained are large compared with those one might expect to arise from any other source. The results agree with theory in form, dependence on temperature, and temperature gradient. The absolute magnitudes differ from the theoretical predictions by a factor of 2. We feel that these results show unequivocally the existence of universal thermopower fluctuations.

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Note added.—This technique has recently been employed to successfully measure the thermopower of quantum point contacts.[9]

Appendix: Calculation of the thermopower fluctuations.—If we take the case when $\delta G_{\rm rms} \leq G$, the gen-

eral expression for the thermopower is

$$\delta S_{\mathrm{rms}} = (G \mid e \mid T)^{-1} \left[\int_0^\infty \int_0^\infty \frac{df}{d\epsilon} \frac{df'}{d\epsilon} (\epsilon - \mu)(\epsilon' - \mu) F(\epsilon - \epsilon') d\epsilon d\epsilon' \right]^{1/2}, \tag{A1}$$

where f is the Fermi function, μ is the chemical potential, and $F(x)$ is the conductivity correlation function. In the case $\mu \gg kT = E_C$ Eq. (2) is obtained and the value of β is found by numerical integration to be insensitive to the form of the correlation function: the values for β being 0.18, 0.17, and 0.16 for $F(x)$ of Gaussian, semielliptic, and inverted parabolic, respectively. These values are much less than those implied in Ref. 4. For $\mu \gg kT \ll E_c$, β is replaced by $\gamma kT/E_c$. γ is now sensitive to the detailed form of $F(x)$ being $\sqrt{2}$ for the Gaussian and inverted parabolic forms but only half this value for the semielliptic form. Furthermore, this expected linear behavior is only accurate for $kT \ll 0.05 E_c$. These results are in agreement with the computer simulations of Kearney.[5]

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