Nonlinear Theory of Intense Laser-Plasma Interactions

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A nonlinear theory of intense laser-plasma interactions is developed and used to describe relativistic optical guiding, coherent harmonic radiation production, and nonlinear plasma wake-field generation. Relativistic optical guiding is found to be ineffective in preventing the leading portion ($\leq$ a plasma wavelength) of a laser pulse from diffracting. Coherent harmonic generation is found to be most efficient for short laser pulses. Optical guiding and harmonic generation may be enhanced by the presence of large-amplitude plasma wake fields. These phenomena may be important in laser-driven plasma accelerators, x-ray sources, and fusion schemes.

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The interaction of ultrahigh-power laser beams[1] with plasmas is rich in a variety of wave-particle phenomena.[2] These phenomena become particularly interesting and involved when the laser power is sufficiently intense to cause the electron oscillation (quiver) velocity to become highly relativistic. Some of the laser-plasma processes addressed in this Letter include the following: (i) relativistic optical guiding[3-5] of the laser beam, (ii) the excitation of coherent radiation at harmonics of the fundamental laser frequency, and (iii) the generation of large-amplitude plasma waves[6-8] (wake fields). These processes are relevant to laser-plasma accelerators, x-ray sources, and laser fusion schemes.

Using a cold-fluid model together with a "quasistatic" approximation, a set of coupled nonlinear equations is derived for the vector potential of the radiation field and the electrostatic potential of the plasma. This fully nonlinear 1D model describes the self-consistent interaction of intense laser pulses with plasmas. The important issue of laser-plasma instabilities,[2,3] however, is not addressed in this paper.

Relativistic optical guiding[3-5] results from the modification of the index of refraction due to the relativistic quiver motion of the electrons by the laser field. Analysis[4] of this effect has shown that as the laser power exceeds a critical threshold, diffraction can be overcome, resulting in optical guiding of the laser beam. Previous analyses of relativistic guiding have included, for the most part, only the transverse electron motion in the plasma response current. Relativistic guiding was believed to occur on a fast time scale (on the order of the inverse laser frequency). However, the present analysis finds this not to be the case. By including the electron-density response and the longitudinal electron motion self-consistently, it is shown that for short laser pulses (pulse lengths less than a plasma wavelength) relativistic optical guiding is significantly diminished. It is found that relativistic guiding occurs only for long pulses with slow rise times (greater than an inverse plasma frequency). The leading edge of a long pulse, however, will continually erode.

As the quiver motion of the electrons in a linearly polarized laser field becomes highly relativistic, the plasma response current will develop harmonic components. This can lead to the excitation of coherent radiation at harmonics of the fundamental laser frequency. In addition, the ponderomotive force of an intense, short-pulse laser (pulse lengths near the plasma wavelength) can generate large-amplitude, plasma-wave wake fields.[6-8]

The 1D fields associated with the laser-plasma interaction can be described by the normalized transverse vector and scalar potentials, $\mathbf{a}(z,t)=|e|\mathbf{A}_\perp/m_0c^2$ and $\phi(z,t)=|e|\Phi/m_0c^2$, respectively. The electrons are assumed to obey the relativistic cold-fluid equations and the ions are assumed to be stationary. Thermal effects may be neglected provided (i) the electron quiver velocity is much greater than the electron thermal velocity, and (ii) the thermal-energy spread is sufficiently small such that electron trapping in the plasma wave is avoided. It proves convenient to perform an algebraic transformation from the laboratory-frame–independent space and time variables (z,t) to the independent variables (ξ,τ), where $\xi=z-ct$, $\tau=t$, and c is the speed of light. In this coordinate system the plasma flows through a nearly stationary (slowly varying in τ) laser pulse. Using the Coulomb gauge, $\nabla\cdot\mathbf{A}=0$, and noting that $\partial/\partial z=\partial/\partial\xi$ and $\partial/\partial t=\partial/\partial\tau-c\,\partial/\partial\xi$, the wave equation is then given by

$$\left(\frac{2}{c}\frac{\partial}{\partial\xi}-c^{-2}\frac{\partial}{\partial\tau}\right)\frac{\partial\mathbf{a}}{\partial\tau}=k_p^2\frac{n}{n_0}\frac{\mathbf{a}}{\gamma}\,, \tag{1}$$

and Poisson's equation is $\partial^2\phi/\partial\xi^2=k_p^2(n/n_0-1)$, where $k_p^2=4\pi|e|^2n_0/m_0c^2$, n is the plasma electron density, and n_0 is the ambient density. The relativistic factor associated with the electrons is $\gamma=(1-\beta_\perp^2-\beta^2)^{-1/2}=(1+a^2)^{1/2}/(1-\beta)^{1/2}$, where $\beta_\perp=v_\perp/c$ and $\beta=v_z/c$ are the normalized transverse and longitudinal electron fluid velocities, respectively. In obtaining the right-hand side of Eq. (1), conservation of transverse canonical momentum has been used, $\gamma\beta_\perp = a$, along with the transverse current, $J_\perp = -|e|nv_\perp = -|e|nca/\gamma$. The longitudinal electron fluid response is given by the momentum equation, $d(\gamma\beta)/dt = c\,\partial\phi/\partial z - (c/2\gamma)\partial a^2/\partial z$, which in the ξ,τ variables becomes

$$\frac{\partial}{\partial\xi}[\gamma(1-\beta)-\phi] = -\frac{1}{c}\frac{\partial}{\partial\tau}(\gamma\beta). \tag{2}$$

Similarly, the continuity equation, $\partial n/\partial t + c\,\partial(n\beta)/\partial z = 0$, becomes $\partial[n(1-\beta)]/\partial\xi = \partial(n/c)/\partial\tau$.

The electron-fluid response can be greatly simplified by noting that, in the ξ,τ coordinates, under certain conditions a quasistatic state will exist in the macroscopic plasma quantities, n, β, and γ. That is, if the laser pulse is sufficiently short, the fields $\mathbf{a}$ and ϕ which drive the plasma are expected to change little during a transit time of the plasma through the laser pulse. Equation (1) implies that the envelope of $\mathbf{a}$ changes on a characteristic time $\tau_e \sim 2\gamma|n_0/n|(\omega/\omega_p)/\omega_p$, where ω is the laser frequency. Assuming $\omega \gg \omega_p$, the time τ_e will be long compared to a plasma period. If the laser-pulse duration, τ_L, is small compared to τ_e, then the quasistatic approximation is valid. In addition, the validity of the 1D model requires that the laser-beam vacuum diffraction time, $\tau_d = \pi r_s^2/\lambda c$, be long compared to τ_e. This is satisfied when the radiation spot size $r_s \gg \lambda_p = 2\pi/k_p$.

Under the quasistatic approximation, the $\partial/\partial\tau$ derivatives may be neglected in the electron-fluid equations. In this case, the fluid equations can be integrated to give $n(1-\beta) = n_0$ and $\gamma(1-\beta)-\phi = 1$. Using these relations, together with the expression for γ, the coupled field equations become

$$\left(\frac{2}{c}\frac{\partial}{\partial\xi} - c^{-2}\frac{\partial}{\partial\tau}\right)\frac{\partial\mathbf{a}}{\partial\tau} = k_p^2\frac{\mathbf{a}}{1+\phi}, \tag{3}$$

$$\frac{\partial^2\phi}{\partial\xi^2} = -\frac{k_p^2}{2}\left(1-\frac{(1+a^2)}{(1+\phi)^2}\right). \tag{4}$$

This coupled set of equations completely describes the 1D nonlinear laser-plasma interaction within the quasistatic approximation. This model is valid for laser pulses of arbitrary polarizations and arbitrary intensities ($|a|^2 \gtrsim 1$). Consistent with the quasistatic assumption, a number of important points can be made concerning intense laser-pulse propagation in plasmas.

The nonlinear index of refraction of the laser beam within the plasma determines, among other things, the optical-guiding properties of the plasma. For the purpose of the present discussion, the laser field $\mathbf{a}$ is assumed to be linearly polarized, $\mathbf{a} = \mathbf{a}_L\exp(ik\xi)$, where $\mathbf{a}_L$ represents the complex amplitude and k is the wave number. The characteristic spatial variation in the laser envelope, $|a_L|$, is assumed to be of the order of L and is long compared to the laser wavelength, $\lambda = 2\pi/k$, i.e., $\partial|a_L|/\partial\xi \simeq |a_L|/L \ll k|a_L|$. From the wave equation, Eq. (3), the refractive index, $\eta = ck/\omega$, is given by

$$\eta = 1 - \frac{\lambda^2/2\lambda_p^2}{1+\phi_s}, \tag{5}$$

where ϕ_s is the slow part of the scalar potential and $\lambda \ll \lambda_p$. In obtaining Eq. (5) from the wave equation, $\partial/\partial\xi$ was replaced with ik and $\partial/\partial\tau$ replaced with $i(ck-\omega)$, where $|ck-\omega| \ll ck$ and $\phi \simeq \phi_s$. Here it has been assumed that $|\phi_f| \ll |\phi_s|$, where ϕ_f is the rapidly varying part of ϕ, which is valid as long as $\lambda \ll \lambda_p$. The fact that $n/\gamma = 1/(1+\phi)$ is predominantly slowly varying, even though n and γ have rapidly varying components, is also suggested by earlier work.[3,9,10]

For a long pulse with a long rise time compared to a plasma period, $L \gg \lambda_p$, the first term on the left-hand side of Eq. (4) can be neglected and ϕ_s can be approximated by $1+\phi_s \simeq (1+|a_L|^2/2)^{1/2}$. Although the present analysis is 1D, one expects that for a slowly varying transverse laser profile, the index of refraction will depend on the transverse coordinates through the laser amplitude $|a_L|$. Since the actual laser-beam amplitude falls off transversely, $\partial|a_L|/\partial r < 0$, so will the refractive index, $\partial\eta/\partial r < 0$. The negative transverse gradient of the refractive index can lead to optical guiding. It is well known that if the refractive index is of the form given by Eq. (5) in the limit $1+\phi_s \simeq (1+|a_L|^2/2)^{1/2}$, a critical laser power necessary for relativistic optical guiding exists and is given by[4] $P_{\rm crit} \simeq 17(\lambda_p/\lambda)^2$ GW.

We next consider a short laser pulse compared to a plasma period, $L \lesssim \lambda_p$. When $|\phi| \ll 1$, Eq. (4) can be solved for arbitrary L.[7] If the pulse envelope is given by $a_L = a_{L0}\sin(\pi\xi/L)$ for $-L \le \xi \le 0$ and $a_L = 0$ otherwise, the scalar potential within the laser pulse, for $L \ll \lambda_p$, is $\phi_s \simeq (a_{L0}k_p/4)^2 g(\xi)$, where

$$g(\xi) = \xi^2 - 2(L/2\pi)^2[1-\cos(2\pi\xi/L)].$$

Note that, even for $|a_{L0}| > 1$, the assumption that $\phi_s \ll 1$ is valid as long as $L \ll \lambda_p$. In the short-pulse limit, the fact that $\phi_s \ll 1$ implies that the optical-guiding effect is reduced significantly, by more than the factor $(\pi^2/2)(L/\lambda_p)^2 \ll 1$. The critical power, therefore, is increased by the inverse of this factor and, in addition, the degree of guiding varies along the pulse. Hence, it is unlikely that relativistic optical guiding can be effectively utilized in short, $L \lesssim \lambda_p$, laser pulses.

Although it may appear that a long laser pulse may undergo guiding, assuming the various laser-plasma instabilities can be controlled, the front of the pulse will diffract. Initially, that portion of the head of a long-rise-time pulse in which the local power is less than $P_{\rm crit}$ will diffract. Once this portion has diffracted away, the pulse will exhibit "short-pulse" diffractive behavior; i.e., the front region ($\sim\lambda_p$) will continue to diffract. The erosion of the front of the pulse will propagate back through the body of the pulse (in the ξ frame) at a velocity on the order of $v_E \simeq c\lambda_p/z_R$, where $z_R = \pi r_s^2/\lambda$ is the vacuum Rayleigh length.

The nonlinearities associated with relativistic effects can provide a source for the generation of coherent radiation at harmonics of the laser frequency. To examine this process, the full radiation field is represented by $\mathbf{a} = \sum \mathbf{a}_j \exp(ijk\xi)$, where the sum is over $j = 1,2,3,\ldots$ and $\mathbf{a}_1 = \mathbf{a}_L(\xi)$ is the envelope of the dominant fundamental laser pulse, $|a_L| \gg |a_j|$, for $j \geq 2$. It is clear from the right-hand side of Eq. (3) that harmonic excitation is solely due to the fast part of ϕ. In particular, since the fundamental component of the radiation field dominates, the fast part of the scalar potential is driven by the fast part of a^2, which is $(a_L^2/2)\cos(2k\xi)$. Equation (4) may be solved for ϕ_f by replacing $\partial/\partial\xi$ with $2ik$ and by approximating ϕ by ϕ_s on the right-hand side. This gives $\phi_f = \hat{\phi}_f \cos(2k\xi)$, where

$$\hat{\phi}_f = -(\lambda/4\lambda_p)^2 a_L^2 (1+\phi_s)^{-2}.$$

Noting that $|\phi_f| \ll |\phi_s|$, the source term in Eq. (3) for the harmonics is

$$\mathbf{S} = k_p^2 \mathbf{a}_L (1+\phi_s)^{-1} \cos(k\xi) \sum_m [Q\cos(2k\xi)]^m, \quad (6)$$

where $Q = (\lambda/4\lambda_p)^2 a_L^2 (1+\phi_s)^{-3}$, and $m = 0,1,2,\ldots$.

As an illustration, consider the excitation of third-harmonic radiation (3ω). Substituting the third-harmonic component of the source, $(m=1)$ in Eq. (6), into the right-hand side of Eq. (3) allows one to solve for the third-harmonic field, $|a_3|$. The ratio of the third-harmonic power to the fundamental laser power is $P_3/P_1 = R^2$, where $R = (\lambda/4\lambda_p)^3 (1+\phi_s)^{-4} a_L^2 \omega_p \tau$ and τ is the laser-plasma interaction time.

Equation (6) shows that the generation of harmonics is a strong function of the plasma wake field ϕ_s in the region of the fundamental laser pulse. For a single long-pulse, large-amplitude laser, $|a_L| \gg 1$, the slow part of the scalar potential is $\phi_s \simeq |a_L|/\sqrt{2}$. In this case, the harmonic content of the source term in Eq. (6) is exceedingly small. Taking τ to be a diffraction time, $\tau_d = \pi r_s^2/\lambda c$ and $r_s \gtrsim \lambda_p$, the third-harmonic power becomes $P_3/P_1 \simeq F^4$, where $F = \sqrt{2}\pi\lambda/4\lambda_p |a_L|$. In the case of a short, $L \ll \lambda_p$, large-amplitude laser pulse, one has $\phi_s \ll 1$ which gives $P_3/P_1 \simeq G^4$, where $G = \sqrt{2}\pi\lambda |a_L|/8\lambda_p$. For $|a_L|^2 \gg 1$, a short pulse is more efficient than a long pulse for harmonic generation.

The nonlinear excitation of plasma wake fields is governed by Eqs. (3) and (4). Figure 1 shows the plasma-density variation $\delta n/n_0 = n/n_0 - 1$, and the axial electric field E_z for a laser-pulse envelope given by $a_L = a_{L0}\sin(\pi\xi/L)$ for $-L \leq \xi \leq 0$, where $L = \lambda_p = 0.03$ cm, $\lambda = 10$ μm, and (a) $a_{L0} = 0.5$ and (b) $a_{L0} = 2$. For $a_{L0} \geq 1$, nonlinear effects become important. In Fig. 1(b) one observes a steepening of the electric field and an increase in the period of the wake field.[8,9,11] In addition, Fig. 1(b) shows that the electrostatic potential ϕ is predominantly slowly varying within the laser pulse even though $\delta n/n_0$ has rapidly varying components.

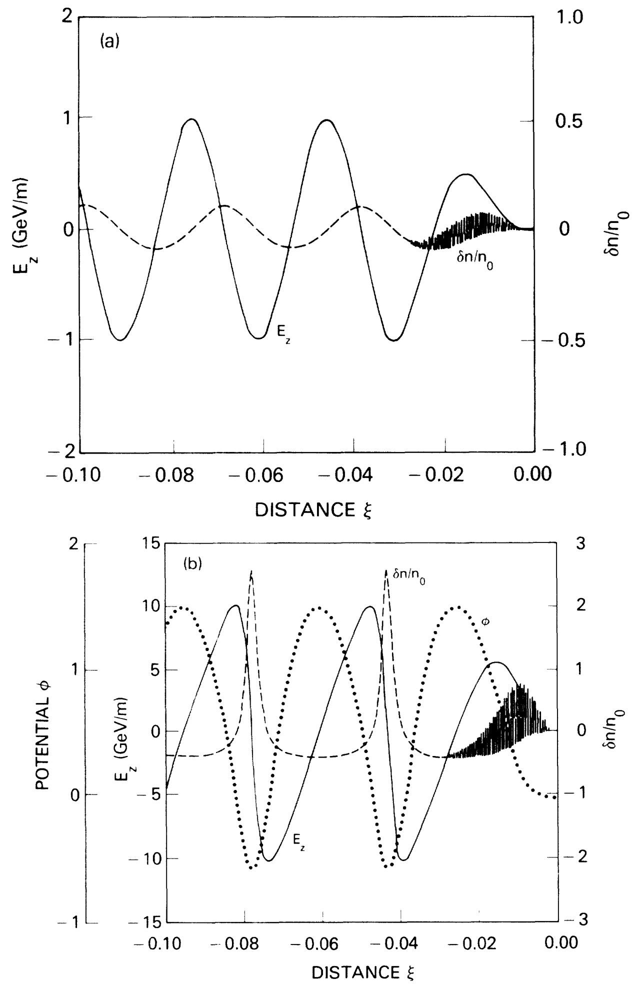

FIG. 1. Density variation $\delta n/n_0 = n/n_0 - 1$ (dashed line), axial electric field E_z in GeV/m (solid line), and electrostatic potential ϕ (dotted line) for a laser pulse located within the region $-L \leq \xi \leq 0$, where $L = \lambda_p = 0.03$ cm and (a) $a_{L0} = 0.5$ and (b) $a_{L0} = 2$.

Aspects of nonlinear wake-field generation[8] may be examined analytically by solving Eq. (4) for a circularly polarized laser pulse with a square-pulse profile, $a_L = a_{L0}$ for $-L \leq \xi \leq 0$ and $a_L = 0$ otherwise. In particular, one finds that the optimal pulse length for maximizing the wake-field amplitude is $L_{\mathrm{op}} = 2\gamma_0 E(\rho)/k_p \rightarrow 2a_{L0}/k_p$ for $a_{L0}^2 \gg 1$, where $E(\rho)$ is the complete elliptic integral of the second kind, $\rho^2 = 1 - 1/\gamma_0^2$ and $\gamma_0^2 = 1 + a_{L0}^2$. This gives a wake field where $\gamma_0^2 \geq 1 + \phi_s \geq 1/\gamma_0^2$ with a maximum axial electric field of $\hat{\mathbf{E}}_{\max} \simeq \gamma_0 - 1/\gamma_0 \rightarrow a_{L0}$ for $a_{L0}^2 \gg 1$, where $\hat{\mathbf{E}} = |e|E_z/m_0c^2k_p$. Note that the maximum axial electric field is $\hat{\mathbf{E}}_{\max} \simeq a_{L0}^2$ for $a_{L0}^2 \ll 1$. Also, the nonlinear wake-field wavelength is $\lambda_p^{\mathrm{NL}} = 4\gamma_0 E(\rho_0)/k_p \rightarrow 4a_{L0}/k_p$ for $a_{L0}^2 \gg 1$, where $\rho_0^2 = 1 - 1/\gamma_0^4$.

The large-amplitude axial electric fields associated with the plasma waves can be utilized to accelerate an injected beam of electrons to high energies.[6-8] In addition to accelerating electrons, the plasma wake field can affect both the primary laser pulse as well as a trailing laser pulse. For example, in the region where $\delta n/n_0 < 0$, the transverse profile of the plasma wake field can lead to a negative transverse gradient of the refractive index,[12] $\partial\eta/\partial r < 0$. Equation (5) indicates that a properly phased trailing laser pulse, which is located at a maximum in ϕ_s, may be optically guided. Harmonic generation can be substantially enhanced in a properly phased trailing laser pulse propagating in a region of the wake field where $-1 \le \phi < 0$, as indicated by Eq. (6). In addition, the sharp axial gradient in $\delta n/n_0$ for a highly nonlinear plasma wake field could induce large frequency shifts in a laser pulse.[13]

Based on the 1D nonlinear quasistatic model of laser-plasma interaction, Eqs. (3) and (4) have been used to analyze (i) relativistic optical guiding, (ii) coherent harmonic production, and (iii) nonlinear wake-field generation. Relativistic optical guiding is shown to depend strongly on the laser-pulse duration. In the long-pulse regime, optical guiding requires a minimum level of total laser power, i.e., P_{crit}. However, the leading portion of the pulse will experience diffraction. In the short-pulse regime, relativistic-guiding effects are greatly diminished by the density response and longitudinal motion of the electrons. As the electron quiver motion becomes highly relativistic, the plasma response current develops harmonics. Analysis of this process has shown that coherent harmonic generation is more effective for short pulses than it is for long pulses with slow rise times. The generation of nonlinear plasma wake fields by intense, short pulses was also examined. Various applications of the plasma wake fields may be possible, including (i) the acceleration of a trailing electron bunch (laser wake-field acceleration), (ii) optical guiding of a trailing laser pulse, and (iii) enhancing the coherent harmonic radiation generated by a trailing laser pulse.

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