Comment on “Zeeman Bifurcation of Quantum-Dot Spectra”

Hansen *et al.* have interpreted the magnetic-field dependence of capacitance spectra of quantum-dot arrays as reflecting the evolution of individual electron states. They model the system as a two-dimensional quantum well with lateral confinement on the scale, independent of bias, of 120 nm.[1] We question whether the experiment has adequate resolution to reveal structure associated with individual levels.

In a circular dot of diameter D, two energies are important, the typical splitting $\delta \approx 16\hbar^2/m^*D^2$ of the one-electron energy levels, usually both orbitally and spin degenerate, and a charging energy $\Delta \equiv e^2/C \approx 16e^2 d_e/\epsilon D^2$. C is the capacitance of the dot to its surroundings, ϵ is the dielectric constant of the medium, and d_e is a length of the order of the distance from the dot to the substrate or gate structure. The ratio of the charging energy to the quantum splitting is $\Delta/\delta \approx e^2 d_e m^*/\epsilon\hbar^2 \approx 5$ for the configuration of the Hansen experiment and is independent of the size of the dot.

If the experiment can be carried out with adequate resolution, each successive electron transfer will contribute a distinct peak to the capacitance spectrum.[2] With $\Delta \gg \delta$, the spacing between peaks will be determined principally by the charging energy Δ, not by the level splitting δ.

Figure 1(a) illustrates a model calculation for a dot with a square-well confining potential (infinite walls) with circular symmetry. Individual energy levels were calculated for the dot whose diameter was taken to be proportional to the square root of the dot capacitance deduced from Ref. 1 by integration of dC/dV_{gate}. A maximum value of 270 nm was chosen for the dot diameter, nearly equal to the lithographically defined size of 300 nm. The one-electron spectrum for the dot was converted to a capacitance spectrum by adding Δ for each electron added to the dot and converting to gate voltage from well voltage using a geometric lever arm of 2 for this structure. This scheme gives an areal density of electrons in the dot equal to that implied by the interpretation of the strong maxima in the 2-T data of Ref. 1 as spin-degenerate Landau maxima. The increase in level density with increasing gate voltage in Fig. 1(a) results from the increase in the diameter of the dot as it is filled and the consequent decrease of both Δ and δ.

To simulate the experiment, this spectrum is convolved with a Gaussian with rms width of 11 meV at the gate (or ~0.3 meV at the well) to destroy resolution on the scale of individual energy levels, giving the density of states (DOS) of Fig. 1(b). For comparison Fig. 1(c) shows the normalized DOS, obtained from the integration of dC/dV_{gate} of the zero-magnetic-field data of Ref. 1.

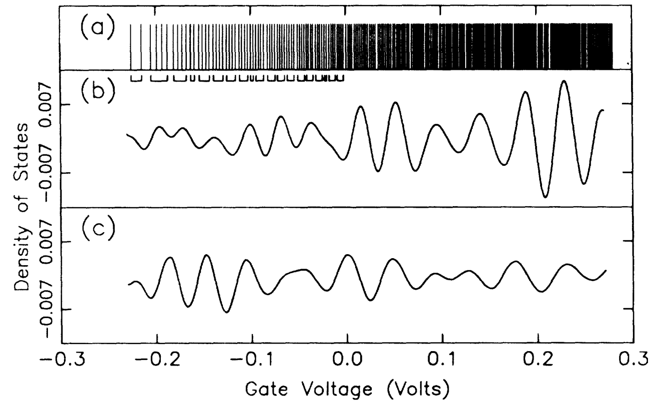

FIG. 1. (a) Each bar represents the gate voltage at which one electron may enter a dot, from the model described in text. Brackets at low gate bias demarcate quantum levels. (b) Density of states (DOS) determined from (a) after convolution with a Gaussian rms width 11 meV. (c) DOS from the data of Ref. 1. In both (b) and (c), the smooth variation has been subtracted, and the results have been normalized to the DOS at 0.26 V.

The two curves show the same qualitative behavior. Comparison with the unbroadened spectrum shows that the first two or three peaks may indeed be associated with filling of the first doublet and the next one or two quartets of the single-particle spectrum, but structure at higher filling is the consequence of fluctuations in the density of states. Each peak is associated, at the highest gate voltages, with about 20–30 electrons. The results of this simulation indicate that over most of the range of gate voltage, the experiment does not resolve individual quantum levels, and changes in structure induced by a magnetic field are not simply interpretable in terms of the behavior of the individual quantum levels.

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