Quantization of the Acoustoelectric Current in a Two-Dimensional Electron System in a Strong Magnetic Field

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An expression for the off-diagonal component of the acoustoelectric tensor is obtained from the momentum-conservation law. The acoustoelectric current along the direction of sound propagation exhibits plateaus as a function of gate voltage if the circuit in the perpendicular direction is open. The product jB has the same plateaus as a function of magnetic field. The values of jB at the plateaus are equidistant and are separated by deep minima. The ratio of the plateau values of the current to the open-circuit voltage is equal to σ_{xy}. This effect can be used for measuring the intensity of a surface acoustic wave propagated through the heterojunction.

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We discuss the effect of the quantization of the Hall conductivity on the acoustoelectric phenomena in a two-dimensional electron gas (2DEG). The principal scheme of a possible experiment is shown in Fig. 1. The heterostructure with the 2DEG near the interface is placed above the surface of a piezoelectric. The piezoelectric and the heterostructure are not coupled acoustically. For example, a small air clearance can be left between them. A surface acoustic wave (SAW) can be excited in the piezoelectric, and it couples to the 2DEG via its electric field.

The wavelength of the SAW is much larger than all the microscopic lengths in the problem. The interaction of a SAW with the 2DEG in the absence of a magnetic field has been considered both theoretically and experimentally by Ingebrigtsen,[1] and Krasheninnikov and Chaplik.[2] Wixforth, Kotthaus, and Weimann[3] have observed giant oscillations of the SAW absorption in a magnetic field, caused by the interaction of the SAW with the 2DEG in a GaAs/GaAlAs heterojunction. We predict here the oscillation and the quantization of the acoustoelectric current created by the SAW drag effect in the regime of the quantum Hall effect. SAW's of two types, the Rayleigh waves (see, e.g., Ref. 4) and the Gulayev-Bleustein waves (see Refs. 5 and 6), are considered here. Before discussing the acoustoelectric current we derive the expression for the absorption coefficient in a strong magnetic field for both types of SAW.

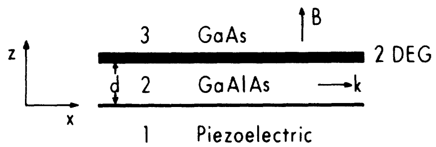

FIG. 1. A heterostructure with the 2DEG placed above the surface of a piezoelectric. The directions of the SAW propagation and of the magnetic field are shown by the arrows.

(1) Sound absorption.—The wave vector $\mathbf{k}$ of the SAW is assumed to be along the x axis, while the magnetic field $\mathbf{B}$ is along the z axis. To find the absorption coefficient Γ one should solve the equation $\mathrm{div}\mathbf{D}=0$ for the electrical displacement $\mathbf{D}$ in the three media (see Fig. 1). The displacement vector $\mathbf{D}$ at $z<0$ has the form

$$\mathbf{D}=\epsilon\mathbf{E}+4\pi\mathbf{P}\,,\qquad P_i=\beta_{i,kl}u_{kl},\quad u_{kl}=\tfrac{1}{2}(\partial u_i/\partial x_k+\partial u_k/\partial x_i)\,, \tag{1}$$

where $\hat{\beta}$ is the piezoelectric tensor, u_{kl} is the strain tensor, ϵ is the dielectric constant of the piezoelectric, and $\mathbf{E}$ and $\mathbf{P}$ are the electric field and the polarization induced by the SAW. We assume that both semiconductors of the heterostructure have the same dielectric constant ϵ_s. Then $\mathbf{D}=\epsilon_s\mathbf{E}$ at $z>0$.

The boundary conditions are as follows: The x component of the electric field E_x is continuous at $z=0$ and $z=d$; the component D_z is also continuous at $z=0$. The condition at $z=d$ has the form

$$D_z(d+0)-D_z(d-0)=4\pi\rho\,. \tag{2}$$

Here ρ is the excess charge density of the 2DEG caused by the SAW. It is connected with the current density $\mathbf{j}=\hat{\sigma}\mathbf{E}$ by the continuity equation:

$$\partial\rho/\partial t+\mathrm{div}\mathbf{j}=0\,. \tag{3}$$

We consider an isotropic semiconductor where the conductivity tensor $\hat{\sigma}$ has the components $\sigma_{xx}=\sigma_{yy}$ and $\sigma_{xy}=-\sigma_{yx}$. Since the wavelength of the SAW is large, the conductivity tensor is independent of the wave vector $\mathbf{k}$ and of the frequency ω of the SAW. Only the diagonal component of the conductivity enters Eq. (3).

We begin with the SAW of the Rayleigh type. If the interaction between the SAW and the 2DEG is small enough and acoustic coupling is absent, the strain tensor u_{kl} can be found from the solution of the elasticity problem with a free surface. The solution has the form $u_{kl}=U_{kl}(z)\exp(ikx-i\omega t)$, where $\omega=sk$ is the angular frequency of the SAW, and s is the velocity of the SAW at the free surfaces. $U_{kl}(z)$ decreases exponentially with increasing $|z|$:

$$U_{kl}(z)=\sum_q U_{kl}^{(q)}\exp(\gamma_q z)\,, \tag{4}$$

where $q=1,2$ denotes the two components of the SAW, and γ_q are the decay constants which are of the order of k.[4]

Substituting Eq. (4) into the second Eq. (1) one gets

$$\mathbf{P}=\sum_q \mathbf{P}^{(q)}\exp(-i\omega t)\,,$$
$$P_i^{(q)}=\beta_{i,kl}U_{kl}^{(q)}\exp(\gamma_q z+ikx)\,. \tag{5}$$

It is convenient to use a potential ϕ, so that $\mathbf{E}=-\nabla\phi$, and to write it in the form $\phi_i=\varphi_i(z)\exp(-i\omega t)$, where $i=1,2,3$ denotes the number of the medium (see Fig. 1). In a piezoelectric it obeys the equation

$$-\epsilon\nabla^2\phi_1+4\pi\,\mathrm{div}\mathbf{P}=0\,. \tag{6}$$

The partial solution of Eq. (6) is $\Phi\exp(-i\omega t)$, where

$$\Phi=\frac{4\pi}{\epsilon}\sum_q\frac{\mathrm{div}\mathbf{P}^{(q)}}{\gamma_q^2-k^2}\,. \tag{7}$$

The general solution is

$$\varphi_1=\Phi+A_1e^{kz+ikx}\,,$$
$$\varphi_2=(A_2^+e^{kz}+A_2^-e^{-kz})e^{ikx}\,, \tag{8}$$
$$\varphi_3=A_3e^{-kz+ikx}\,.$$

The constants A_i can be found from the boundary conditions.

The power absorbed is $\mathrm{Re}(j_xE_x^*/2)$ while the absorption coefficient $\Gamma=\mathrm{Re}(j_xE_x^*/2I)$, where I is the intensity of the SAW. One gets

$$\Gamma=\chi k\exp(-2kd)\xi/(1+\xi^2f^2)\,, \tag{9}$$

where $\chi=2\pi s|\Phi-\Psi|^2/I(\epsilon+\epsilon_s)$, $\xi=4\pi\sigma_{xx}/s(\epsilon+\epsilon_s)$,

$$\Psi=\frac{4\pi i}{\epsilon}\sum_q\frac{\mathrm{rot}_y\mathbf{P}^{(q)}}{\gamma_q^2-k^2}\,,$$
$$f=\frac{\epsilon_s\cosh(kd)+\epsilon\sinh(kd)}{\epsilon_s\exp(kd)}\,. \tag{10}$$

The functions Ψ and Φ in Eq. (9) are taken at $z=0$.

The dimensionless constant χ characterizes the interaction between the SAW and the 2DEG. Its order-of-magnitude estimate is $4\pi\beta^2/(\epsilon+\epsilon_s)\eta s^2$, where β is the relevant component of the piezoelectric tensor, and η is the density of the piezoelectric. We have assumed that $\chi\ll 1$. In the spirit of the Ingebrigtsen approach[7] it can be expressed through $s-s_m$, where s_m is the velocity of the SAW at the surface covered with a thin metallic film. In the case of the metallic surface the electric field of the SAW vanishes. Thus $s-s_m$ is equal to the contribution of the piezoelectric effect to the velocity of the SAW. One can show that this difference is equal to $\chi_0 s/2$, where

$$\chi_0=\chi(\epsilon+\epsilon_s)/(\epsilon+1)\,.$$

So we get

$$\chi=2\frac{\epsilon+1}{\epsilon+\epsilon_s}\frac{s-s_m}{s}\,. \tag{11}$$

Equations (9)-(11) are similar to those used by Wixforth, Kotthaus, and Weimann.[3]

Now we turn to the Gulayev-Bleustein waves (GBW). The interaction of this SAW with the 2DEG without magnetic field has been considered previously.[2] As follows from the above derivation, in the long-wavelength limit one should replace the conductivity σ by the component σ_{xx}, to take the magnetic field into account. For the case of the piezoelectric of the $6mm$ class with the axis L_6 parallel to $0x$, one gets (cf. Ref. 2)

$$\Gamma=\frac{2\varkappa^4k\xi[1-1/(1+\delta)(1+\xi^2)]}{(1+\delta)(1+\xi^2)}\,, \tag{12}$$

where $\delta=\epsilon/\epsilon_s$, $\varkappa^2=4\pi e_{15}^2/\epsilon C_{44}$, and e_{15} and C_{44} are the piezoelectric and elastic moduli, respectively. The decay constant γ of the GBW is of the order of $k\varkappa^2$ and is much smaller than k. It is assumed here that $\gamma d\simeq k\varkappa^2d\ll 1$.

Equations (9) and (12) are quite similar. At small conductivities ($\xi\ll 1$), $\Gamma\propto\xi$, while at high conductivities, $\Gamma\propto 1/\xi$. The physical reason is as follows. At small ξ absorption increases with the conductivity because in this case the 2DEG does not change essentially the piezoelectric field of the SAW. Thus the losses are proportional to the conductivity, i.e., to ξ. At high conductivities the piezoelectric field is screened by the 2DEG and is proportional to σ_{xx}^{-1}. Thus the Joule losses are also proportional to σ_{xx}^{-1}, i.e., to ξ^{-1}.

In the quantum-Hall-effect regime, ξ oscillates with magnetic field or with gate voltage between very small (at the plateaus) and very large values. Consequently, the absorption coefficient exhibits giant oscillations with the magnetic field. Each maximum of σ_{xx} causes two maxima of Γ, which are on the wings of the σ_{xx} peaks. This picture has been observed by Wixforth, Kotthaus, and Weimann.[3]

(2) Acoustoelectric current.—The two-dimensional current density in the presence of both the SAW and a

static electric field can be written in the form

$$j_i = \sigma_{il} E_l + e^{-1} \Lambda_{il} \dot{Q}_l \,. \tag{13}$$

Here

$$\dot{Q}_i = (k_i/k) \Gamma I/s \tag{14}$$

is the momentum transferred to the 2DEG from the SAW per second and unit area, I is the intensity of the SAW, and Λ_{ik} is the acoustoelectric tensor. We have assumed the 2DEG to be isotropic.

The expression for Λ_{yx} can be obtained from the momentum-conservation law. An electron acquiring the momentum p_x shifts in the y direction by the distance $y_0 = -cp_x/eB$. Thus

$$\dot{Q}_x = -A^{-1} \sum_m eB\dot{y}_{0m}/c \,, \tag{15}$$

where A is the area of the sample, and the sum is performed over all the electrons. The current density j_y caused by the SAW is $j_y = -eA^{-1}\sum_m \dot{y}_{0m}$. Making use of Eqs. (14) and (15) one gets

$$\Lambda_{yx} = -ec/B \,. \tag{16}$$

It is difficult to give an accurate estimate for the component Λ_{xx}. We think, however, that the ratio $\Lambda_{xx}/\Lambda_{xy}$ is of the order of the ratio σ_{xx}/σ_{xy}. Within the magnetic field intervals corresponding to the plateaus of the Hall conductivity Λ_{xx} is much smaller than Λ_{xy}.

If a sample has two contacts in the x direction, and two contacts in the y direction, one can use four possible geometries to study acoustoelectric phenomena: (i) $j_y = 0$, $E_x = 0$; (ii) $j_x = 0$, $E_y = 0$; (iii) $j_x = 0$, $j_y = 0$; (iv) $E_x = 0$, $E_y = 0$. We think that the first one is the most interesting. Supposing $j_y = 0$, $j_x = j$ we get from Eq. (9)

$$E_y = -\dot{Q}_x \Lambda_{yx}/e\sigma_{yy} \,,$$
$$j = -\dot{Q}_x (\sigma_{xy}\Lambda_{yx} - \sigma_{yy}\Lambda_{xx})/e\sigma_{yy} \,. \tag{17}$$

In the regions of the plateaus of σ_{xy} one can omit the second term in the parentheses of Eq. (17). Then

$$j = -\dot{Q}_x \Lambda_{yx} \sigma_{xy}/e\sigma_{yy} \,. \tag{18}$$

Thus

$$j/E_y = \sigma_{xy} \,. \tag{19}$$

One can see that the ratio on the left-hand side of Eq. (19) should be quantized like the Hall conductivity. Note that Eq. (19) relates the acoustoelectric current in the direction of the sound propagation to the acoustoelectric field in the perpendicular direction [geometry (i)]. One can show that no such relation exists for any of the other geometries listed above. Moreover, for all of these the acoustoelectric current and voltage tend to zero with temperature in the regions of the plateaus of σ_{xy}, but this is not so for geometry (i). Indeed, substituting Eq. (9) for the absorption coefficient into Eqs. (14) and (18) one gets

$$j = \frac{4\pi c\chi I k}{s^2(\epsilon_s + \epsilon)B} \frac{\sigma_{xy}\exp(-2kd)}{1+\xi^2 f^2} \,. \tag{20}$$

The expression for the case of the GBW has the form

$$j = \sigma_{xy} \frac{8\pi x^4 c I k}{s^2(\epsilon + \epsilon_s)B} \frac{1 - 1/(1+\delta)(1+\xi^2)}{(1+\delta)(1+\xi^2)} \,. \tag{21}$$

The product jB as a function of magnetic field B exhibits plateaus in the regions where $\xi \ll 1$, which corresponds to the plateaus of the Hall conductivity. The differences between the values of the product jB at the neighboring plateaus are equal. In the gated structures the current density j shows the equidistant plateaus as a function of the gate voltage. There are deep minima between the plateaus where the Hall conductivity has steps.

The acoustoelectric current in the plateau regions, as given by Eqs. (20) and (21), does not tend to zero as the temperature goes to zero, while σ_{xx} and Γ tend to zero in these regions. So the current is nondissipative here. If one takes into account an external resistance in the circuit, one gets expressions for the current that are different from Eqs. (20) and (21) in such a way that the Joule losses at this resistance tend to zero with σ_{xx}. The reason is that the absorption of the SAW, connected with σ_{xx}, is the only source of the energy for the acoustoelectric current. One can show that Eqs. (20) and (21) are valid until the external resistance is much smaller than $\sigma_{xx}/\sigma_{xy}^2$.

According to Eqs. (19)-(21) we get a simple expression for the acoustoelectric field E_y in the plateau regions. For the case of the Rayleigh wave it reads

$$E_y = \frac{4\pi c\chi k \exp(-2kd)}{s^2(\epsilon_s + \epsilon)B} I \,. \tag{22}$$

A similar expression is valid for the GBW. The product BE_y is the same for different plateaus. The order-of-magnitude estimate gives $E_y \cong 6$ V/cm for $\chi = 3\times 10^{-4}$ (see Ref. 3), $s = 3\times 10^5$ cm/sec, $B = 50$ kG, $\omega = 3\times 10^8$ sec^{-1}, $I = 10^{-3}$ W/cm, $kd \ll 1$. So the acoustoelectric effect is very big.

The ratio E_y/I, as obtained from Eq. (22), contains quantities which could be easily measured independently. Thus one can determine precisely the intensity I of the SAW from the measurements of the acoustoelectric field or current. Usually it is very difficult to measure I, because the conversion factor between the input electric signal intensity and the acoustic intensity is unknown and can be estimated only very roughly. On the other hand, it is important to know the intensity for many acoustic measurements made on the same structure, such as measurements of the other components of the acoustoelectric tensor Λ_{il}.

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[1]K. A. Ingebrigtsen, J. Appl. Phys. **41**, 454 (1970).

[2]M. V. Krasheninnikov and A. V. Chaplik, Zh. Eksp. Teor. Fiz. **75**, 1907 (1978); **76**, 1812 (1979) [Sov. Phys. JETP **48**, 960 (1978); **49**, 921 (1979)].

[3]A. Wixforth, J. P. Kotthaus, and G. Weimann, Phys. Rev. Lett. **56**, 2104 (1986).

[4]G. W. Farnell, in *Physical Acoustics,* edited by W. P. Mason and R. N. Thurston (Academic, New York and London, 1970), Vol. 6.

[5]J. L. Bleustein, Appl. Phys. Lett. **13**, 412 (1968).

[6]Yu. V. Gulayev, Pis'ma Zh. Eksp. Teor. Fiz. **9**, 63 (1969) [JETP Lett. **9**, 37 (1969)].

[7]K. A. Ingebrigtsen, J. Appl. Phys. **40**, 2681 (1969).