Nuclear Charge Asymmetry and Charge Dependence and the ^{3}H-^{3}He Binding-Energy Difference

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We solved a set of Faddeev equations for some realistic two-nucleon potentials, taking into account the two-pion-exchange three-nucleon interaction, the Coulomb interaction, and the charge-independence- and charge-symmetry-breaking (CIB and CSB) interactions. After 32 case studies, we find a very good linear relationship between ^{3}H and ^{3}He binding energies, from which we deduce a model-independent value for the Coulomb-energy difference (648 ± 4 keV for finite-size protons) and the CIB and CSB nuclear effects. With other small effects, these effects reasonably account for the ^{3}H-^{3}He binding-energy difference.

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After years of efforts to theoretically obtain the triton binding energy of 8.48 MeV,[1-4] we are coming to a time for studying ^{3}He with full Coulomb interactions. At the same time, we study the charge-independence-breaking (CIB) and charge-symmetry-breaking (CSB) effects that have attracted nuclear and particle physicists for over fifty years.[5-7] All previous estimates of the ^{3}He Coulomb effect were less than the ^{3}H-^{3}He mass difference of 764 keV, and this discrepancy is mainly due to the CSB effects.

We review previous ^{3}He Coulomb-energy calculations: While a model-independent formula[8,9] yields 638 keV,[9-11] recent calculations[12,13] suggest a value of about 650 keV after the proton finite-size correction is made. The calculated Coulomb energy in perturbation theory is well correlated with the calculated triton binding energy,[12] and a large Coulomb energy is obtained for a two-nucleon potential that yields a large triton binding energy.[13,14] Since we need at least 34 angular momentum states (channels) to obtain the correct triton binding energy,[1-4] previous three-channel[15,16] and fourteen-channel calculations[16] should not be enough to calculate the binding energy of ^{3}He.

A recent high-precision determination of a_{nn}, the nn $^1S_0$ scattering length, gives $a_{nn}=-18.7\pm0.6$ fm,[17,18] instead of the value $a_{nn}\sim-16$ fm.[7] A precise determination of the nuclear a_{pp} is difficult.[19] In this Letter, we utilize the long-accepted value $a_{pp}=-17.1\pm0.2$ fm.[13,18,20-22] We discuss a recently given value of $a_{pp}=-17.9\pm0.8$ fm (Refs. 23 and 24) at the end. (Although the difference of these two values of a_{pp} is small, this difference has a significant influence on the CSB effect in three-nucleon systems.) In any case, these values show that the nn potential is more attractive than the pp potential, in conformity with the ^{3}H-^{3}He mass difference, and the $\sim$1-fm difference for $\Delta a=|a_{nn}|-|a_{pp}|$ is in agreement with the ρ-ω-dominance prediction of the $^1S_0$ CSB force effect.[21,25] The study of CSB is interesting also in relation to the Nolen-Schiffer anomaly of mirror nuclei.[26,27] Study of the simplest mirror nuclei ^{3}H-^{3}He is very important to spell out this anomaly.

The purposes of this Letter are to present a model-independent value of the Coulomb energy of ^{3}He, and the CIB and CSB effects in ^{3}H-^{3}He. We solve the Coulomb-modified Faddeev equation[28] with and without CIB and CSB nuclear potentials. We let the Coulomb interaction in the modified Faddeev equations include the finite-size effect of proton and assume a dipole charge form factor of $G_E^p(q^2)=[\Lambda^2/(q^2+\Lambda^2)]^2$ with $\Lambda=840$ MeV for the finite-size proton. For ^{3}H, the Coulomb potential is switched off. Utilizing a number of calculated results obtained from various realistic potentials for various numbers of channels, we deduce a model-independent conclusion as in the case of the asymptotic normalization constants versus the triton binding energy.[29]

To obtain the Coulomb energy of ^{3}He, we solve the Coulomb-modified Faddeev equation for various realistic interactions with or without the Tucson-Melbourne[30] (TM) π-π-exchange three-nucleon force. For realistic two-nucleon (NN) interactions, we employ the Reid soft-core[31] (RSC), de Tourreil-Rouben-Sprung[32] (TRS), Paris[33] (PARIS), Argonne-14[34] (AV), and Bonn-r[35] (BONN) potentials. All of these realistic NN interactions are charge independent. They are parametrized to fit either a_{pp} or a_{np}. We take $\Lambda_\pi=700$ MeV for the cutoff mass of the πNN form factor in TM, as this value reproduces the triton binding energy.[4,36] We solve the Faddeev equation including the total isospin $T=\frac{3}{2}$ components by the method of continued fractions.[37]

The calculated binding energies of the triton versus ^{3}He are plotted on the upper line of Fig. 1. The 32 circles represent the results of 6- (including $^1S_0$ and $^3S_1$-$^3D_1$ for the interacting pair), 28- ($J\le2$), 38- ($J\le3$), and 52- ($J\le4$) channel calculations, for the above potentials. Here J denotes the total angular momentum of the interacting pair. A remarkable linear relationship between these binding energies is represented by the

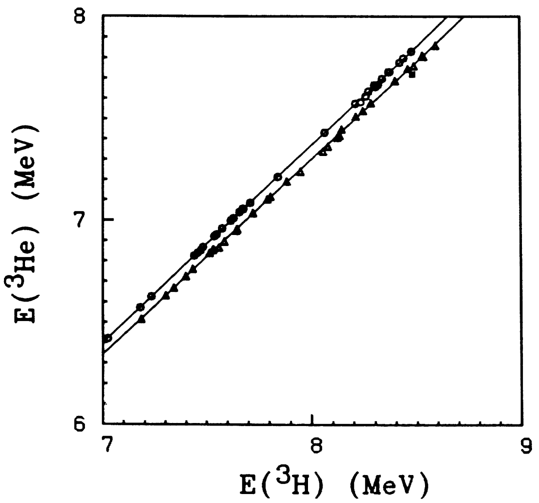

FIG. 1. The 32 circles are the binding energies of ^{3}He with Coulomb forces plotted vs the binding energies of ^{3}H for the following cases (2NP means the two-nucleon potential): RSC 6, 28; [RSC+TM] 6, 28; 2NP 6, 28, 38, 52 for 2NP=AV, PARIS, TRS, and BONN; [2NP+TM] 6, 28, 38, 52 for 2NP=AV, PARIS, and TRS. The 32 triangles are the results with CIB and CSB forces. The experimental point is shown as a square.

TABLE I. The contributions of charge-asymmetric effects to the ^{3}H and ^{3}He binding-energy difference in keV.

Charge-asymmetry effects	δE
Static Coulomb ($E_{C,\mathrm{MI}}$)	648 ± 4
Magnetic interaction	10 ± 1
Momentum-dependent electromagnetic	12 ± 1
Vacuum polarization	4
Orbit-orbit interactions	9 ± 1
Kinetic energy due to *n-p* mass difference	11
$\delta E_{\mathrm{other}}$	46 ± 3
CIB and CSB forces ($^1S_0$)	75 ± 7
CSB other than $^1S_0$	2
Uncertainty from V_{phe}	1 ± 1
δE_{CSB}	78 ± 8
Total (theory)	772 ± 15
Experiment	764

equation

$$E(^3\mathrm{He})=0.9684E(^3\mathrm{H})-0.3799\pm0.0044\ \mathrm{MeV}.$$

Putting in the experimental binding energy of $E_{\mathrm{expt}}(^3\mathrm{H})=8.482$ MeV, we obtain a model-independent value for ^{3}He, $E_{\mathrm{MI}}(^3\mathrm{He})=7.834\pm0.004$ MeV, and from the difference the model-independent Coulomb energy of ^{3}He including the finite-size effects of the proton, $E_{C,\mathrm{MI}}=0.648\pm0.004$ MeV. This value of $E_{C,\mathrm{MI}}$ agrees with the Los Alamos perturbation result $E_C=0.652$ MeV in Ref. 12. However, we see that this value of $E_{C,\mathrm{MI}}$ is less than the experimental ^{3}H-^{3}He mass difference $\Delta=0.764$ MeV. As for the finite-size effect of proton, we found by perturbation calculations that it contributes -0.033 ± 0.003 MeV to the mass difference.

Before discussing the effects of CIB and CSB of the nuclear potentials, we should consider the contribution of other effects: (1) the magnetic interactions,[6] (2) the momentum-dependent electromagnetic interaction due to the relativistic corrections,[6] (3) the vacuum-polarization potential acting on the *pp* pair,[38] (4) the orbit-orbit interactions,[39] and (5) the kinetic energy due to the *n-p* mass difference.[40] We use an approximate scaling relation for the magnetic-moment form factor $G_M^p/\mu_p=G_M^n/|\mu_n|=G_E^p$. We estimated these effects to the mass difference by averaging first-order perturbation values obtained from the wave functions generated from TRS-52(TM700), PARIS-52(TM700), and AV-52(TM700). The results are shown in Table I. The sum of these small effects is $\delta E_{\mathrm{other}}=0.046\pm0.003$ MeV. Now, our task is to account for the remainder; $\Delta-E_{C,\mathrm{MI}}-\delta E_{\mathrm{other}}=0.070\pm0.007$ MeV.

So far, we have overestimated the *NN* force effect in the calculation of $E(^3\mathrm{He})$ by using the charge-independent V_{NN}, where $V_{pp}=V_{nn}$, in spite of the fact that $\Delta a>0$, and accordingly, V_{pp} is less attractive than V_{nn}. To correct it, we introduce the CSB and CIB nuclear forces to reproduce the experimental values of a_{nn}, a_{pp}, and a_{np} for the $^1S_0$ two-body interactions. We shall use these forces in the three-nucleon calculations since this $^1S_0$ state comprises 90% of the two-body $\tau=1$ component of trinucleon systems.

The CIB forces are caused by several reasons. The mass differences of charged and neutral mesons are responsible to the long-range CIB forces.[21,22,41] The CIB potentials due to the pion mass difference is given by Eq. (17) in Ref. 22, and that arising from the vector-meson mass difference is given by Eq. (18) in Ref. 22. To represent all other complicated effects of the CIB interactions, we added a small phenomenological short-range Woods-Saxon potential acting on the $^1S_0$ state,

$$V_{\mathrm{phe}}(r)=V_0/\{1+\exp[(r-R)/a]\},$$

where $R=0.5$ fm, $a=0.2$ fm, and $V_0=6.5$ MeV. This phenomenological potential is introduced to shift the values of $|a_{np}|$ and $|a_{nn}|$ about 2–3 fm.

The best studied origins of the CSB effects are the ρ^0-ω mixing, π^0-η mixing, and the *n-p* mass difference in the one-pion-exchange potential.[7,21,42,43] The CSB potential due to the ρ^0-ω mixing is given by Eq. (8) in Ref. 21, in which we take β as a parameter and adjust for each potential so that the experimental $\Delta a=1.5$ fm is reproduced. The CSB potentials from the π^0-η mixing and the n-p mass difference are given by Eq. (25) of Ref. 42 and Eq. (6) of Ref. 43, respectively. The various boson-exchange parameters are taken from Refs. 21 and 42. With these CIB and CSB potentials, the realistic potentials yield the ${}^1S_0$ scattering lengths (in units of fm) $a_{np}=-23.63$ (expt, -23.74), $a_{nn}=-19.13$, and $a_{pp}=-17.63$ with $\beta=1.21$ for the PARIS potential for which the original charge-independent a_{NN} is -17.63 fm. The value of Δa is crucial for CSB rather than a_{nn} and a_{pp} themselves.

We briefly review recent calculations for the CIB in the triton: Friar, Gibson, and Payne[44] calculated it to first order in the difference $V_{nn}-V_{np}^s$ by replacing the ${}^1S_0$ interaction by the potential $\frac{2}{3}V_{nn}+\frac{1}{3}V_{np}^s$. The effect of $T=\frac{3}{2}$ components is estimated to have a small probability of $\sim10^{-5}$. Brandenburg *et al.*[13] used the averaged t matrix $\frac{1}{3}t^{\tau=1}(n,p)+\frac{2}{3}t^{\tau=1}(n,n)$ for ^{3}H and $\frac{1}{3}t^{\tau=1}(n,p)+\frac{2}{3}t^{\tau=1}(p,p)$ for ^{3}He. The Coulomb contribution in ^{3}He was estimated in first-order perturbation theory, using the five-channel ^{3}He wave function without including the Coulomb effect. The total isospin $T=\frac{3}{2}$ component was neglected.

Contrary to these calculations, we performed 52-channel calculations including $T=\frac{3}{2}$ components with the CIB forces in the ${}^1S_0$ state, in addition to the forces already included in the calculations to see the Coulomb effect. As a result, we see that the CIB force makes an increase (decrease) in the binding energy of ^{3}H and ^{3}He of about 0.1 MeV (0.2 MeV) for RSC, TRS, and PARIS (AV and BONN), and furthermore there is about 0.015 MeV from the TM force effect in the same direction as for each NN force. In any case, the CIB force affects the binding energies of both ^{3}H and ^{3}He in the same direction, and results in only a few-keV increase of the mass difference δE of ^{3}H-^{3}He. $V_{\rm phe}$ affects the mass difference by 1 ± 1 keV: e.g., for TRS+TM, $\delta E=0.728$ (0.729) MeV for $V_0=0$ (6.5) MeV.

The contributions to the increasing mass from various CIB forces are estimated by first-order perturbation calculations, using the 52-channel triton wave function for the AV potential [AV-52(TM700)]. As seen in Table II, the long-range CIB force caused by the pion mass difference is most important, contributing about 60% of the whole CIB effect.

We solved the 52-channel Coulomb-modified Faddeev equation further by adding the CSB forces. To get a model-independent conclusion on these CIB and CSB force effects, we plotted the calculated values on the lower line in Fig. 1. Again a very good linear relationship,

$$E({}^3\mathrm{He})=0.9582E({}^3\mathrm{H})-0.3687\pm0.0073\ \mathrm{MeV},$$

is obtained. If we put the experimental energy of the triton in $E({}^3$H) of this equation, we obtain the model-independent binding energy of ^{3}He, $E_{\rm MI}^{\rm CSB}({}^3{\rm He})=7.759\pm0.007$ MeV, which includes the CIB and CSB effects in addition to the Coulomb contribution. Thus, the effects of the CIB and CSB forces from the ${}^1S_0$ state is given by the difference of $E_{\rm MI}({}^3{\rm He})$ and $E_{\rm MI}^{\rm CSB}({}^3{\rm He})$. It is $\delta E_{\rm CSB}({}^1S_0)=0.075\pm0.007$ MeV. The sum of $\delta E_{\rm CSB}({}^1S_0)$, the Coulomb energy $E_{C,\rm MI}$, and $\delta E_{\rm other}$ amounts to 0.769 ± 0.014 MeV, which is very close to the experimental ^{3}H-^{3}He mass difference. Also, first-order perturbation estimates for various contributions are shown in Table II, where we see that the ρ-ω mixing gives the largest contribution as expected by various authors.[21,25] This table lists only the contributions from the ${}^1S_0$ state. For CIB, the contributions to δM from other states are, in keV, ${}^3P_0$ (27), ${}^3P_1$ (-16), ${}^3P_2$-${}^3F_2$ (29), and ${}^1D_2$ (1), totaling 41 keV. However, as we have mentioned, CIB effects have almost no contribution to δE due to the same interactions of nn and pp pairs. For CSB, the contributions to δE from other states are, in keV, ${}^3P_0$ (0.58), ${}^3P_1$ (0.5), ${}^3P_2$ (0.5), ${}^1D_2$ (0.47), ${}^3F_2$ (0.1), ${}^3F_3$ (0.02), and ${}^3H_4$ (0.04), totaling 2.21 keV.

Collecting all results in Table I, we see that the sum of these effects almost accounts for the experimental ^{3}H-^{3}He binding-energy difference. The sum of various less important CSB effects that we have not taken into account, such as those due to the meson-photon exchange,[7,41] electromagnetic Δ mass splitting,[7] and quark effects as well as heavy-meson exchanges,[41] is expected to modify our results only marginally. [However, we remark that if we accept the value $a_{pp}=-17.9$ fm,[23,24] our calculation yields $E_{\rm MI}^{\rm CSB}({}^3{\rm He})=7.788\pm0.006$ MeV, 29 keV larger than in the case of $a_{pp}=-17.1$ fm.]

The contributions from the class-IV CSB forces[45] should be negligible since this force acts only between the singlet and triplet states of the np pair: For $J=1$, ρ-ω mixing (-0.0658) and n-p mass difference (-0.0413) for ^{3}H, in keV, total -0.107 (-0.114 for ^{3}He). The totals are 0.192 (0.187) for $J=2$, 0.0168 (0.0163) for $J=3$, and 0.015 (0.0145) for $J=4$ for ^{3}H (^{3}He). The differences for ^{3}H and ^{3}He are due to the difference of the wave functions.

Finally, the percentage of the total isospin $T=\frac{3}{2}$ components is calculated taking AV-52(TM700) as an example. We obtain 10^{-3} for ^{3}He, if only the Coulomb force is included. We have 4.0×10^{-4} and 4.0×10^{-3} for

TABLE II. The perturbation calculation of CIB and CSB force effects from the ${}^1S_0$ state on the binding energy of ^{3}H for AV-52(TM700) in keV. δM (δE) denotes the increase in mass (in binding-energy difference).

δM(CIB)		δE(CSB)	
π mass difference	126	ρ^0-ω mixing	58
ρ mass difference	23	π^0-η mixing	11
Phenomenon	73	np mass difference	3
Total	222	Total	72

^{3}H and ^{3}He, respectively, if the CIB and CSB forces are included. The matrix elements of the Coulomb force $\langle T|V_C|T'\rangle$ in MeV are 0.663 ($T=T'=\frac{1}{2}$), 8.7×10^{-4} ($T=\frac{1}{2}$, $T'=\frac{3}{2}$), and 2.7×10^{-5} ($T=T'=\frac{3}{2}$). These values have already been included in our Faddeev solutions.

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