Transverse Interactions and Transport in Relativistic Quark-Gluon and Electromagnetic Plasmas

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In calculating the transport properties of relativistic electromagnetic or quark-gluon plasmas microscopically, one encounters the problem of divergent constituent cross sections arising from the exchange of transverse photons or gluons. This paper shows how inclusion of Landau damping of the virtual quanta exchanged provides an effective long-wavelength cutoff. The dominance of Landau damping over a possible magnetic mass in QCD is demonstrated in the weak-coupling limit. The viscosities of a pure gluon plasma and of a quark-gluon plasma are calculated in weak coupling from a variational solution to the Boltzmann equation.

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Calculation of the properties of relativistic plasmas, whether electromagnetic or quark-gluon, requires that one include the long-range transverse, or "magnetic," interactions of the constituents of the plasma. The scattering of two particles via either longitudinal or transverse photon exchange, or gluon exchange in a quark-gluon plasma, has a small-angle divergence, as in Rutherford scattering. Debye screening eliminates the divergence from longitudinal exchange, but the transverse electromagnetic or color-magnetic interactions are not simply screened. The latter are generally neglected in nonrelativistic plasmas because particle velocities are much less than the speed of light. However, if one naively includes them, for example, in calculating transport processes in the early Universe prior to the h[illegible]nization transition, or in neutron stars or white dwar[illegible]ne finds vanishing transport coefficients. A similar problem is encountered in calculating the approach to local equilibrium in the deconfined matter expected in ultrarelativistic heavy-ion collisions. (In addition, the long-range magnetic interactions cause a breakdown of the Landau-Fermi liquid theory in degenerate systems—even nonrelativistic electrons in a metal, albeit at exponentially small temperatures.) The question is, what physics leads to finite answers for observable quantities?

The physics of screening, for both electromagnetic and QCD interactions, is qualitatively different for the longitudinal and transverse parts of the interaction. For the longitudinal, or "electric" part, the interaction is cut off, in both cases, at a distance $\sim q_D^{-1}$, where q_D is the Debye wave number. In a quark-gluon plasma at zero baryon density,[1-3] $q_D^2=(N_f+2N_c)g^2T^2/6$, where N_f is the number of quark flavors, N_c is the number of colors, g is the QCD coupling constant, and T is the temperature. Static magnetic fields in a normal QED plasma, e.g., a metal, are not screened out at large distances; magnetic fields are screened only in a superconductor. On the other hand, as a result of its nonlinear structure QCD may possibly develop a spontaneous magnetic mass or screening wave number[4,5] $q_m \sim g^2T$. However, as we demonstrate in this paper, in both QED and QCD magnetic interactions at finite frequencies, and hence at finite temperatures, are screened at large distances $\sim q_D^{-1}$ due to the Landau damping of the virtual photons, or gluons, whose exchange produces the transverse interactions.[6] This effect in thermal quark-gluon plasmas overwhelms that of a magnetic mass for small g.

We illustrate how the Rutherford-type divergences of magnetic interactions are treated by calculating the viscosity of a quark-gluon plasma in the weak-coupling limit. Extensions to other physical situations will be discussed in separate publications. The viscosity is a sum of contributions from quark and gluon degrees of freedom. We first consider the viscosity of a pure gluon gas. The physics is most simply described by the Boltzmann equation for gluons, studied in the rest frame of the plasma.[7-9] In the presence of a small time-independent velocity field in the x direction, $\mathbf{u}(y)$, varying with y, the Boltzmann equation is

$$p_{1x}v_{1y}\frac{\partial n_1}{\partial \epsilon_1}\frac{\partial u_x}{\partial y}=2\pi\nu_g\sum_{\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4}|M|^2[n_1n_2(1+n_3)(1+n_4)-(1+n_1)(1+n_2)n_3n_4]\delta(\epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4)\delta_{\mathbf{p}_1+\mathbf{p}_2,\mathbf{p}_3+\mathbf{p}_4}, \quad (1)$$

where ϵ_i, $\mathbf{p}_i$, $\mathbf{v}_i$, and n_i are the gluon energy, momentum, velocity, and occupation number, and the factor $\nu_g=16$ represents the sum over target gluon spin and color. We work in units in which $\hbar=c=1$. The squared matrix element $|M|^2$ for scattering gluons from states one and two to states three and four is given by $|\mathcal{M}|^2/16\epsilon_1\epsilon_2\epsilon_3\epsilon_4$, where $|\mathcal{M}|^2$ is the invariant gluon-gluon scattering matrix element squared, summed over spins and colors in the final state, and averaged over initial spins and colors. To lowest order in g the bare interaction gives[10]

$$|\mathcal{M}|^2=\frac{9}{2}g^4\left(3-\frac{ut}{s^2}-\frac{us}{t^2}-\frac{ts}{u^2}\right), \quad (2)$$

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where s, t, and u are the usual Mandelstam variables. For small t, $|\mathcal{M}|^2$ diverges as $\frac{9}{2}g^4s^2/t^2$, since for massless particles $s+t+u=0$; therefore for zero energy transfer it has the Rutherford-type divergence $1/q^4$, where $\mathbf{q}$ is the momentum transfer.

We now include the effects of the plasma. We write the spatial momenta of the gluons as $\mathbf{p}_1=\mathbf{p}+\mathbf{q}/2$, $\mathbf{p}_2=\mathbf{p}'-\mathbf{q}/2$, $\mathbf{p}_3=\mathbf{p}-\mathbf{q}/2$, $\mathbf{p}_4=\mathbf{p}'+\mathbf{q}/2$. For small $\mathbf{q}$, the energy transfer ω is $\simeq\mathbf{v}\cdot\mathbf{q}\simeq\mathbf{v}'\cdot\mathbf{q}$, where $\mathbf{v}$ and $\mathbf{v}'$ are the velocities corresponding to $\mathbf{p}$ and $\mathbf{p}'$. Since the divergence occurs at small momentum and energy transfers, we shall focus only on the small-q contributions to the scattering. The most singular part of the bare matrix element (2) for small q and ω is the $t^{-2}=(\omega^2-q^2)^{-2}$ term; with the inclusion of screening effects in the random-phase approximation (one-loop order),[11] the singular part becomes

$$|M|^2=\frac{9}{2}g^4\left|\frac{1}{q^2q_D^2\chi_l(\mu)}-\frac{(1-\mu^2)\cos\phi}{q^2(1-\mu^2)+q_D^2\chi_t(\mu)}\right|^2. \quad (3)$$

Here $\mu=\omega/q$, and χ_l and χ_t, given by

$$\chi_l(\mu)=1-\frac{\mu}{2}\ln\left|\frac{\mu+1}{\mu-1}\right|,$$
$$\chi_t(\mu)=\frac{\mu^2}{2}+\frac{\mu(1-\mu^2)}{4}\ln\left|\frac{\mu+1}{\mu-1}\right|, \quad (4)$$

represent the effect on the longitudinal and transverse gluons of interactions with the plasma. (Inclusion of massless quarks only modifies q_D.) The angle ϕ, shown in Fig. 1, is that between the plane containing $\mathbf{p}_1$ and $\mathbf{p}_3$ and that containing $\mathbf{p}_2$ and $\mathbf{p}_4$.

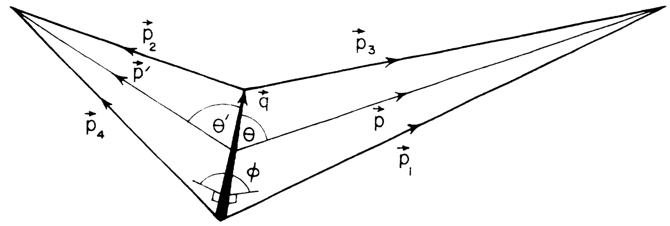

FIG. 1. The vectors involved in the scattering.

For $\omega\ll q$, χ_l is close to 1; the denominator in the electric part of the interaction behaves as $q^2+q_D^2$, which shows that the interaction is screened out at distances $\sim q_D^{-1}$. On the other hand, $\chi_t\simeq -i\pi\mu/4$ and the denominator in the magnetic part of the interaction is approximately $q^2-i\pi q_D^2\mu/4$; consequently, magnetic disturbances are screened at an ω-dependent length scale $l\sim(q_D^2\omega)^{-1/3}$. The leading finite-frequency contributions to χ_t are imaginary because they are due to Landau damping—the absorption of a gluon accompanied by scattering of a gluon, quark, or antiquark. (For $\omega\ll q$, decay of gluons into $q\bar{q}$ pairs is forbidden kinematically.) In the case considered here the gluons are virtual while Landau[12] considered the decay of on-energy-shell plasma oscillations. This screening at finite frequencies is the analog of the anomalous skin effect in pure normal metals; its importance in QCD plasmas was emphasized by Weldon.[2]

To calculate the viscosity we first linearize the Boltzmann equation, writing the distribution function as

$$n_i=n_i^{\rm LE}+(\partial n_i^0/\partial\epsilon_i)\Phi_i(\partial u_x/\partial y),$$

where $n_i^{\rm LE}=(e^{(\epsilon_i-\mathbf{p}_i\cdot\mathbf{u})/T}-1)^{-1}$ is the local equilibrium distribution function; one finds

$$p_{1x}v_{1y}=2\pi v_g\sum_{\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4}|M|^2\frac{n_2(1+n_3)(1+n_4)}{(1+n_1)}\delta(\epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4)\delta_{\mathbf{p}_1+\mathbf{p}_2,\mathbf{p}_3+\mathbf{p}_4}(\Phi_1+\Phi_2-\Phi_3-\Phi_4), \quad (5)$$

which we may write in the compact form $|X\rangle=I|\Phi\rangle$. The distribution functions n on the right-hand side of Eq. (5) are now equilibrium ones, with $\mathbf{u}=0$. The viscosity η is given in terms of Φ by[13] $\eta=-v_g\sum_{\mathbf{p}}p_xv_y(\partial n/\partial\epsilon)\times\Phi_{\mathbf{p}}\equiv\langle X|\Phi\rangle$. This summation defines the scalar product operation. By rotational symmetry $\Phi_{\mathbf{p}}$ must be of the form $f(p)\hat{p}_x\hat{p}_y$.

We calculate the viscosity from the equivalent expression $\eta=\langle X|\Phi\rangle^2/\langle\Phi|I|\Phi\rangle$; since for an arbitrary "trial function" Ψ the quantity $\langle X|\Psi\rangle^2/\langle\Psi|I|\Psi\rangle$ gives a lower bound on η, this latter expression lends itself to a variational treatment. Evaluation of the multidimensional integrals in $\langle\Phi|I|\Phi\rangle$ is simplified considerably since for weak coupling only small momentum transfers $\mathbf{q}$ are important. Integrals over the magnitudes p and p' decouple from those over q and the relative angles between $\mathbf{p}$, $\mathbf{p}'$, and $\mathbf{q}$. The integral over q and $\mu=\hat{\mathbf{q}}\cdot\hat{\mathbf{p}}=\hat{\mathbf{q}}\cdot\hat{\mathbf{p}}'$ has the form $\int d\mu\, h(\mu)\int dq\, q^3|M|^2$, where $h(\mu)$ is a sum of a purely longitudinal contribution, a purely transverse contribution, and an interference term between longitudinal and transverse. The longitudinal integral over q is cut off by the Debye screening wave vector, and to logarithmic accuracy in g^2, the leading order, gives $\ln(q_{\max}/q_D)$, where $q_{\max}\sim T$ is the maximum momentum transfer. The contribution from transverse exchange is

$$(\cos\phi)^2\int d\mu\, h(\mu)\int dq\, q^3/|q^2+q_t^2(\mu)|^2,$$

where $q_t^2(\mu)=q_D^2\chi_t(\mu)/(1-\mu^2)$; the integral over q, to logarithmic accuracy, gives

$$\ln|q_{\max}/q_t(\mu)|=\ln(q_{\max}/q_D)-\ln|q_D/q_t(\mu)|.$$

On integrating over μ, the second term here gives no logarithm. Similarly, the logarithmic contribution from the cross term is $-2\cos\phi\ln(q_{\max}/q_D)$, so that the net result of the q integration is $(1-\cos\phi)^2\ln(q_{\max}/q_D)$. Therefore to logarithmic accuracy one finds the same result as if q_t^2 were simply replaced by q_D^2, even though as $\mu\to 0$ the μ-dependent cutoff in the transverse amplitude tends to zero. This simplification reflects the fact that processes in which μ is small do not play a dominant role. Indeed, one can calculate the leading terms in transport coefficients by using the bare interaction cutoff at q_D, without explicit decomposition into longitudinal and transverse parts, even though the underlying physics is very different in the two cases.

Were a magnetic mass term q_m^2 (assumed q and ω independent) present in the denominator of the transverse scattering amplitude in Eq. (3), the $\ln|q_D/q_t(\mu)|$ term above would be altered to $\frac{1}{2}\ln|(\chi_t+q_m^2/q_D^2)/(1-\mu^2)|$. The effect of a magnetic mass is therefore only of the order $(q_m/q_D)^2\sim g^2$ in the added constant term; it does not contribute to the logarithm.

Gluon-gluon scattering is symmetrical under interchange of the t and u channels, and therefore small-u processes also contribute to the logarithmic term. However, in summing over the momenta $\mathbf{p}_3$ and $\mathbf{p}_4$, one must insert a factor $\frac{1}{2}$ to avoid overcounting identical final states; thus for gluon-gluon scattering one can equivalently neglect both the u-channel singularity and the factor $\frac{1}{2}$. For small s the apparent $\sim s^{-2}$ singularity does not contribute to the logarithm since for small s the gluons are almost collinear, and therefore t and u are also small.

Calculating all integrals to leading logarithmic order without approximation, we find that

$$\langle\Phi|I|\Phi\rangle=\frac{2}{\pi^3}g^4T^3\ln\left(\frac{T}{q_D}\right)\times\int_0^\infty\left(f^2+\frac{f_1^2}{10}+\frac{2}{5}ff_1\right)\left(\frac{\partial n}{\partial\epsilon}\right)dp\,, \quad (6)$$

where $f_1(p)=p^3d[f(p)/p^2]/dp$. The absence of a double integral $\sim\int dp\,dp'f(p)f(p')$ is due to an accidental cancellation dependent entirely on the form of the interaction.

From the variational principle one can derive a Fokker-Planck equation for f, and thus obtain its functional form, but here for simplicity we make variational estimates of the viscosity, assuming $f\sim p^\nu$. The resulting viscosity of the pure gluon gas, η_g^0, for $\nu=2$, is

$$\eta_g^0=\frac{2^9\times 15\times\zeta(5)^2}{\pi^5}\frac{T^3}{g^4\ln(T/q_D)}\simeq 0.342\frac{T^3}{\alpha^2\ln\alpha}\,, \quad (7)$$

where $\alpha=g^2/4\pi$. Equation (7) differs from that for the optimal ν ($\simeq 2.104$) by 0.37%. In the simplest variational estimate one usually takes a trial function $|\Psi\rangle=|X\rangle$, corresponding to $\nu=1$, which gives a viscosity 0.492 times that with $\nu=2$. The result obtained by Hosoya and Kajantie[7] using a relaxation-time approximation is 0.36 times Eq. (7). We may define a viscous relaxation time by writing the viscosity as $\eta=\langle X|X\rangle\tau_\eta$, a form motivated by the relaxation-time approximation, $|X\rangle=|\Phi\rangle/\tau_\eta$; then the variational estimate for the relaxation rate is

$$\frac{1}{\tau_{\eta,g}}=\frac{1}{\eta_g^0}\langle X|X\rangle=\frac{\pi^7}{2^4\times 3^3\times 5^3\times\zeta(5)^2}Tg^4\ln(g^{-1})\simeq 4.11T\alpha^2\ln(\alpha^{-1})\,, \quad (8)$$

where the numerical evaluations are for $\nu=2$.

We now include the contributions to the viscosity of quarks and antiquarks, which transport momentum as well as scatter quarks and gluons. The calculations are similar to those for gluons alone, except that the vectors $|X\rangle$ and $|\Phi\rangle$ must be labeled by particle type. For simplicity we take zero baryon number, so that the quark and antiquark distribution functions are identical. The independent deviation functions are $f_q(p)$ for quarks and antiquarks, and $f_g(p)$ for gluons. The general expressions for qq and $\bar{q}\bar{q}$ scattering depend on whether or not the flavors are identical. The situation we consider, however, possesses a simplifying feature: The singular contributions to the interaction are the same for all pairs of quarks and antiquarks since, as with gluons, the factor 2 from the separate singularities in t and u for q (or $\bar{q}$) with identical flavors is canceled by the extra $\frac{1}{2}$ in the final states. Hence all q's and $\bar{q}$'s are taken into account by considering quarks of a single flavor, but with an additional degeneracy factor $2N_f$. The absence of cross terms in the momentum integrals that we found in (6) is a property of the angular parts of the integrals, and is independent of the Bose or Fermi nature of the scattered particles. Consequently, the Boltzmann equations for $f_g(p)$ and $f_q(p)$ decouple, and the total viscosity may be written as $\eta=\eta_q+\eta_g$, a sum of quark and gluon contributions. For simplicity we assume that the deviation functions are both proportional to p^ν, so that they differ by only a multiplicative constant. For $\nu=2$ and N_f quark flavors one finds

$$\eta_g=\frac{\eta_g^0}{1+N_f/6},\quad \eta_q=\frac{3^5\times 5^2}{2^9\times 7}N_f\eta_g\simeq 1.70N_f\eta_g\,. \quad (9)$$

The ratio η_q/η_g includes a factor $(\frac{15}{16})^2/\frac{7}{8}$ from the differences between Fermi and Bose integrals, a factor $3N_f/4$ which is the ratio of the number of q plus $\bar{q}$ helicity states, $12N_f$, to that of the gluons, $\nu_g=16$, and a factor $\frac{4}{9}$ in the denominator, which is the ratio of $|M|^2$ for qg and gg scattering. In fact, the $|M|^2$ for qq, qg, and gg scattering are all proportional, with factors $\frac{8}{9}:2:\frac{9}{2}$, respectively, a geometric progression resulting from the factorization of the single Feynman diagram that contributes in the weak-coupling limit. We see that scattering from quarks increases scattering rates by a factor $1+N_f/6$ for both quarks and gluons; the first term represents scattering by gluons, while $N_f/6$ (given by the ratio of helicity states times $\frac{4}{9}$ in the matrix elements, times $\frac{1}{2}$ for a Fermi rather than Bose integral) arises from scattering by quarks. This ratio is independent of the scattered particle because of the dominance of small-momentum-transfer processes, together with the above factorization. We also see that quarks contribute significantly more to the viscosity than do gluons, a result in accord with Ref. 7.

The total viscosity is thus given, for $N_f=2$, by

$$\eta \simeq 232 \left(\frac{T}{200\ \mathrm{MeV}} \right)^3 \frac{1}{\alpha^2 \ln(1/\alpha)}\ \mathrm{MeV\,fm^{-2}}c^{-1}. \quad (10)$$

The viscosity of the plasma is generally larger, for $\alpha \lesssim 1$, than that of hot-nuclear matter, estimated to be[14] $\eta \sim 82[T/(200\ \mathrm{MeV})]^{1/2}\ \mathrm{MeV\,fm^{-2}}c^{-1}$. The viscous relaxation time in weak coupling for quarks is a factor $(\frac{45}{28})^2 \simeq 2.58$ times that for gluons; the gluon viscous relaxation time, from Eqs. (8) and (9), is at least of order 1 fm/c. While these results provide a first estimate of the time needed to equilibrate a quark-gluon plasma produced in an ultrarelativistic heavy-ion collision, it should be kept in mind that they are derived in the weak-coupling limit; relaxation times in the strongly interacting nonperturbative regime could be considerably shorter.

The crucial result of this paper is that Landau damping of transverse gluons or photons provides the physical basis for treating the long-wavelength divergence associated with transverse exchange processes. We note that Landau damping plays an even more important role in degenerate systems than in thermal plasmas. For the electron gas, Holstein, Norton, and Pincus showed that as a consequence of transverse photon exchange the specific heat at low temperatures behaves as $T\ln T$ rather than linearly as in a normal Fermi liquid.[15] Also, in the absence of screening, integrals over q in the evaluation of transport coefficients diverge at small q either linearly (in the case of viscosity) or cubically (in the case of thermal conductivity),[16] rather than logarithmically as we found above. As a consequence of the singular nature of the interaction for small momentum transfers, the angular and energy integrals in the calculation of collision rates cannot be decoupled in the standard manner,[13] and the transport coefficients have a different temperature dependence than in the usual Landau theory.

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