Effect of the Electromagnetic Environment on the Coulomb Blockade in Ultrasmall Tunnel Junctions

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The current-voltage characteristic of an ultrasmall tunnel junction is calculated for arbitrary frequency dependence of the impedance presented to the junction by its electromagnetic environment. It is shown that the Coulomb blockade of tunneling is washed out by quantum fluctuations of the charge on the junction capacitor except for ultrahigh-impedance environments. Two simple cases where the environment can be treated as an inductor or resistor are examined in detail. Effects of finite temperatures are discussed.

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New effects have been predicted[1] to arise in ultrasmall tunnel junctions with capacitance C such that the charging energy of a single electron $e^2/2C$ exceeds the characteristic energy k_BT of thermal fluctuations. Simple energy considerations suggest that the tunneling of a single electron is completely blocked when the junction capacitor holds a charge less than $e/2$. A large body of experimental data now exists[2-4] that seems to support the theoretical ideas underlying this Coulomb blockade of tunneling. The effect is clearly seen in multijunction configurations[2] while the existence of elementary charging effects in single tunnel junctions[3,4] is still questionable.

An obvious objection against existing theories of the Coulomb blockade in single junctions comes from considering the effective junction capacitance. Should it not include contributions from the on-chip electromagnetic environment of the junction like the leads and pads? They might easily enlarge the capacitance to values where elementary charging effects become unobservably small. One point of view[5] is that the tunneling electron probes the electromagnetic environment at distances $r < c\tau_t$, where τ_t is the traversal time of the electron passing through the potential barrier and c is the speed of charge propagation in the electrical circuit surrounding the junction. On the other hand, recent experimental work[4] suggests that the electron probes distances $r < \hbar c/\Delta E$, where $\Delta E = \max(eV, k_BT)$, V being the dc voltage across the junction.[6]

In this Letter we show that the quantum-mechanical nature of the electromagnetic environment can severely reduce Coulomb charging effects in single junctions. For a tunneling electron to change effectively the charge on the junction capacitor and thus lead to the Coulomb blockade effect it has to excite electromagnetic modes of the coupled system formed by the junction and its electromagnetic environment. Since the energy $\hbar\omega$ of these modes is quantized, they will not be excited unless the voltage V across the junction reaches $\hbar\omega/e$. The larger the impedance of the environment the stronger the junction couples to low-frequency modes. Thus, charging effects will usually only be observable when the junction is placed in a very-high-impedance environment or for large voltages. The situation here is reminiscent of the well-known Mössbauer effect[7] where naive reasoning suggests that γ quanta are emitted with a shifted frequency due to the recoil of the nucleus. An analogy can be drawn between a change of the momentum of the nucleus and a change of the local charge on the capacitor of a tunnel junction so that the occurrence of a frequency shift corresponds to the Coulomb blockade. The Mössbauer effect arises because the probability to excite crystal modes coupled to the nucleus is small when the average recoil energy does not match the energy of the dominant crystal modes. From another point of view, the change of the momentum of the nucleus due to recoil is small compared with its spontaneous zero-point fluctuations. Hence, recoilless transitions are favored. Likewise, we will find that the Coulomb charging effect in single junctions is washed out by quantum fluctuations of the electric charge in practical cases where the impedance of the environment is not well above the resistance quantum $R_Q = h/2e^2$.

Our treatment of electron tunneling in a normal junction imbedded in an electrical circuit is based on the assumption that the tunneling Hamiltonian takes the form

$$H_T = \sum_{\sigma k q} T_{kq} c_{k\sigma}^\dagger c_{q\sigma} \Lambda_e + \text{H.c.}\,,$$

where $T_{kq} c_{k\sigma}^\dagger c_{q\sigma}$ is the usual tunneling term[8] which transfers a quasielectron from one side of the junction to the other and where Λ_e is an operator changing the charge Q on the capacitor plates of the junction: $\Lambda_e Q \Lambda_e^\dagger = Q - e$. Here, Q is assumed to be an operator with a continuous spectrum. In contrast with the conventional treatment of tunneling, our scheme takes into account—albeit in a minimal fashion—the rearrangement of the electric charge density on the junction during a tunneling

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event. Introducing the phase[9,10] $\phi(t)=(e/\hbar)\int_{-\infty}^{t}V(t')\,dt'$ as the integral over the voltage across the junction, Λ_e can be expressed as $\Lambda_e=e^{i\phi}$ by making use of the commutation relation $[Q,\phi]=ie$. We further assume that Q and ϕ commute with the quasielectron creation and annihilation operators.

As a simple model for the electromagnetic environment we may consider the circuit depicted in Fig. 1(a) where the leads attached to the junction are represented through a series inductance L and a shunt capacitance C_s. (For the case of a general circuit, see below.) The practical observations of Coulomb charging effects requires junction capacitances less than a few femtofarads and the leads attached to such a junction will easily produce shunt capacitances that are several orders of magnitude larger, i.e., $C_s \gg C$. The change of the charge Q_s on the shunt capacitor caused by a tunneling event is therefore entirely negligible and the current-biased junction depicted in Fig. 1(a) can thus effectively be replaced by the voltage-biased junction shown in Fig. 1(b).[11] For this latter model, the total Hamiltonian includes the usual kinetic and chemical-potential terms for quasielectrons, the tunneling Hamiltonian H_T, and the Hamiltonian of the electromagnetic environment,

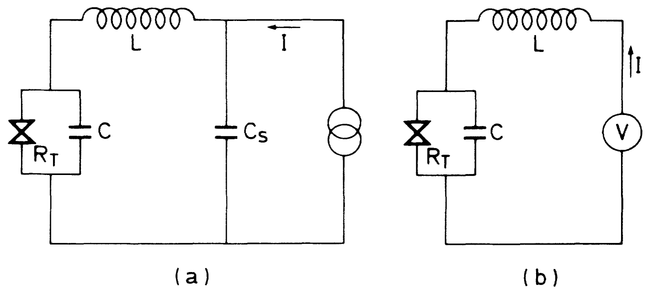

FIG. 1. (a) A current-biased tunnel junction coupled to an external circuit with impedance of leads modeled by a series inductor L and a shunt capacitor C_s. (b) An equivalent voltage-biased junction in the limit $C_s \gg C$.

$$H_{\rm em}=Q^2/2C+(\hbar^2/2e^2L)\phi^2-QV\,,$$

which describes the Coulomb charging energy on the capacitor and the magnetic energy of the self-inductance of the leads. Then, assuming a constant tunneling matrix element T_{kq}, one finds for the tunneling current, along the standard line of reasoning,[12]

$$I=\frac{1}{eR_T}\int_{-\infty}^{+\infty}dE\int_{-\infty}^{+\infty}dE'\{f(E)[1-f(E')]P(E+eV-E')-[1-f(E)]f(E')P(E'-E-eV)\}\,, \tag{1}$$

where $1/R_T$ is the usual tunneling conductance which is proportional to $|T|^2$ and $f(E)=[1+\exp(\beta E)]^{-1}$ is the Fermi function with $\beta=1/k_BT$. The novel feature here is the appearance of the function

$$P(E)=(2\pi\hbar)^{-1}\int_{-\infty}^{+\infty}dt\exp[J(t)+iEt/\hbar]\,, \tag{2}$$

in which

$$J(t)=\langle[\phi(t)-\phi(0)]\phi(0)\rangle$$

is the equilibrium phase correlation function. $P(E)$ gives the probability that a tunneling electron creates an excitation with energy E of the electromagnetic environment described by $H_{\rm em}$. In the conventional treatment where the coupling to the environment is disregarded one has $P(E)=\delta(E)$ and Eq. (1) reduces to the well-known Ohmic law $I=V/R_T$. Because of $C\dot\phi=(e/\hbar)Q$, the equilibrium correlation function $J(t)$ is intimately connected with the spontaneous charge fluctuations on the junction capacitor arising from the coupling to the electromagnetic environment. Using properties of the Fermi function and the detailed balance symmetry obeyed by $P(E)$, the result (1) may be written as

$$I=\frac{1}{eR_T}\int_{-\infty}^{+\infty}dE\,E\frac{1-e^{-\beta eV}}{1-e^{-\beta E}}P(eV-E)\,. \tag{3}$$

For the circuit shown in Fig. 1(b), we have

$$J(t)=\rho\{\coth(\tfrac{1}{2}\beta\hbar\omega_L)[\cos(\omega_Lt)-1]-i\sin(\omega_Lt)\}\,,$$

where $\omega_L=(LC)^{-1/2}$ is the oscillation frequency of the environmental mode described by the Hamiltonian $H_{\rm em}$ and $\rho=\pi/2CR_Q\omega_L$ is the ratio of the single-electron charging energy $e^2/2C$ and the mode excitation energy $\hbar\omega_L$. This leads to an I-V characteristic of the form

$$I=\frac{1}{eR_T}\exp[-\rho\coth(\tfrac{1}{2}\beta\hbar\omega_L)]\times\sum_{n=-\infty}^{+\infty}\epsilon_n\frac{\sinh(\frac{1}{2}\beta eV)}{\sinh(\frac{1}{2}\beta\epsilon_n)}I_n\left(\frac{\rho}{\sinh(\frac{1}{2}\beta\hbar\omega_L)}\right),$$

where $\epsilon_n=eV-n\hbar\omega_L$ and $I_n(x)$ is the modified Bessel function. Each term corresponds to a tunneling channel where the electron creates or absorbs n quanta of the environmental mode. For zero temperature this result simplifies to read[13]

$$I=\frac{1}{eR_T}e^{-\rho}\sum_{n=0}^{n_{\max}}\frac{\rho^n}{n!}(eV-n\hbar\omega_L)\ \text{for}\ V>0\,,$$

where $n_{\max}$ is the largest integer below $eV/\hbar\omega_L$. At $T=0$ a tunneling electron can only excite the environment and create at most $n_{\max}$ quanta. It is important to note that the differential conductance dI/dV displays a series of steps at voltages $V_n=n\hbar\omega_L/e$. Usually, for low voltages, the I-V characteristic will be dominated by the elastic channel ($n=0$). This behavior is analogous to the presence of a strong elastic line in the Mössbauer γ spectrum. Inelastic processes are negligible when the mode energy $\hbar\omega_L$ exceeds the single-electron charging energy, that is, for $\rho \ll 1$. This will mostly be the case since typical series inductances will be well below $(\hbar^2/e^4)C$ even for ultrasmall junctions. At least for low voltages the Coulomb charging effects are then suppressed. Finite temperatures make the deviation from Ohmic behavior even less pronounced. Another way of understanding the suppression of the Coulomb blockade effect is to estimate the spontaneous charge fluctuations on the junction capacitor arising from the coupling to the external circuit. From the Hamiltonian $H_{\rm em}$ we find

$$\langle \delta Q^2 \rangle = \tfrac{1}{2}\hbar (C/L)^{1/2} \coth(\tfrac{1}{2}\beta\hbar\omega_L),$$

which at $T=0$ exceeds e^2 unless $\rho \gg 1$.

For the case of a general electromagnetic environment with arbitrary frequency dependence the coupled junction-circuit system amy be characterized by the diagram in Fig. 1(b) with the series inductance L replaced by a general frequency-dependent impedance $Z(\omega)$. Since the electromagnetic environment is dissipative but linear (there is no other junction in the circuit), we may follow Caldeira and Leggett[14] and treat this environment as if it were an infinite collection of LC oscillators. The Hamiltonian $H_{\rm em}$ is then replaced by a corresponding Hamiltonian with an infinite number of environmental modes with a spectral density determined by $Z(\omega)$. The result (3) for the I-V characteristic remains valid for the general case. The environmental influence is again described through the function $P(E)$, Eq. (2), which gives the probability that the tunneling electron transfers the energy E to the circuit. The phase correlation function now takes the form

$$J(t) = \int_0^\infty \frac{d\omega}{\omega} \frac{\mathrm{Re} Z_t(\omega)}{R_Q} \{\coth(\tfrac{1}{2}\beta\hbar\omega)[\cos(\omega t) - 1] - i\sin(\omega t)\}, \tag{4}$$

where

$$Z_t(\omega) = \frac{1}{i\omega C + Z^{-1}(\omega)}$$

is the total impedance of the junction in parallel with the environmental impedance. The formulas (2)-(4) allow for the determination of the I-V characteristic for arbitrary frequency dependence of the electromagnetic environment at finite temperatures.

Let us first extract the I-V characteristic at large voltages. There only the behavior of $P(E)$ for large energies E is relevant, which in turn is related to the short-time behavior of the phase correlation function. Using $J(t) = -(i\pi/2C)t - (e/\hbar C)^2 \langle \delta Q^2 \rangle t^2$ for $t \to 0$, we find $I = R_T^{-1}(V - e/2C)$ for $eV \gg k_B T, e^2/2C$. This describes an offset of magnitude $e/2C$, the so-called Coulomb gap. For low voltages the behavior of the I-V characteristic depends on the spectrum of charge fluctuations at frequencies below $eV/\hbar$. Only when the impedance $\mathrm{Re} Z_t(\omega)$ in this frequency range exceeds the quantum resistance will the offset still be noticeable.

As a concrete example let us consider a tunnel junction connected to a long dissipative transmission line with characteristic impedance R. In that case $Z(\omega) = R$ (see inset of Fig. 2), and we recognize that the phase correlation function (4) behaves like the mean-square displacement of a quantum Brownian particle.[15] At zero temperature the phase correlation grows logarithmically for large t, $J(t) \sim -\alpha \ln(\omega_R |t|)$, where $\alpha = R/R_Q$ and $\omega_R = 1/RC$. This implies a power-law decay proportional to $t^{-\alpha}$ of the Fourier transform of $P(E)$. As a consequence, for $\alpha < 1$, the function $P(E)$ has a singularity at $E=0$,

$$P(E) = \frac{\exp(-\alpha\gamma)}{\Gamma(\alpha)} \frac{1}{E} \left(\frac{E}{\hbar\omega_R} \right)^\alpha \Theta(E) \text{ for } E \to 0,$$

where $\gamma = 0.5772\ldots$ is Euler's constant. We see that quasielastic transitions with small energy transfer E to the environment are most probable. Using (3), this yields for the I-V characteristic

$$I = \frac{\exp(-\alpha\gamma)}{\Gamma(\alpha+2)} \frac{V}{R_T} \left(\frac{e|V|}{\hbar\omega_R} \right)^\alpha \text{ for } V \to 0,$$

which is nonanalytic at $V=0$ and shows a suppression of the Coulomb gap for $\alpha < 1$. The same low-voltage behavior of the characteristic is obtained for a general frequency-dependent impedance $Z(\omega)$ with a finite Ohmic component $R = Z(0)$. At finite temperatures, the logarithmic growth of the phase correlation function is replaced by a linear growth, $J(t) \sim -(\pi\alpha/\hbar\beta)|t|$ for $t \to \pm\infty$, and $I(V)$ is now analytic at $V=0$, but at low temperatures and for voltages between $k_B T/e$ and e/C the behavior is close to the zero-temperature case. Figure 2 displays numerically integrated I-V curves for various values of R/R_Q at $T=0$. Clearly, the Coulomb gap only survives at low voltages for resistance values of the order of or larger than the resistance quantum R_Q.

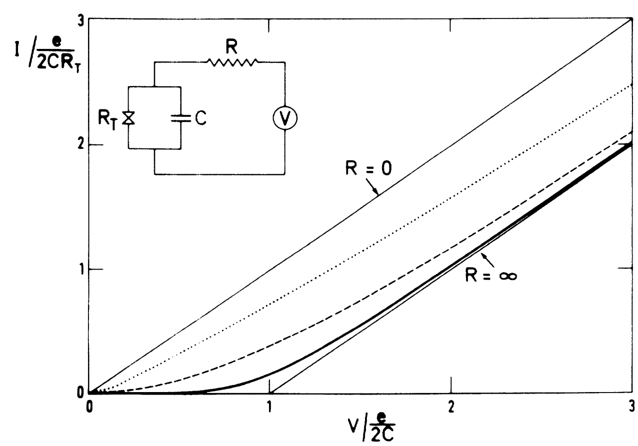

FIG. 2. The I-V characteristic of a tunnel junction coupled to an environment characterized by a resistance R (see inset) for $R/R_Q = 0, 0.1, 1, 10$, and ∞.

In conclusion, by a quantum-mechanical treatment of the electromagnetic environment of a tunnel junction we have calculated the I-V characteristic as a function of the environment impedance $Z(\omega)$ [Eqs. (2)-(4)]. We have shown that the junction capacitance C will be revealed in the I-V characteristic as a voltage offset $e/2C$ only if the impedance of the environment $Z(\omega)$ at frequencies below $e^2/2\hbar C$ exceeds the resistance quantum R_Q. The influence of the electromagnetic environment is not associated with the finite duration of the electron tunneling process. In fact, the simple model employed here entirely disregards the finite traversal time. We thus predict that in experimental setups designed to provide a high-impedance environment for a single junction,[16] the Coulomb gap should be observable. We have stressed that the suppression of the Coulomb gap for a junction in a low-impedance electrical circuit occurs for much the same reason as the absence of a frequency shift of γ radiation from nuclei embedded in a crystal. In linear junction arrays, the presence of islands that can accommodate only an integer number of elementary charges is manifest in a Coulomb gap which can be observed even for low-impedance environments.

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