Evolution of Weinberg's Gluonic *CP*-Violation Operator

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The renormalization-group evolution of Weinberg's purely gluonic dimension-6 *CP*-violating operator is computed to first order in the QCD coupling constant, including its mixing with the quark color electric dipole moment. Our result for the anomalous dimension of the quark operator agrees with several previous calculations. For the gluonic operator, we find the anomalous dimension to have the same magnitude but opposite sign as a previous calculation. This significantly relaxes the constraints imposed on extensions of the standard model by experimental measurements of the neutron electric dipole moment.

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It was recently pointed out by Weinberg[1] that there is a *CP*-violating operator of dimension 6 that is constructed out of gluon fields only. It can therefore give a contribution to the neutron electric dipole moment that is not suppressed by any light-quark masses or mixing angles. This gluonic operator $\mathcal{O}_G$ can be induced as a term in the low-energy effective Hamiltonian by exchange of heavy Higgs bosons,[1,2] by the exchange of gluinos in supersymmetric models,[3] or by exchange of gauge bosons in left-right symmetric models.[4] The consideration of the operator $\mathcal{O}_G$ allows the constraints on *CP* violation from experimental limits on the neutron electric dipole moment to be tightened significantly. A recent calculation of the renormalization-group evolution of $\mathcal{O}_G$ found it to have a large anomalous dimension,[5] so its coefficient is magnified enormously by renormalization-group evolution from the heavy mass scale down to the hadronic scale. By including this factor, which can be as large as 800, and imposing a naturalness condition on the source of *CP* violation, one can rule out a number of extensions of the standard model.[1,3,4]

Since the renormalization of the operator $\mathcal{O}_G$ has such a dramatic effect, it is important to check the calculation of its anomalous dimension and to investigate the effects of mixing with other operators. In this Letter, we calculate the renormalization-group evolution of $\mathcal{O}_G$ to first order in the QCD coupling constant, including its mixing with the quark color electric dipole moment operator $\mathcal{O}_q$. Our result for the anomalous dimension of $\mathcal{O}_q$ agrees with several previous calculations. For the gluonic operator $\mathcal{O}_G$, we find the anomalous dimension to have the same magnitude but opposite sign as the previous calculation. This changes an enhancement factor of 800 into a suppression by 800, significantly relaxing the constraints imposed on extensions of the standard model. We also show that renormalization-group mixing between $\mathcal{O}_G$ and $\mathcal{O}_q$ can be neglected, but that the color electric dipole moment of a heavy quark can induce significant corrections to the coefficient of $\mathcal{O}_G$ by the matching conditions at the heavy-quark threshold.

The purely gluonic dimension-6 *CP*-violating operator discovered by Weinberg[1] is

$$\mathcal{O}_G(\mu) = -\tfrac{1}{6} f^{abc} \epsilon^{\mu\nu\lambda\rho} G^a_{\mu\sigma} G^{b\sigma}_{\nu} G^c_{\lambda\rho} , \tag{1}$$

where μ is the renormalization scale and our convention is $\epsilon^{0123} = +1$. Under renormalization-group evolution, it mixes with the quark color electric dipole moment operator

$$\mathcal{O}_q(\mu) = \tfrac{1}{4} \epsilon^{\mu\nu\lambda\rho} G^a_{\mu\nu} \bar{q} \sigma_{\lambda\rho} T^a q , \tag{2}$$

which also contributes to the neutron electric dipole moment. The effective low-energy *CP*-violating Hamiltonian obtained by integrating out particles at a large mass scale M will include the terms

$$\mathcal{H}_{\rm CPV} = C_G(\mu)\mathcal{O}_G(\mu) + \sum_q C_q(\mu)\mathcal{O}_q(\mu) . \tag{3}$$

The μ dependence must cancel between the coefficients and the operators. Once the evolution of the operators as a function of μ is calculated, the μ dependence of the coefficients is determined. The initial values of the coefficients at the scale $\mu = M$ must be determined by a separate calculation. Note that the operator in the first term in (3) could equally well be replaced by $g_s(\mu)^3 \times \mathcal{O}_G(\mu)$, where $g_s(\mu)$ is the running coupling constant of QCD. This changes the anomalous dimension of the operator significantly, but there is a compensating change in the coefficient.

The operator $\mathcal{O}_q$ defined in (2) is the only hadronic dimension-5 *CP*-violating operator. Since there are no operators with which it can mix, it must be an eigenstate under renormalization-group evolution. Its anomalous dimension is the same as that of the operator $G^a_{\mu\nu}\bar{q} \times \sigma^{\mu\nu} T^a q$,[6] and has been calculated by several groups.[7] We have repeated the calculation with the same result, so we only quote the final answer. The evolution of the operator $\mathcal{O}_q$ is

$$\mu \frac{\partial}{\partial \mu} \mathcal{O}_q(\mu) = \frac{\alpha_s(\mu)}{4\pi} \gamma_{qq} \mathcal{O}_q(\mu), \qquad \gamma_{qq} = \tfrac{23}{3} C_A - 10 C_F - \tfrac{2}{3} N_f , \tag{4}$$

where $C_A = 3$ and $C_F = \frac{4}{3}$ are the Casimirs for the adjoint and fundamental representations of SU(3) and N_f is the number of light quarks at the scale μ. To leading order in $\alpha_s = g_s^2/4\pi$, the running coupling constant $g_s(\mu)$ satisfies $\mu(\partial/\partial\mu)g_s(\mu) = -\beta(\alpha_s/4\pi)g_s(\mu)$, where $\beta = \frac{1}{3}\times(11C_A - 2N_f)$.

To first order in α_s, the gluonic operator $\mathcal{O}_G$ can only mix with the quark operators $\mathcal{O}_q$. Up to total derivatives, there are no other gauge-invariant CP-violating operators of dimension 6 or less that involve the gluon field. Ignoring total derivatives since they cannot contribute to an effective action, the renormalization-group equation to order α_s is

$$\mu\frac{\partial}{\partial\mu}\mathcal{O}_G(\mu) = \frac{\alpha_s(\mu)}{4\pi}\left[\gamma_{GG}\mathcal{O}_G(\mu) + \gamma_{Gq}\sum_q m_q(\mu)\mathcal{O}_q(\mu)\right]. \quad (5)$$

To leading order in α_s, the running quark mass satisfies $\mu(\partial/\partial\mu)m_q(\mu) = \gamma_m(\alpha_s/4\pi)m_q(\mu)$, where $\gamma_m = -6C_F$.

The previous result for γ_{GG} was calculated using the background-field method.[5] We use a different method which requires the calculation of fewer diagrams. We compute a particular term in the matrix element of the operator $\int d^4x\,\mathcal{O}_G$ between two initial-state and two final-state on-shell gluons. Since this is a scattering amplitude and therefore gauge invariant, we avoid the complications of mixing with gauge-noninvariant operators. The μ dependence of the operator can be determined by calculating this scattering amplitude to first order in α_s with an ultraviolet cutoff μ on the loop momentum. Actually it is not necessary to compute the complete scattering amplitude, because it has a pole in the invariant mass q^2 of two external gluon legs, and the residue of this pole is also gauge invariant. The residue factorizes into a three-gluon vertex with perturbative corrections and the CP-violating vertex with perturbative corrections. Thus we can reduce the calculation of the anomalous-dimension coefficient γ_{GG} to the calculation of the divergent part of the matrix element of $\int d^4x\,\mathcal{O}_G$ between two on-shell gluons, with the third gluon leg off its mass shell. The third leg can be treated as if it were on its mass shell. If it has momentum q and Lorentz index ν, terms proportional to q^2 can be dropped because they do not contribute to the residue of the pole in q^2. Terms proportional to q^ν can also be dropped because they vanish after contracting with the three-gluon vertex.

Aside from wave-function renormalization on the external lines, there are four topologically distinct diagrams that contribute to this amplitude and they are shown in Fig. 1. Each diagram contains quadratic divergences which cancel after summing over the cyclic permutations of the external lines. Only the diagrams 1(a) and 1(c) give contributions that survive after the sum over cyclic permutations. Thus the problem has been reduced to the calculation of two diagrams.

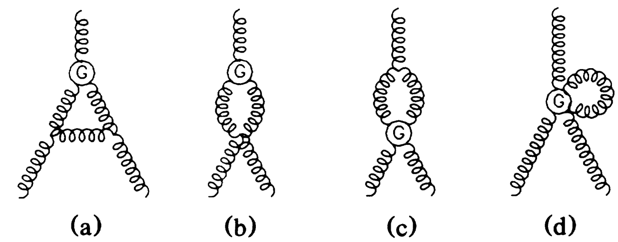

FIG. 1. Diagrams that determine the anomalous-dimension coefficient γ_{GG}. The circle with G inside represents the gluonic CP-violating operator $\mathcal{O}_G$.

If the Feynman rule for the CP-violating vertex is derived straightforwardly from the expression (1), the calculation of the diagrams involves complicated algebraic manipulations of the Levi-Cività tensor $\epsilon^{\mu\nu\lambda\rho}$. This can be avoided by using Dirac traces to automatically handle the algebra for the Levi-Cività tensor. For example, the Feynman rule for the CP-violating vertex with three external gluon legs with incoming momenta p,q,r, Lorentz indices μ,ν,λ, and color indices a,b,c can be written in an elegant form as the Dirac trace

$$-\tfrac{1}{16}f^{abc}\mathrm{Tr}([p,\gamma^\mu][q,\gamma^\nu][r,\gamma^\lambda]\gamma_5), \quad (6)$$

where inside the trace $p = p_\mu\gamma^\mu$, etc.

We calculated the diagrams in Feynman gauge using an ultraviolet momentum cutoff μ. After using the mass-shell conditions for the external lines, the divergent parts of the one-loop correction to our matrix element are indeed proportional to (6). From the coefficient $\gamma_{GG}\alpha_s\ln(\mu)/4\pi$ we read off the diagonal anomalous-dimension coefficient γ_{GG} in (5),

$$\gamma_{GG} = -C_A - 2N_f. \quad (7)$$

The contributions in Feynman gauge from the individual diagrams are $11C_A/2$ from diagram 1(a), $-23C_A/2$ from diagram 1(c), and $5C_A - 2N_f$ from wave-function renormalization.

The off-diagonal coefficient γ_{Gq} in (5) is computed in a similar way. We calculate the divergent parts of the matrix element of $\int d^4x\,\mathcal{O}_G$ between two on-shell quarks, with one external off-shell gluon line. The external gluon can be treated as if it was on shell, allowing us to drop terms proportional to q^2 or q^ν. The only diagram that need be calculated is one in which two gluons from the operator attach to the quark line. The divergent part of the diagram is proportional to $m_q\bar{u}(p')[q,\gamma^\nu]\gamma_5T^a u(p)$, where $q = p' - p$. Reading off the anomalous dimension coefficient, we find $\gamma_{Gq} = 2C_A$.

If we use an alternative basis of operators for the effective Hamiltonian in (3), such as $\mathcal{O}_1(\mu) = g_s(\mu)^3\times\mathcal{O}_G(\mu)$ and $\mathcal{O}_2(\mu) = g_s(\mu)m_q(\mu)\mathcal{O}_q(\mu)$, the anomalous-dimension matrix is changed. The diagonal coefficients analogous to γ_{GG} in (5) and γ_{qq} in (4) are

$\gamma_{11}=\gamma_{GG}-3\beta=-12C_A$ and $\gamma_{22}=\gamma_{qq}-\beta+\gamma_m=4C_A-16C_F$.

As we will justify later, the mixing of the operators $\mathcal{O}_G$ and $\mathcal{O}_q$ can be ignored. The solution to the renormalization-group equations for the coefficient functions is then

$$C_G(\mu)=\left[\frac{g_s(\mu)}{g_s(M)}\right]^{\gamma_{GG}/\beta}C_G(M)\,, \qquad (8)$$

$$C_q(\mu)=\left[\frac{g_s(\mu)}{g_s(M)}\right]^{\gamma_{qq}/\beta}C_q(M)\,. \qquad (9)$$

With initial conditions $C_q(M)=0$ and $C_G(M)=c\times g_s(M)^3/M^2$, where c is a dimensionless coefficient, the CP-violating Hamiltonian (3) becomes

$$\mathcal{H}_{\mathrm{CPV}}=\frac{c}{M^2}g_s(M)^3\left[\frac{g_s(M)}{g_s(\mu)}\right]^{39/23}\mathcal{O}_G(\mu)$$
$$=\frac{c}{M^2}\left[\frac{g_s(M)}{g_s(\mu)}\right]^{108/23}g_s(\mu)^3\mathcal{O}_G(\mu)\,. \qquad (10)$$

We have assumed five flavors of light quarks between the scales M and μ. The exponent in the second line of (10) is the same as in Ref. 5 but of opposite sign. Thus instead of a large enhancement of the coefficient of $g_s(\mu)^3\times\mathcal{O}_G(\mu)$, we find a large suppression. Although some of the suppression comes from the evolution of $\mathcal{O}_G(\mu)$, a larger part comes from having expressed the small coupling constant $g_s(M)$ in terms of the much larger coupling constant $g_s(\mu)$ at the hadronic scale. For this reason, it is more natural to take $\mathcal{O}_G(\mu)$ as the operator, as in the first line of (10), leaving the factor of $g_s(M)^3$ in the coefficient. Then there is only a modest suppression from the evolution of the operator, but its coefficient is small due to the fact that the operator is generated at a scale M where the QCD coupling is small. We note that to reproduce the result of Ref. 5, the anomalous-dimension coefficient γ_{GG} in (7) would have to be $23C_A-2N_f$.

The operator $\mathcal{O}_q$ does not affect $\mathcal{O}_G$ through its renormalization-group evolution, but it has a significant effect in another way. As one evolves down through the mass scale of a heavy quark Q, the color electric dipole moment operator $\mathcal{O}_Q(\mu)$ for that quark induces a shift in the coefficient of the gluonic operator $\mathcal{O}_G$.[8] The shift can be determined by calculating the diagrams in Fig. 2. After summing over permutations of the external lines, only diagram 2(a) survives and gives a finite contribution proportional to $\alpha_s(m_Q)/m_Q$. Matching matrix elements of the operator $C_G(\mu)\mathcal{O}_G(\mu)+C_Q(\mu)\mathcal{O}_Q(\mu)$ just above the threshold $\mu=m_Q$ with the corresponding matrix elements of $C_G(\mu)\mathcal{O}_G(\mu)$ just below m_Q, we find that the shift is

$$C_G(m_Q^-)=C_G(m_Q^+)+C_Q(M)\left[\frac{g_s(m_Q)}{g_s(M)}\right]^{\gamma_{qq}/\beta}\frac{1}{8\pi}\frac{\alpha_s(m_Q)}{m_Q}\,. \qquad (11)$$

Thus this contribution to $C_G(\mu)$ at the hadronic scale involves an enhancement from the evolution of $\mathcal{O}_Q$ from M down to m_Q, followed by a suppression from the evolution of $\mathcal{O}_G$ from m_Q down to μ.

This two-step process is in fact the simplest way to understand the results of Refs. 2 and 4.[8] Their initial conditions on C_G came from calculating two-loop diagrams in which the quark loop involved both the t quark and the b quark. One can divide the calculation into two steps, the first being the generation of the operator $\mathcal{O}_b(m_t)$ from a one-loop diagram at the scale m_t. In the second step, the operator $\mathcal{O}_b(m_b^+)$ induces the operator $\mathcal{O}_G(m_b^-)$ by (11). This approach has the advantage of summing up all the leading logarithms of the form $\alpha_s\ln(m_t/m_b)$.

We now justify our earlier claim that operator mixing can be ignored. We need only consider mixing of $\mathcal{O}_G$ with the heavy-quark operators $\mathcal{O}_Q$, because hadronic matrix elements involving light-quark operators will be suppressed by the light-quark mass. If $C_G(M)$ is nonzero, renormalization-group evolution of $\mathcal{O}_G$ down to the scale m_Q will generate a contribution to $C_Q(m_Q^+)$ on the order of $C_G(M)m_Q$. Applying the matching condition at the heavy-quark threshold, we find that the shift in C_G is of the order of $C_G(M)\alpha_s(m_Q)$. This shift is suppressed by a power of $\alpha_s(m_Q)$ compared to $C_G(m_Q^+)$, and should not be included unless one also computes the order-α_s corrections to the initial conditions and to the diagonal evolution of $\mathcal{O}_G$.

Finally, we discuss the implications of our result for the constraints imposed on extensions of the standard model by measurements of the neutron electric dipole moment. The results that are most affected are those that relied on an enormous renormalization factor due to evolution from the t-quark mass scale down to the hadronic scale. In particular, one cannot yet rule out maximal CP violation in either the Higgs sector[1] or in supersymmetric models.[3] If one accepts the estimates that gave the enhancement factor of 800 in Refs. 1 and 5,

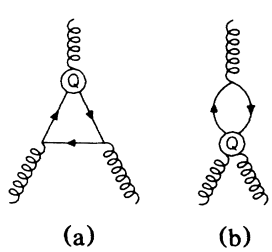

FIG. 2. Diagrams that determine the shift in the coefficient $C_G(\mu)$ at the threshold for a heavy quark Q. The circle with Q inside represents the color electric dipole moment operator $\mathcal{O}_Q$.

then it should be replaced by a suppression factor of 800. One should remember, however, that these estimates are extremely sensitive to the lower end point for the renormalization-group evolution, and the suppression factor could easily be overestimated by an order of magnitude. If the gluonic operator $\mathcal{O}_G$ is generated at the b-quark scale, as in Refs. 2 and 4, the suppression factor will be reduced by another factor of 5. We conclude that simple extensions of the standard model with maximal CP violation are not yet ruled out. Nevertheless the purely gluonic dimension-6 CP-violation operator remains a powerful probe of the mechanism for CP violation, and should serve as a stimulus for further improvements on measurements of the neutron electric dipole moment.

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