Supersymmetry and Large Transition Magnetic Moment of the Neutrino

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We show that the most general minimal supersymmetric standard model with $l_e - l_{\mu}$ symmetry $(l_i = i$ th lepton number) and a discrete horizontal symmetry between e and μ families can generate a large transition magnetic moment for the neutrino while leading to a naturally small mass. A magnetic moment of order $\mu_v \approx 0.3 \times 10^{-10} \mu_B$ implies $m_{\nu_e} = m_{\nu_{\mu}} \ge 1$ eV. The photino, which is expected to be the lightest supersymmetric particle, is unstable in this scheme, decaying dominantly into three-lepton final states ($\mu + \bar{\nu}_e \tau^-$, $e^+ \bar{\nu}_{\mu} \tau^-$ + charge conjugate).

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The apparent discrepancy between the observed flux of solar neutrinos on Earth^{1,2} and the predictions of the standard solar model³ can be resolved if the neutrino is endowed with a magnetic moment⁴ of the order of $(0.3-1) \times 10^{-10} \mu_R$. Two types of magnetic-moment interactions can serve the purpose by rotating v_e emitted in the solar core into a species undetectable in the existing experiments: (i) The Dirac-type magnetic moment that rotates v_{e_l} to v_{e_R} ; (ii) the transition magnetic moment that rotates v_{e_L} to $v_{\mu_R}^c$. The latter case, whose viability has been established,⁵ has the advantage that there are no severe bounds on it from SN 1987A observations.⁶ This mechanism, however, requires the $v_e - v_\mu$ mass splitting to be less than 10^{-7} eV². This condition is automatically satisfied if, for instance, $l_e - l_\mu$ ($l_i = i$ th lepton number) is an unbroken symmetry. If the present hints of an anticorrelation of the neutrino flux with the sunspot number is borne out by future experiments, it will provide a dramatic confirmation of this proposal.

The theoretical problem with a large magnetic moment of the neutrino is that it generally implies a large neutrino mass (e.g., $\mu_v \approx 10^{-10} \mu_B$ implies $m_{\nu_e} \approx a$ few keV), in conflict with the observed upper limit from tritium β decay. One way to overcome this difficulty⁷ is to introduce an SU(2)_v symmetry of neutrino interactions under which ν_L and ν_L^c form a doublet. Under this SU(2)_v symmetry, the Dirac mass term $v^T C v^c$ $= \frac{1}{2} (v^T C v^c + v^c {}^T C v)$ transforms as a triplet whereas the magnetic-moment interaction

$$v^{T}C\sigma_{\alpha\beta}v^{c}F^{\alpha\beta} = \frac{1}{2} (v^{T}C\sigma_{\alpha\beta}v^{c} - v^{c} {}^{T}C\sigma_{\alpha\beta}v)F^{\alpha\beta}$$

transforms as a singlet. Therefore, in the limit of exact $SU(2)_{\nu}$ symmetry, the neutrino is massless, but can have a magnetic moment. Several attempts have recently been made to incorporate this idea in the context of renormalizable gauge models.^{8,9} In particular, in Ref. 9, it was proposed to identify the $SU(2)_{\nu}$ symmetry with a gauged horizontal $SU(2)_{H}$ symmetry operating between the electron and the muon generations. In this Letter we report on some new results on this subject.

First, we point out that instead of the continuous $SU(2)_v$ symmetry, a two-element global discrete symmetry operating on (v, v^c) is sufficient for the purpose.

Consider the following discrete symmetry D under which

$$\begin{pmatrix} v \\ v^c \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} v \\ v^c \end{pmatrix}.$$
 (1)

Under this symmetry, the mass term is odd, whereas the magnetic moment is even. Therefore, this allows a large magnetic moment of the neutrino while keeping m_v small. This symmetry, D, has the advantage that it can be spontaneously (or softly) broken at the electroweak scale without causing phenomenological problems.

Next, we show that the most general minimal $SU(2)_L \times U(1)_Y$ supersymmetric standard model $l_e - l_{\mu}$ conservation and this extra global *D* symmetry between *e* and μ generations provides a realization of this scheme. No additional fermions or Higgs bosons other than those present in the minimal supersymmetric standard model are required. We emphasize that it is the combination of $l_e - l_{\mu}$ symmetry and *D* symmetry which facilitates a light neutrino with a large transition magnetic moment. $l_e - l_{\mu}$ symmetry alone forbids Majorana mass terms of the type $v_e^T C v_e$ and $v_{\mu}^T C v_{\mu}$, but allows a "Dirac" mass term of the type $v_e^T C v_{\mu}$, which of course vanishes in the *D*-symmetric limit. The mass splitting between v_e and v_{μ} in this case is exactly zero, thereby satisfying $\Delta m_v^2 \le 10^{-7} \text{ eV}^2$ automatically.

Description of the model.—Our model is the minimal supersymmetric extension of the standard model¹⁰ with *R*-parity-violating terms¹¹ in such a way that the lepton number $l_e - l_{\mu}$ remains unbroken and the additional discrete horizontal symmetry *D* between $e - \mu$ generations is respected. Let $L_{e,\mu,\tau}$ denote the leptonic chiral superfields transforming as (2, -1) under SU(2)_L ×U(1)_Y and e^c, μ^c, τ^c those transforming as (1, +2). Under the discrete symmetry *D*, these fields transform as

$$\begin{pmatrix} L_e \\ L_\mu \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} L_e \\ L_\mu \end{pmatrix},$$

$$\begin{pmatrix} e^c \\ \mu^c \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} e^c \\ \mu^c \end{pmatrix},$$

$$L_r \rightarrow L_r, \quad \tau^c \rightarrow \tau^c.$$

$$(2)$$

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The quark superfields, denoted by Q_a , u_a^c , and d_a^c (a=generation index), do not transform under D. As usual, we keep two Higgs superfields, $H_u(2,1)$ and $H_d(2,-1)$.

The most general superpotential invariant under $l_e - l_{\mu}$ and D (and baryon number) is given by (suppressing the generation index for quarks)

$$W = \sum_{i=1}^{2} W_{i} = h_{u}QH_{u}u^{c} + h_{d}QH_{d}d^{c} + h_{d}^{i}QL_{\tau}d^{c} + f(L_{e}L_{\tau}e^{c} + L_{\mu}L_{\tau}\mu^{c}) + f^{i}L_{e}L_{\mu}\tau^{c} + h_{\tau}L_{\tau}H_{d}\tau^{c} + h_{\mu}(L_{e}H_{d}e^{c} + L_{\mu}H_{d}\mu^{c}) + m_{H}H_{u}H_{d} + m_{H}^{i}H_{u}L_{\tau}.$$
(3)

Since L_{τ} and H_d have identical transformation properties, one can choose a basis where L_{τ} has zero vacuum expectation value. The presence of the m'_H term in Eq. (3), however, results in a mixing of v_{τ} with the Higgsino \tilde{H}^0_u , which in turn mixes with the gauginos. v_{τ} thus picks up a tree-level mass of order m'_H^2/M , where M is typically the Z-ino mass. Cosmological constraints on stable neutrinos which require $m_{v_{\tau}} \leq 100$ eV imply that m'_H should be less than 10 MeV or so.

For supersymmetry breaking, we choose only soft terms so that the nonrenormalization theorem for the superpotential still holds:

$$-\mathcal{L}_{\text{soft}} = \sum_{i} \mu_{i}^{2} \phi_{i}^{*} \phi_{i} + \left[\sum_{i=1}^{9} \int d^{2}\theta \, \theta^{2} A_{i} W_{i} + \text{H.c.} \right]$$
$$+ \frac{1}{2} \sum_{i=1}^{3} M_{i} \lambda_{i}^{T} C \lambda_{i} .$$
(4)

Here the sum over ϕ_i goes over all the spin-zero members of the chiral superfields and includes the term $\tilde{L}_{\tau}^{\dagger}H_d + \text{H.c.}$ The second term in Eq. (4) denotes the most general soft-breaking term that has the same structure as the superpotential, and the last term represents gauge-invariant Majorana masses for the gauge fermions. The signs of the scalar mass terms are appropriately chosen so that at the minimum of the potential the following fields acquire vacuum expectation values $(\text{VEV's}): \langle H_u^0 \rangle = v_u, \langle H_d^0 \rangle = v_d, \langle \tilde{v}_{\tau} \rangle = 0. \langle \tilde{v}_{\tau} \rangle = 0$ is not an assumption, but just the choice of our basis. $l_e - l_{\mu}$ conservation prevents $\tilde{v}_{e,\mu}$ from acquiring VEV's.

Since all the scalar fields which acquire VEV's trans-

form trivially under the discrete symmetry D, even after the spontaneous breaking of gauge symmetry, D is left intact, leading to zero neutrino mass and a nonzero magnetic moment. However, in this limit the electron and muon are degenerate. In order to lift the degeneracy we add to the soft-supersymmetry-breaking Lagrangian all possible dimension-two terms which break the D symmetry softly:

$$-\mathcal{L}_{\text{soft}}^{\prime} = \delta M_{\tilde{\mu}}^{2} \tilde{L}_{\mu}^{\dagger} \tilde{L}_{\mu} + \delta M_{\tilde{\mu}^{c}}^{2} \tilde{\mu}^{c*} \tilde{\mu}^{c}.$$
(5)

We shall assume that the mass terms of Eq. (5) are much smaller than their counterparts in Eq. (4). The terms in Eq. (5) will not only lift the degeneracy of e and μ , but will also generate nonzero neutrino mass. The challenge is then to show that the soft breaking of Dsymmetry generates sufficient splitting between e and μ masses, but does not induce unacceptably large neutrino mass. We find it remarkable that indeed there exists a range of parameters of the model where the abovementioned criterion is satisfied.

Let us now turn to the discussion of neutrino mass and magnetic moment and its intimate relation to the $e-\mu$ mass splitting. After the spontaneous breaking of the gauge symmetry by the nonvanishing VEV's of the neutral Higgs bosons, the scalar fields \tilde{e} and \tilde{e}^c will mix. Similarly $\tilde{\mu}$ and $\tilde{\mu}^c$ will also mix. There are two sources for such mixing: (i) cubic terms in the soft-supersymmetry-breaking Lagrangian of Eq. (4), and (ii) the F terms from the superpotential, which for the color singlet fields takes the form

$$V_{F} = |f\tilde{L}_{\tau}\tilde{e}^{c} + f'\tilde{L}_{\mu}\tilde{\tau}^{c} + h_{\mu}H_{d}\tilde{e}^{c}|^{2} + |f\tilde{L}_{\tau}\tilde{\mu}^{c} + f'\tilde{L}_{e}\tilde{\tau}^{c} + h_{\mu}H_{d}\tilde{\mu}^{c}|^{2} + |f(\tilde{L}_{e}\tilde{e}^{c} + \tilde{L}_{\mu}\tilde{\mu}^{c}) + h_{\tau}H_{d}\tilde{\tau}^{c} + m'_{H}H_{u}|^{2} + |f\tilde{L}_{e}\tilde{L}_{\tau} + h_{\mu}\tilde{L}_{e}H_{d}|^{2} + |f\tilde{L}_{\mu}\tilde{L}_{\tau} + h_{\mu}\tilde{L}_{\mu}H_{d}|^{2} + |f'\tilde{L}_{e}\tilde{L}_{\mu} + h_{\tau}\tilde{L}_{\tau}H_{d}|^{2} + |m_{H}H_{d} + m'_{H}\tilde{L}_{\tau}|^{2} + |h_{\tau}\tilde{L}_{\tau}\tilde{\tau}^{c} + h_{\mu}(\tilde{L}_{e}\tilde{e}^{c} + \tilde{L}_{\mu}\tilde{\mu}^{c}) + m_{H}H_{u}|^{2}.$$
(6)

In combination with the f and f' terms of Eq. (3), this mixing leads to a nonvanishing transition magnetic moment of the neutrino via the diagrams of Fig. 1. In the D-symmetric limit the $\tilde{e}\tilde{e}^c$ and the $\tilde{\mu}\tilde{\mu}^c$ mixing matrices are identical. Consequently, the magnitudes of the contributions from Figs. 1(a) and 1(b) are equal. The two, however, have an opposite sign at one of the vertices. Therefore we have $m_{v_e v_{\mu}} = 0$. For the effective magnetic-moment interaction, the two diagrams add, leading to a nonzero $\mu_{v_e v_{\mu}}$. If we denote by (m_1^2, m_2^2, θ) the two mass-squared eigenvalues and the mixing angle corre-

sponding to the $\tilde{e}\tilde{e}^c$ mixing matrix (which is the same for $\tilde{\mu}\tilde{\mu}^c$ mixing in the *D*-symmetric limit), the transition magnetic moment is given by

$$\mu_{v_e v_{\mu}} = \frac{ff'e}{8\pi^2} m_{\tau} \sin 2\theta \left\{ \frac{1}{m_1^2} \left[\ln \left(\frac{m_1^2}{m_{\tau}^2} \right) - 1 \right] - \frac{1}{m_2^2} \left[\ln \left(\frac{m_2^2}{m_{\tau}^2} \right) - 1 \right] \right\}.$$
 (7)



FIG. 1. Diagrams responsible for neutrino mass and magnetic moment. A photon line should be attached to the internal lines in the case of magnetic moment.

This clearly can be in the interesting range of $(0.3-1) \times 10^{-10} \mu_B$. As a numerical example, assume $f = 10^{-2}$, f' = 0.3, $m_1 = 50$ GeV $\ll m_2$, and $\sin 2\theta = 0.3$, in which case $\mu_{v_e v_\mu} \approx 0.5 \times 10^{-10} \mu_B$.

In the presence of *D*-symmetry-breaking terms [Eq. (5)], the $\tilde{e}\tilde{e}^c$ and $\tilde{\mu}\tilde{\mu}^c$ mass matrices are no longer equal. The cancellation in the mass of the neutrino between Figs. 1(a) and 1(b) now becomes incomplete. Parame-



FIG. 2. Dominant diagram for $e-\mu$ mass splitting with the exchange of the gaugino.

trizing the mass-squared eigenvalues and the mixing angle in the $\tilde{\mu}\tilde{\mu}^c$ sector by $(m_1^2 + \delta m_1^2, m_2^2 + \delta m_2^2, \theta + \delta \theta)$ and working to the lowest order in $\delta m_{1,2}^2$, the nonzero neutrino mass generated is found to be¹²

$$m_{\nu_{e}\nu_{\mu}} = \frac{ff'}{16\pi^2} m_{\tau} \sin 2\theta \left[\frac{\delta m_2^2}{m_2^2} - \frac{\delta m_1^2}{m_1^2} + \left(\frac{\delta m_2^2 - \delta m_1^2}{m_2^2 - m_1^2} \right) \ln \left(\frac{m_1^2}{m_2^2} \right) \right].$$
(8)

Demanding that $\mu_{v_e v_\mu} \ge 10^{-11} \mu_B$ and $m_{v_e v_\mu} \le 10$ eV requires the quantity in the square brackets of Eq. (8) to be less than 5×10^{-3} (for $m_1 \approx 50$ GeV). We now show that realistic $e -\mu$ mass splitting can be generated consistent with this constraint.

The dominant contribution to the $e-\mu$ mass splitting arises from the diagram of Fig. 2 involving the exchange of the gaugino. This contribution is given by

$$m_{e} - m_{\mu} = \frac{\tilde{g}^{2}}{32\pi^{2}} m_{\tilde{Z}} \sin 2\theta \left\{ \frac{\delta m_{1}^{2}}{m_{1}^{2} - m_{\tilde{Z}}^{2}} \left[\frac{m_{\tilde{Z}}^{2}}{m_{1}^{2} - m_{\tilde{Z}}^{2}} \ln \left(\frac{m_{1}^{2}}{m_{\tilde{Z}}^{2}} \right) - 1 \right] - \frac{\delta m_{\tilde{Z}}^{2}}{m_{\tilde{Z}}^{2} - m_{\tilde{Z}}^{2}} \left[\frac{m_{\tilde{Z}}^{2}}{m_{\tilde{Z}}^{2} - m_{\tilde{Z}}^{2}} \ln \left(\frac{m_{\tilde{Z}}^{2}}{m_{\tilde{Z}}^{2}} \right) - 1 \right] \right. \\ \left. + \left(\frac{\delta m_{\tilde{Z}}^{2} - \delta m_{1}^{2}}{m_{\tilde{Z}}^{2} - m_{1}^{2}} \right) \left[\frac{m_{1}^{2} \ln(m_{1}^{2}/m_{\tilde{Z}}^{2})}{m_{1}^{2} - m_{\tilde{Z}}^{2}} - \frac{m_{\tilde{Z}}^{2} \ln(m_{\tilde{Z}}^{2}/m_{\tilde{Z}}^{2})}{m_{\tilde{Z}}^{2} - m_{\tilde{Z}}^{2}} \right] \right\}.$$

$$(9)$$

Here \tilde{g}^2 denotes the coupling of the gaugino to the leptons. Because of the mixing in the neutral gaugino-Higgsino sector, this coupling is not completely specified. We shall assume \tilde{g} to be of the same order as the weak gauge coupling. Note that the quantity in the curly braces of Eq. (9) is not quite the same as the one constrained to be less than 5×10^{-3} in Eq. (8). Even if we demand that they be of the same order, we see that realistic $e -\mu$ mass splitting can be generated provided $m_{\tilde{Z}} \approx 1-10$ TeV.

Once the $e-\mu$ mass splitting occurs, the neutrino mass receives a new contribution from the diagrams of Fig. 1 with the internal particles replaced by their respective superpartners. This contribution is proportional to $m_e - m_\mu$ and therefore vanishes in the *D*-symmetric limit. The contribution is given by

$$\delta m_{\nu_e \nu_\mu} \approx (f f' / 16 \pi^2) \sin 2\gamma (m_e - m_\mu) ,$$

where $\sin \gamma$ parametrizes the mixing of $\tilde{\tau}$ and $\tilde{\tau}^c$. This can easily be less than 10 eV, as it involves a new mixing angle γ which we choose to be much smaller than the mixing angle θ . (This will naturally be the case if, for

example, $\tilde{\tau}$ is a few times heavier than \tilde{e} .)

Phenomenological implications.—(i) Since τ lepton number is not conserved, v_{τ} receives a tree-level contribution to its mass. We have chosen the m_{H}' term of Eq. (3) to be small so that $m_{v_{\tau}} \leq 100$ eV is satisfied. At the one-loop level, there are new contributions to v_{τ} mass via diagrams such as in Fig. 3. This can be estimated to be

$$m_{v_{\tau}} \approx (f^2/16\pi^2) \sin 2\theta m_{\mu} \ln(m_{\mu}^2/m_1^2)$$
.

Demanding this to be less than 100 eV requires (for $\sin 2\theta = 0.3$, $m_1 = 50$ GeV) $f \le 10^{-2}$. There are also



FIG. 3. One-loop graph generating v_{τ} mass. There are also other graphs with intermediate quarks and squarks.

similar diagrams involving intermediate quarks and squarks with analogous constraints on the coupling matrix h'_d .

(ii) Because of $l_e - l_{\mu}$ symmetry present in the model, rare processes such as $\mu \rightarrow 3e$, $\mu \rightarrow e\gamma$, $\mu^+e^- \rightarrow \mu^-e^+$, $\tau \rightarrow \mu e\bar{e}$, $\tau \rightarrow 3e$, $K_L \rightarrow \mu\bar{e}$, $K \rightarrow \pi\mu\bar{e}$, etc., are all forbidden. Processes such as $K^- \rightarrow \mu^- v_e$, $e^- v_{\mu}$, $\pi^- \mu^- v_e$, and $\pi^0 e^- v_{\mu}$ arise in this model due to the presence of h'_d , f', and $\tilde{\tau}\tilde{\tau}^c$ mixing, but they occur with a negligible branching ratio since the mixing angle γ and the coupling h'_d are required to be small.

(iii) The photino, which is expected to be the lightest supersymmetric particle, is unstable in our scheme due to explicit R-parity-violating interactions. The dominant decay modes are $\tilde{\gamma} \rightarrow \mu^- \tau^+ v_e$, $e^- \tau^+ v_\mu$, $\mu^+ \tau^- \bar{v}_e$, and $e^+\tau^-\bar{v}_{\mu}$. The hadronic decay modes are suppressed since the coupling h'_d is constrained to be small from the bound on v_{τ} mass. There is no such constraint on the coupling f' of Eq. (3) from v_{τ} mass which implies that the above decay modes dominate. In fact, for the $\mu_{v_e v_\mu}$ magnetic moment to be large, we need $f' \ge 10^{-1}$ or so. The lifetime of $\tilde{\gamma}$ is expected to be around (10^{-22}) sec) $[m_z/(100 \text{ GeV})]^5$. The branching fraction for each of the leptonic mode is about 25%. In the e^+e^- annihilation, the decay of $\tilde{\gamma}$ will lead to multilepton final states of the type $e^+e^-\tau^+\tau^-$, $e^{\pm}\tau^{\pm}e^{\mp}\tau^{\pm}$, $\mu^+\mu^-\tau^+\tau^-$, $\mu^{\mp}\tau^{\pm}\mu^{\mp}\tau^{\pm}$, $e^{\mp}\tau^{\pm}\mu^{\mp}\tau^{\pm}$, and $e^{\mp}\tau^{\pm}\mu^{\pm}\tau^{\mp}$ with missing energy corresponding to the neutrinos. The events with same-sign dilepton (ee or $\mu\mu$) as well as the ones with $e\mu$ will provide spectacular signatures with very little background from the standard-model physics.

(iv) The exchange of $\tilde{\tau}^c$ provides a new contribution to μ decay. The amplitude for this process has the traditional V-A form, which means that only the decay rate is affected. Consistency with universality implies $f'^2/m_{\tilde{\tau}^c}^2 \leq 0.04G_F$, which is easily satisfied if $m_{\tilde{\tau}^c} \geq 500$ GeV or so.

To conclude, a transition magnetic moment $\mu_{v_e v_\mu}$ of order $(0.3-1) \times 10^{-10} \mu_B$ required to resolve the solarneutrino puzzle can be generated in the minimal supersymmetric extension of the standard model. $l_e - l_{\mu}$ symmetry and the horizontal discrete *D* symmetry operating on the *e*- μ families guarantee that the neutrino mass is naturally small. Since the discrete symmetry *D* is broken softly in the Lagrangian, the model has no cosmological domain-wall problem. Observation of the photino decay into three-lepton final states will be a crucial test of the model in the laboratory. This work was supported by a grant from the National Science Foundation.

Note added.— After the completion of this work, we received a preprint by Ecker, Grimus, and Neufeld¹³ which has used a different discrete symmetry to generate a large transition magnetic moment of the neutrino in a multi-Higgs-boson extension of the standard model with new fermions. We have also been informed of a work by Chang, Keung, and Senjanovic¹⁴ which obtains similar results.

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¹²Note that $\delta\theta$ is not an independent parameter; it is fixed in terms of θ , $m_{1,2}^2$, and $\delta m_{1,2}^2$. The last term in Eq. (9) is the contribution from $\delta\theta$.

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