## Resistive Behavior of High- $T_c$ Superconductors: Influence of a Distribution of Activation Energies

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A model combining thermally activated motion of flux lines, viscous flux flow at high current density, and a distribution of activation energies is shown to reproduce the characteristic features of currentvoltage curves of high- $T_c$  superconductors. In particular, the recent data of Koch *et al.* and Zeldov *et al.* can be explained without invoking a continuous phase transition (freezing into a superconducting vortex-glass phase) or a logarithmic current dependence of the activation energy.

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Since the discovery of high- $T_c$  superconductivity a wealth of information has been gathered about the resistive transition near  $T_c$  in applied magnetic fields. In early experiments the R(T) curves showed a smooth, structureless transition from a normal-state value above  $T_c$  to essentially zero at lower temperatures. Tinkham<sup>1</sup> first succeeded in describing qualitatively the broadening of R(T) curves in a magnetic field.

However, as the quality of single crystals and thin films improved a knee became apparent in the R(T)curves in high magnetic fields. This characteristic feature has recently been interpreted by Hebard and Palstra,<sup>2</sup> Batlogg *et al.*,<sup>3</sup> and Malozemoff *et al.*<sup>4</sup> as a crossover from thermally activated flux creep at low temperatures to viscous flux flow just below  $T_c$ .

To gain more information on the resistive behavior of high- $T_c$  superconductors, Koch *et al.*<sup>5</sup> and Zeldov *et al.*<sup>6</sup> investigated in detail the temperature dependence of the *I-V* curves, respectively, the  $\rho$ -*I* curves, of laser-ablated epitaxial films. For a given magnetic field Koch et al. found the I-V isotherms to exhibit power-law behavior at a given temperature,  $T_g$ , which is identified as a secondorder phase transition between a normal and a true superconductor (with resistivity identically zero). Further, the value of the critical exponents determined from the experimental data are indicative of a transition into a vortex-glass superconductor. On the same type of epitaxial films Zeldov et al.<sup>6</sup> observed a power-law behavior over three decades in resistivity versus current-density curves at temperatures between 77 and 81 K and current densities between  $10^8$  and  $10^{10}$  A/m<sup>2</sup>. These data are shown by Zeldov et al. to be consistent with a thermally activated flux-creep model with a current dependent activation energy  $U(T,H,j) = U(T,H) \ln(j/j_0)$ , where  $j_0$ is the current for which U approaches zero. The parameter  $j_0$  is both field and temperature dependent. In this work we show that both assumptions, (i) the existence of a continuous phase transition to a glassy superconductor and (ii) a logarithmic current dependence of the activation energy, are unnecessary if one takes into account that even good quality epitaxial films of YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> are

probably still fairly disordered systems.

In the spirit of our previous model<sup>7,8</sup> on the temperature dependence of giant flux creep in polycrystalline and single-crystalline YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>, we present here a model which takes into account a distribution  $m(U^*)$  of activation energies  $U^*$  for thermally activated flux motion<sup>9</sup> and incorporates a continuous crossover to a flux-flow regime at high current densities. As will be shown in a subsequent paper<sup>10</sup> it also leads to a knee in the R(T)curves in high magnetic fields.

To illustrate the model we consider the situation where a sample consists of an ensemble of domains in parallel. Each domain is characterized by a temperature- and field-dependent activation energy U(T,B) which separates adjacent pinning regions in a given domain. In the flux-creep regime the drift velocity of flux lines is given by the usual expression<sup>11</sup>

$$v_c = v_0 e^{-U(T,B)/kT} \sinh(Aj/kT), \qquad (1)$$

where  $v_0$  is a velocity prefactor related to the attempt frequency for flux-line hopping, *j* is the local current density, and *Aj* is the change in the energy of a flux line associated with the Lorentz force acting on a vortex.<sup>12</sup>

In the flux-flow regime the velocity of flux lines is limited by a viscous drag. In the simple Stephen-Bardeen<sup>13</sup> model it is given by

$$v_f = \frac{\rho_n}{B_{c2}(T)} j, \qquad (2)$$

where  $\rho_n$  is the normal-state resistivity and  $B_{c2}(T)$  the upper critical field at temperature T, i.e.,  $B_{c2}(T) = B_{c2}(0)[1 - (T/T_c)^2]$ . One can go smoothly from the weak-driving-force (flux-creep) regime to the strongdriving-force (flux-flow) regime by assuming that the motion of a flux line is governed by flux creep [Eq. (1)] over a distance  $L_c$  and by flux flow over a distance  $L_f$ . The average velocity  $\langle v \rangle$  is then given by

$$\langle v \rangle = \frac{L_c + L_f}{L_c / v_c + L_f / v_f} \,. \tag{3}$$

Near  $T_c$  in zero field or, in fields near  $B_{c2}(T)$ , the sys-

tem is in the flow regime as the activation energy U vanishes when  $B = B_{c2}(T)$ . Consequently  $L_c(B_{c2}(T)) = 0$ . For simplicity we assume that  $L_c \propto B_{c2}(T) - B$  with  $L_c \ll L_f$  and by using Eqs. (1) and (2) we obtain finally that the electric field E set up by the motion of the flux lines in a magnetic field B is

$$E = \left\{ S \exp\left(\frac{U}{kT}\right) \left[ \sinh\left(\frac{Aj}{kT}\right) \right]^{-1} + \frac{B_{c2}}{Bj\rho_n} \right\}^{-1}, \quad (4)$$

with  $S \equiv (L_c/L_f)v_0^{-1}B^{-1} = S_0(1 - B/B_{c2})$ .

Until now we have only considered flux motion within a single domain. A sample consisting of a large number of pinning regions with different activation energies can be characterized by a distribution function<sup>7,8</sup>  $m(U^*)$ which is defined in such a way that  $m(U^*)dU^*$  is the fraction of domains in the sample with pinning energies between  $U^*$  and  $U^* + dU^*$  at T=0 and B=0. The actual activation energy U(T,B) for a given domain at temperature T in a field B is written as

$$U(T,B) = U^* b(T) [1 - B/B_{c2}(T)]$$
(5)

if one assumes that U(T,B) vanishes linearly with *B*. As discussed in Refs. 7 and 8,  $b(T)=1-(T/T_c)^4$  gives an excellent fit to the magnetization-relaxation experiments carried out by Yeshurun and Malozemoff.<sup>14</sup> The Lorentz-force term, on the other hand, was found to be temperature independent. In this work we also take *A* independent of the domain under consideration. Then the largest current compatible with flux creep in the absence of thermal fluctuation is U(T,B)/A. Domains with large pinning energies have thus also large critical currents. As all domains are in parallel, the electric field and consequently the drift velocity of flux lines are uniform over the whole sample. The average current densi-



FIG. 1. Electric field vs current-density isotherms for T=84.9 to 89.9 K at 0.5-K intervals. The magnetic field is 0.5 T. The values of the parameters entering the calculation are  $U_0^*=120$  meV,  $\gamma=1.4$ ,  $A=3\times10^{-10}$  meV/Am<sup>-2</sup>, and  $S_0=4\times10^{-7}$  m/V. For comparison with experimental data see Koch *et al.* (Ref. 5).

ty is given by

$$\langle j \rangle = \int_{U_{\min}^{*}}^{U_{\max}^{*}} j(U^{*}, E, T, B) m(U^{*}) dU^{*},$$
 (6)

where  $j(U^*, E, T, B)$  is the current density in a domain with activation energy  $U^*$  (at T=0 and B=0) obtained by solving Eq. (4) for j. For the distribution  $m(U^*)$  we take a log-normal function<sup>7</sup>

$$m(U^*) = \left(\frac{\gamma}{\pi}\right)^{1/2} \frac{e^{-1/4\gamma}}{U_0^*} \exp\left\{-\gamma \left[\ln\left(\frac{U^*}{U_0^*}\right)\right]^2\right\}$$
(7)

as this type of function resembles strongly the distribution derived from giant-flux-creep experiments between 4 and ~80 K by means of the Hagen-Griessen inversion scheme.<sup>7</sup> The integration limits in Eq. (6) are determined by the requirement that pinning regions cannot be arbitrarily small. To be physically meaningful, in our model, the size of a pinning region must be larger than the coherence length. This implies that for a filmstrip of typically 20- $\mu$ m width (as used by Koch *et al.* and Zeldov *et al.*) and  $\xi \approx 5$  Å,  $m(U^*) \gtrsim 10^{-5}$ . In Eq. (6),  $U_{\min}^*$  and  $U_{\max}^*$  are determined by the condition  $m(U_{\min}^*) = m(U_{\max}^*) = 10^{-5}$ .

We have now all the ingredients to calculate currentvoltage curves or resistivity-current curves  $(\rho = E/\langle j \rangle)$ . Of the four parameters  $U_0$ ,  $\gamma$ , A, and  $S_0$  which enter the calculation, only  $S_0$  is difficult to estimate beforehand. For the other parameters we expect from our previous analysis of flux-creep experiments<sup>6,7</sup> that  $U_0^* = 50$  meV,  $A \approx 10^{-11}$  meV/A m<sup>-2</sup> and  $\gamma$  is of order unity. The corresponding value of the critical current at 0 K is then  $j_c(0) \approx 5 \times 10^{12}$  A/m<sup>2</sup>. As  $S_0$  is the only true unknown parameter it turned out to be easy to find a set of parameters  $U_0^*$ ,  $\gamma$ , A, and  $S_0$  which reproduces the experimental data of Koch *et al.* and Zeldov *et al.* For  $S_0$  we found that  $S_0 = 4 \times 10^{-7}$  m/V. From the definition of  $S_0 = L_c/L_f v_0 B$ , and typical values  $(B=1 \text{ T}, v_0 = 10^3)$ 



FIG. 2. Same as Fig. 1 but for an applied field of 4 T. Here  $U_0^* = 38$  meV.



FIG. 3. Resistivity vs current-density isotherms in a magnetic field of 4 T for T=77 K up to 89 K (with 1-K interval). Here  $U_0^* = 60$  meV.

m/s), it follows that  $L_c/L_f \approx 10^{-4}$  which is consistent with the approximation made in deriving Eq. (4). For  $B_{c2}(0)$  we took 60 T, in agreement with recent high-field results.<sup>15</sup>

The *E*-*j* curves in Fig. 1 for an applied field of 0.5 T exhibit the same trend as the experimental data, being steep at low temperature and having a slope equal to 1 just below  $T_c$ , which we have taken to be 91.5 K. Around 85 K the log*E*-vs-log*j* curves are virtually linear over five decades in voltage although no power-law expression has been assumed in the present model. A fit of equal quality has also been found for an applied field of 4 T with the same parameters  $\gamma$ , *A*, and  $S_0$  (see Fig. 2). Only  $U_0^*$  had to be decreased from 120 meV at 0.5 T to 38 meV at 4 T. Such a decrease in activation energy is consistent with previous observations.

The  $\rho$ -j curves in Fig. 3 are also in good agreement with the experimental data.<sup>6</sup> At relatively low current densities  $\rho$  is constant as the argument of the sinh term in Eq. (1) or (4) is smaller than 1. Around 79 K the  $\rho$ vs-j curves exhibit a power-law behavior over almost three decades in  $\rho$ . It is important to stress here that the power-law behavior of both E-*j* curves and  $\rho$ -*j* curves at a certain temperature is not arising from a phase transition or a postulated logarithmic dependence of U on j. Our model incorporates only flux flow, flux creep, and a distribution of activation energies. It is interesting to mention that the exact shape of  $m(U^*)$  is not critical for the quality of the fits in Figs. 1-3. To illustrate this point we indicate in Fig. 4 the contribution of the various regions of the sample to the total current  $\langle j \rangle$ . From Eq. (6) this contribution is just  $j(U^*, E, T, B)m(U^*)$ . As expected, at temperatures not far from  $T_c$  the main contribution to the current is due to a relatively small fraction of domains with large  $U^*$ . In this temperature regime the exact shape of  $m(U^*)$  for  $*U \lesssim 100$  meV is



FIG. 4. (a) Distribution  $m(U^*)$  of activation energies  $U^*$  at T=0 and B=0 corresponding to the calculated data in Fig. 1. (b) Contribution of domains with activation energy  $U^*$  to the total current  $\langle j \rangle$  for various temperatures and an electric field E=0.001 V/m.

thus not relevant. Of course with decreasing temperature more and more domains contribute to  $\langle j \rangle$ . At T=0, to a very good approximation

$$\langle j \rangle = \frac{1}{A} \int_{U_{\min}^*}^{U_{\max}^*} U^* m(U^*) dU^* \cong \frac{U_0^*}{A} \exp\left(\frac{3}{4\gamma}\right). \quad (8)$$

Finally, as will be shown elsewhere,<sup>10</sup> the calculated resistivity R(T) exhibits a knee as has recently been reported by Malozemoff *et al.*<sup>4</sup> and Batlogg *et al.*<sup>3</sup> among others.

In conclusion, we have shown that a model with (i) a smooth transition from a flux-creep regime to a flux-flow regime and (ii) a distribution of energies for thermally activated flux motion reproduces the main features of the resistive investigations of Koch *et al.* and Zeldov *et al.*, including a power-law behavior over several decades.

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we use  $U^*$  to avoid confusion with the electric field E.

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