Magnetic Domain Patterns as Self-Organizing Critical Systems

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In practice, large specimens of ferromagnetic materials settle into one of a large number of metastable states, not necessarily into the energetically lowest state. We suggest that they tend to select marginally stable states, in a manner similar to the process recently proposed by Bak, Tang, and Wiesenfeld. We apply this idea to the formation of the so-called zigzag walls that separate oppositely magnetized domains in a magnetic recording tape.

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A few years ago, Bak, Tang, and Wiesenfeld[1,2] (BTW) proposed that complex dissipative physical systems starting from some unstable state, will not, in general, tend towards the state of lowest energy. Instead, they will organize themselves into a metastable state that is "only just" stable. Possible realizations of these principles by actual physical systems have been proposed, among them the distribution of earthquakes,[3] and, as a demonstration of the original proposal, the shape of sandpiles.

In support of their ideas, the authors of Refs. 1 and 2 cite an idealized model of a sandpile with grains in the form of cubes so that the sides of the pile form a stepped pattern. Perhaps the most primitive, one-dimensional version of their model would be the following: If the vertical height of a step exceeds that of one block, the excess block is assumed to tumble down. States with steps of width exceeding that of one block, would appear to be more stable; however, if "sand" is added to the pile, that excess width will be reduced until the width is exactly one. Deviations from that minimally stable, or "critical," state will return to a critical state in a certain universal manner.

Problems in micromagnetics of ferromagnetic specimen are usually attacked either by mathematical analysis of the magnetization field, by large-scale computation, or by a combination of the two. As far as pure analysis is concerned, the solutions, where they can be found, yield configurations corresponding to either the ground state or the simplest metastable states. The statistical element in the pattern selection of realistic specimens is absent in such analyses. A statistical element can, of course, be supplied in a computational treatment, but it is not easy to obtain qualitative insight in this way. Here we propose that relatively trivial symbolic dynamics, in the spirit of the BTW proposal, can adequately mimic a real situation. We chose as a prime example the case of a so-called "head-on" domain wall such as is found in a recording tape. The wall separates two regions, magnetized equally and oppositely along the length of the tape. The tape may be either a continuous film with easy magnetization direction along the tape, or it may consist of elongated magnetic particles, with long axes aligned parallel to the tape. For simplicity consider first an unphysical "tape" in the form of a thick slab. Continuity of the magnetic flux b requires that the field $h_{1,2}$ on the two sides obey $h_1 - 4\pi m = h_2 + 4\pi m$, so that $h_1 - h_2 = 8\pi m$, where m, in the case of the continuous film, is the saturation magnetization M, and, in the particulate case, is fM, where f (<1) is the volume fraction occupied by the particles. We can choose $h_2 = -h_1 = h = 4\pi m$, and note that on each side, the field h opposes the magnetization direction. In the case of the continuous medium, the straight perpendicular wall is stable only if $\kappa M > 4\pi M$, where κM is the anisotropy field of the material. In the particulate case, the effective κ equals N_T, the transverse demagnetizing factor of the elongated particles, and the stability condition for the straight vertical wall is then $N_T > 4\pi f$. (Of course, f must not be so close to unity that exchange forces between neighboring particles become effective.)

For long cylinders N_T is 2π (the case assumed from hereon), and so the straight wall is unstable for fill factors between 0.5 and 1. On the other hand, neglecting domain rotation (see remarks at end), a straight wall, inclined at angle $\theta_{crit} = \arcsin(2f)^{-1}$ to the tape direction, will be marginally stable for $0.5 < f < 1$. However, the *lowest*-energy state is obviously one with $\theta = 0$, at least in the particulate case where exchange is absent. For the continuous film, with exchange neglected, and $\kappa < 4\pi$, we have $\theta_{crit} = \arcsin(\kappa/4\pi)$, but the lowest state still has $\theta = 0$. When exchange is taken into account in some crude form (for example, exchange energy directly proportional to the total wall area, an approximation valid if the general scale of the zigzag is much larger than a domain-wall thickness), the lowest-energy state has a *finite* θ, the tape being much longer than it is wide. However, θ_{crit}, in that case, too, is larger than that finite θ. In the actual physical tape, the discontinuity is a line charge, and the magnetic fields are perpendicular to the line, drop off like 1/distance from the line, and, as in the case of the slab, oppose the magnetization direction everywhere. Hence the same reasoning as for the slab continues to apply. Once again neglecting the domain rotation, the critical angle is now approximately $\theta_{\rm crit} = \arcsin(a/2f\Delta)$, where a is of the order of the interparticle spacing, and Δ is the thickness of the film. For the continuous thin film, $1/2f$ is replaced by $\kappa/4\pi$, and a is of the order of a domain-wall thickness. This problem has in the past been considered in a kind of energy-balance model by Freiser.[4,5]

Suppose that a perfect recording head initially imposes a straight vertical wall, and conditions are such that this wall is unstable. The wall will collapse, but will not reach the lowest-energy state. Instead, it settles into a zigzag pattern, where the predominant slope of the teeth is $\tan(\theta_{\rm crit})$. First, consider this problem on the basis of traditional micromagnetics of a continuous film. Let $H \equiv \nabla\psi$ denote the magnetic field, and m the magnetization (constrained to the x-y plane of the tape). Then, since the divergence of the total flux vanishes,

$$\nabla^2\psi = -4\pi(\partial m_x/\partial x + \partial m_y/\partial y) . \qquad (1)$$

Let $E_A = -\frac{1}{2}(\kappa_1 m_x^2 + \kappa_2 m_y^2)$ be the anisotropy energy per unit volume. Then, the effective field acting on m has components $\mathcal{H}_x = H_x + \kappa_1 m_x$, $\mathcal{H}_y = H_y + \kappa_2 m_y$. For stability, m must point along $\mathcal{H}$, and since the saturation magnetization M of the m must be constant, we have (with $\mathcal{H} = |H|$)

$$m_x = M\mathcal{H}_x/\mathcal{H} \text{ and } m_y = M\mathcal{H}_y/\mathcal{H} . \qquad (2)$$

It is easily verified that Eqs. (1) and (2) are the Euler-Lagrange equations obtained by extremizing the expres-

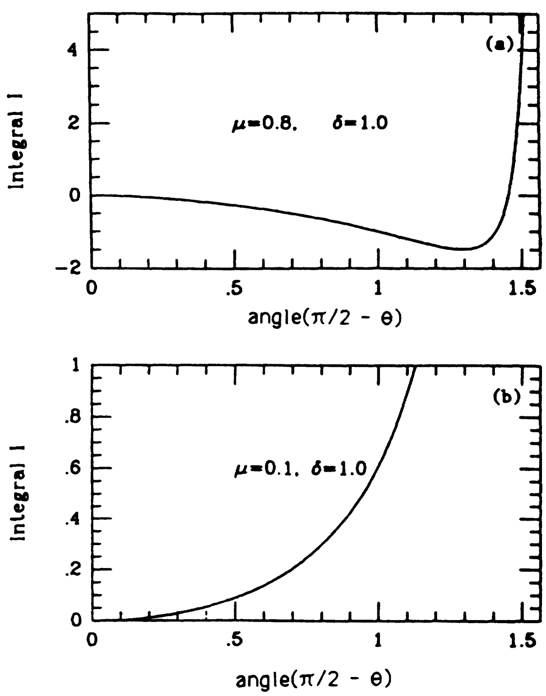

FIG. 1. Integral I as a function of angle θ for different μ and δ, where $\mu = \xi/(J\kappa_1)^{1/2}$, $\delta = 2\Delta/\xi$, and ξ is the period of the zigzag wall. (a) Here the parameters give minimum I when θ is about 1.3 (J and/or κ small). (b) For these parameters a straight wall at right angles to the tape occurs (J and/or κ large). μ and δ are dimensionless. In our units M^2, $\kappa_1 M^2$, and $J(\nabla M)^2$ are energies per unit volume. Thus J has the dimensions of (length)^2.

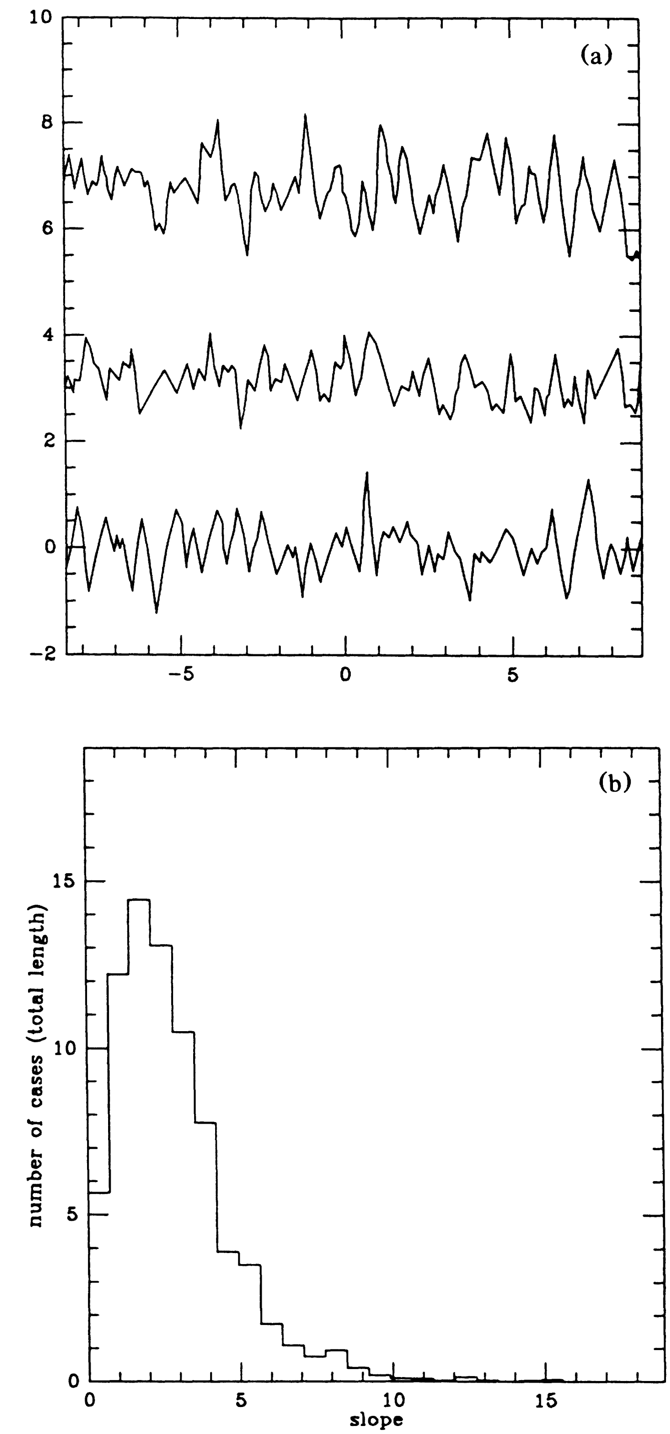

FIG. 2. (a) Experimental zigzag wall (taken from Ref. 7). (b) Histogram of slopes, which is obtained by digitizing (a).

sion (with Δ the film thickness)

$$\int \tfrac{1}{2}(\nabla\psi)^2 dx\,dy\,dz + \int [4\pi\mathcal{H}M - 2\pi E_A]\,dx\,dy\,\Delta\,, \qquad (3)$$

with respect to ψ and m. Further, for sufficiently coarse zigzag patterns, exchange may be accounted for by adding $(J\kappa_1)^{1/2}M^2\Delta L$ to Eq. (3), where L is the total length of wall, and extremizing the sum. Then, at the end of the calculation, κ_2 is allowed to go to zero. A zigzag trial solution, with angle and wavelength of that shape as Ritz parameters, gives the results shown in Fig. 1. It is seen that the smaller J, the more nearly both θ and the wavelength approach zero, as expected.

Figure 2(a) shows experimental zigzag wall sections of a continuous-film tape,[6,7] taken from Ref. 7, and Fig. 2(b) is the histogram of slopes obtained by digitizing Fig. 2(a). To arrive at this shape by purely variational means would require the use of large numbers of Ritz parameters to determine the many possible shapes that render the integral (3), plus the exchange term, stationary and locally stable.

Instead, to produce the "noisy" zigzag observed, we use the following algorithm designed to move the pattern from an unstable ($\kappa < 4\pi$ or $f > 0.5$) straight vertical wall to a marginally stable configuration.

We divide the vertical wall into a large number of small segments, and refer to the magnetic moment on each side of such a segment as a "spin." Starting with the unstable wall, flip one of each pair of opposing spins whose heads meet at the wall, choosing the one on the right or on the left totally randomly. The wall has now assumed a random zigzag shape denoted by $y = y(n,1)$, where $y(n,1)$ is the new coordinate of the head of the nth spin, measured from its original position along the wall as zero. Next, process the pair of spins whose heads meet at the point $(n, y(n,1))$ according to the following rule: Let

$$s(n,1) = \frac{|y(n+1,1) - y(n,1)|^2}{1 + |y(n+1,1) - y(n,1)|^2} + \frac{|y(n-1,1) - y(n,1)|^2}{1 + |y(n-1,1) - y(n,1)|^2}\,. \qquad (4)$$

If $s(n,1) < s_{\text{crit}}$, then flip one of them, which of the two being decided at random. The new domain wall is denoted by $y(n,2)$. Next, update the system using the same rule. At the $(m+1)$st update, flip one of the spins whose heads meet at $(n, y(n,m))$ if $s(n,m) > s_{\text{crit}}$; otherwise leave that pair alone. Eventually, the pattern attains a shape that changes no further. The quantity $p = s_{\text{crit}} - s(n,m)$ is a crude imitation of the net field due to the magnetic surface charge, minus the anisotropy field. Flipping occurs only if $p > 0$. A more refined algorithm would employ for s a weighted average slope

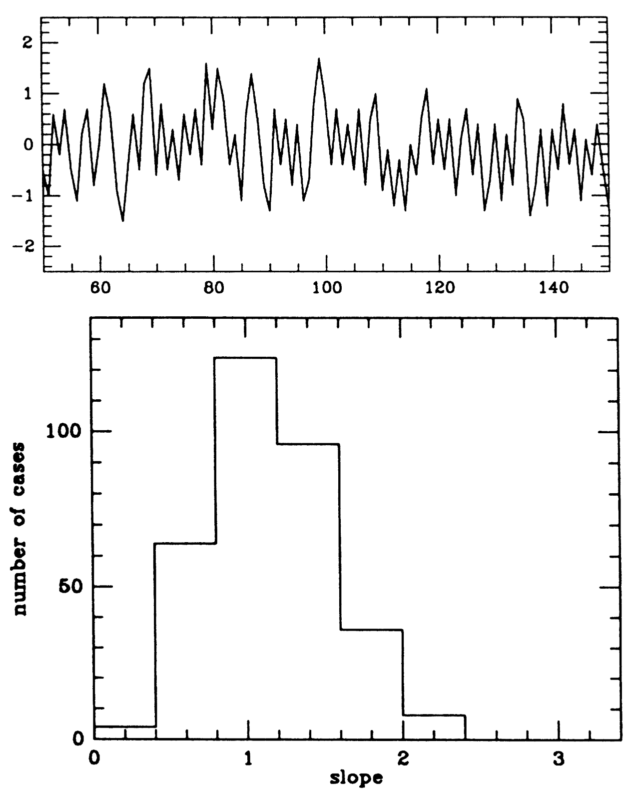

FIG. 3. Wall pattern and histogram of slopes when $s_{\text{crit}} = 1.2$ based on model without exchange.

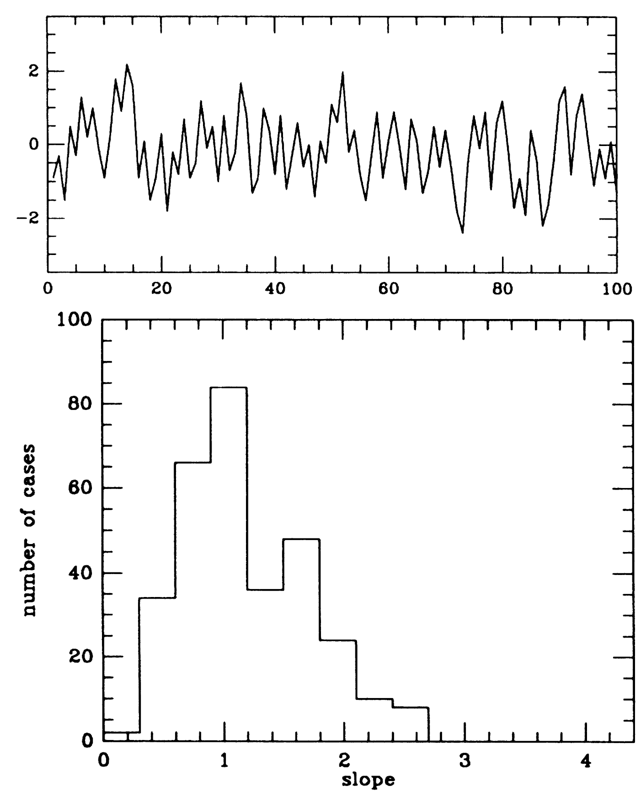

FIG. 4. Wall pattern and histogram of slopes when $s_{\text{crit}} = 1.2$ based on model with exchange taken into account.

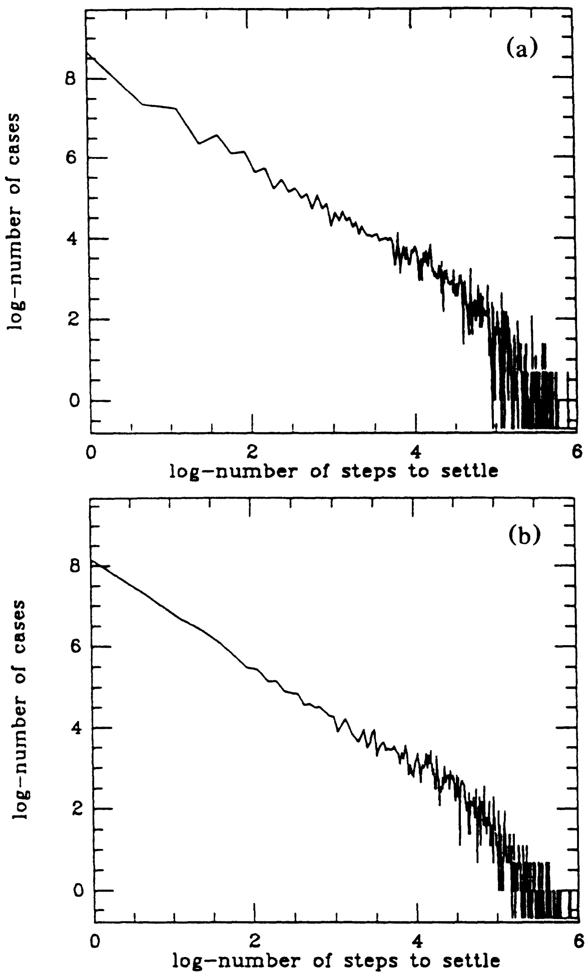

FIG. 5. Plot of "response" of settled pattern to a small perturbation. (a) and (b) correspond to models with 20 and 100 cells, respectively.

over greater length of the pattern.

Figure 3 shows the settled pattern using our algorithm for the case $s_{\rm crit}=1.2$, and the corresponding histogram of the slope p, for zero exchange. Figure 4 shows these results with exchange incorporated in the algorithm in the form of diminished probability of flips that increase the total wall length.

We also Fourier analyzed both the experimental and "theoretical" patterns, and found a white spectrum in either case [of course, only up to wave numbers equal to 1/(lattice spacing)].

Figure 5 shows the "response" to a local deviation from a settled pattern. For all different initial patterns examined, the log-log plot of the number of cases needing a given number of steps to resettle versus that number of steps is a straight line of slope approximately 1.15, whether exchange is included or not. Also, the slope depends only weakly on the nature of the initial deviation. No appreciable change is found between results for 20 and 100 (or even 300) cells. The matter of finite-size scaling is still under study. We have also examined hysteretic effects by modifying the algorithm to allow for a net applied field. These results will be discussed in a future submission. Finally, domain rotation is neglected here, although exact numerical simulations show that it causes the magnetization to deviate somewhat from the uniform head-on configuration assumed here, particularly near the vertices of the zigzag. This will change the form of $\theta_{\rm crit}$, but not our algorithm. We feel that this has little qualitative effect on the results.

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[1]P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).

[2]P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. A **38**, 364 (1988).

[3]J. M. Carlson and J. S. Langer, Phys. Rev. Lett. **62**, 2632 (1989).

[4]M. J. Freiser, IBM J. Res. Dev. **23**, 330 (1979).

[5]E. Köster, in *Magnetic Recording*, edited by C. D. Mee and E. D. Daniel (McGraw-Hill, New York, 1987), Vol. 1, p. 180.

[6]H. C. Tong, R. Ferrier, P. Chang, J. Tzeng, and K. L. Parker, IEEE Trans. Magn. **20**, 1831 (1984).

[7]T. C. Arnoldussen and H. C. Tong, IEEE Trans. Magn. **22**, 889 (1986).