Natural-Parity States in Superdeformed Bands and Pseudo SU(3) Symmetry at Extreme Conditions

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The structure of recently discovered identical superdeformed bands in ^{151}Tb and ^{152}Dy and in ^{150}Gd and ^{151}Tb are discussed in terms of the strong-coupling approach. Based on the experimental evidence that the superdeformed core of ^{152}Dy is extremely insensitive to the polarization effects induced by the odd particle, the bands are shown to exhibit the presence of the pseudo SU(3) symmetry at extreme conditions of large elongations and high spins.

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The spectroscopy of superdeformed (SD) states is a sensitive test of the concept of the nuclear mean field. The increasing availability of high-spin data on SD states[1] (about fourteen SD decay sequences are known in the $A \approx 150$ mass region) makes possible both tests of the detailed shell structure at large elongations and studies of exotic orbitals for different particle numbers and rotational frequencies. The SD bands observed so far are by no means identical. Indeed, they show a rich variety of moments of inertia as a function of particle number and angular momentum characteristic[2,3] of the occupation of intruder orbitals originating from high-N oscillator shells. Therefore, the recent discovery[4] of *identical* γ-ray sequences both in ^{151}Tb and ^{152}Dy and in ^{150}Gd and ^{151}Tb is completely unexpected. This Letter discusses some consequences of these striking experimental results.

In our analysis we employ the simple approximation in which the independent-particle motion of one or more valence particles (with angular momentum **j**) is coupled to a rotating deformed core (with angular momentum **R** and moment of inertia $\mathcal{J}$) forming the total angular momentum $\mathbf{I}=\mathbf{R}+\mathbf{j}$. If the coupling of the odd particle to the core is much stronger than the perturbation of the single-particle motion by the Coriolis interaction, the odd particle will follow the core deformation adiabatically. This strong-coupling limit is expected to work particularly well for superdeformed nuclei where the splitting of the Nilsson levels ($\propto \beta_2$) is large and the Coriolis interaction ($\propto \hbar^2/2\mathcal{J}$) is small. For axial symmetry the γ-ray energy for $\Delta I=2$ in-band transitions, $E_\gamma(I)=E(I)-E(I-2)$, can be obtained from the eigenenergies of the particle-rotor Hamiltonian.[5] In first-order perturbation theory,

$$E_\gamma(I)=(\hbar^2/\mathcal{J})[2I-1+(-1)^{I+1/2}a\delta_{K,1/2}], \qquad (1)$$

where **K** is the projection of **j** onto the symmetry axis and a is the decoupling parameter, can be calculated from the intrinsic wave function of the valence particles.[5] It is instructive to consider Eq. (1) for three limiting values of a.

(i) $a=0$ (e.g., when $K>\frac{1}{2}$).—The signature splitting disappears and transition energies follow the simple rule

$$0.5[E_\gamma(R+\tfrac{1}{2})+E_\gamma(R-\tfrac{1}{2})]=E_\gamma^{\text{core}}(R), \quad R=2,4,\ldots .$$

(ii) $a=1$.—The sequences in the odd-A nucleus with favored ($r=-i$, $I=\frac{1}{2},\frac{5}{2},\ldots$) and unfavored ($r=+i$, $I=\frac{3}{2},\frac{7}{2},\ldots$) signatures are degenerate, and $E_\gamma(I=|R\pm\frac{1}{2}|)\approx E_\gamma^{\text{core}}(R)$, $R=2,4,\ldots$.

(iii) $a=-1$.—Similar to case (ii), except the $r=+i$ sequence is favored and $r=-i$ is unfavored. The rotational band consists of doublets $(\frac{1}{2},\frac{3}{2}),(\frac{5}{2},\frac{7}{2}),\ldots$ and $E_\gamma(I=|R\pm\frac{1}{2}|)\approx E_\gamma^{\text{core}}(R)$, $R=R+1=1,3,5,\ldots$.

Examples of these limits are known from low-spin data. Many high-K rotational bands with $a=0$ have been established. However, only a few cases are known with $a=1$, the classic example being the negative-parity band in ^{19}F based on a proton hole in the $[101]\frac{1}{2}$ Nilsson state[6] ($j\approx\frac{1}{2}$, $a=1.1$). A typical example[7] of the $a=-1$ limit is the $K=\frac{1}{2}$ band in ^{177}Lu built upon the $[411]\frac{1}{2}$ single-proton state ($a=-0.91$). The deviations from the empirical relations between E_γ(odd) and E_γ^{core} given above for the pure limits are sizable (1%–3%) and are of the order of the $A^{5/3}$ variations expected for a macroscopic rotor. Some part of this deviation can be attributed[5] to reduced pair correlations for the odd-mass nuclei.

Because of the large single-particle SD gaps at $Z=66$ and $N=86$ the nucleus ^{152}Dy is expected to be a very good "doubly magic" SD core. Moreover, the pairing correlations in the SD band of ^{152}Dy are very weak[3] which leads to a "rigidlike" rotational pattern. The recent experimental data on ^{151}Tb (Refs. 4 and 8), ^{151}Dy (Ref. 9), and ^{153}Dy (Ref. 10) allow a test of the stability of this core with respect to the addition of a valence particle (or hole). In the nucleus ^{153}Dy two strongly coupled bands with no signature splitting are known. Calculations[3,10] suggest that they can be built upon the high-Ω $[514]\frac{9}{2}$ orbital (see Fig. 1). The effective core for the

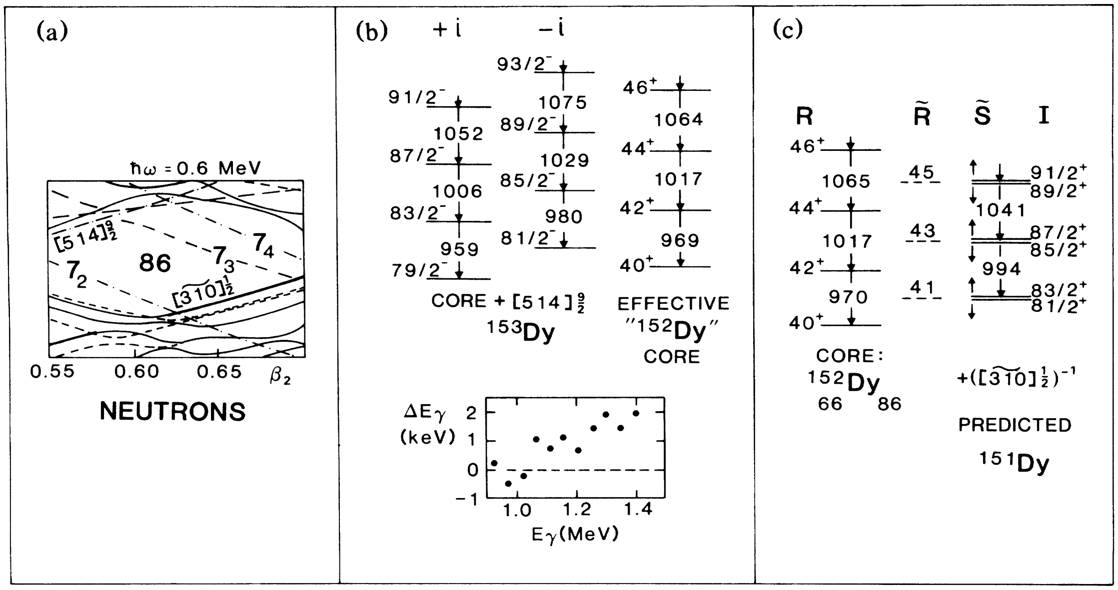

FIG. 1. (a) Calculated single-neutron Routhians at a rotational frequency of 0.6 MeV as functions of β_2 (see Ref. 3 for more details concerning this plot). (b) The effective "^{152}Dy" core transitions, extracted from the $[514]\frac{9}{2}$ bands in ^{153}Dy, and compared to the experimental transition energies in the SD band in ^{152}Dy. (c) The experimental ^{152}Dy core transitions shown together with the predicted $([\tilde{3}\tilde{1}\tilde{0}]\frac{1}{2})^{-1}$ SD band in ^{151}Dy.

$|{}^{152}\mathrm{Dy}\otimes\nu[514]\frac{9}{2}\rangle$ bands can be calculated assuming the coupling scheme (i) discussed above. The resulting γ transitions in the effective "^{152}Dy" core for $42\leq I\leq 46$ are shown in Fig. 1 together with the deviations, ΔE_γ, from the ^{152}Dy spectrum, which are around 1 keV. This shows that the change in the moment of inertia due to adding the oblate-driving $[514]\frac{9}{2}$ neutron to the ^{152}Dy core is $\Delta\mathcal{J}/\mathcal{J}\approx 10^{-3}$.

Because of the large energy splitting between the unique-parity high-j intruder subshells and the natural-parity states, the natural-parity states can be classified according to the pseudo SU(3) representations.[11-13] In the pseudo SU(3) limit the natural-parity Nilsson orbitals form a pseudo oscillator spectrum, labeled by the pseudo asymptotic quantum numbers $\tilde{N}=N-1$, $\tilde{n}_z=n_z$. Moreover, the pairs of Nilsson levels $[Nn_z\Lambda]\,\Omega=\Lambda+\frac{1}{2}$, $[Nn_z\Lambda+2]\,\Omega=\Lambda+\frac{3}{2}$ can be considered as pseudo spin-orbit doublets $[\tilde{N}\tilde{n}_z\tilde{\Lambda}]\,\Omega=\tilde{\Lambda}\pm\frac{1}{2}$ with $\tilde{\Lambda}=\Lambda+1$. In this formalism the single-particle angular momentum can be expressed as the sum of pseudo orbital angular momentum and pseudo spin, $\mathbf{j}=\tilde{\mathbf{l}}+\tilde{\mathbf{s}}$. The pseudo orbital angular momentum of the valence particles is strongly coupled to the angular momentum of the core, forming the total pseudo orbital angular momentum $\tilde{\mathbf{R}}=\mathbf{R}+\tilde{\mathbf{l}}$, and the pseudo spins are then added to form the total angular momentum, $\mathbf{I}=\tilde{\mathbf{R}}+\tilde{\mathbf{s}}$. Consequently, the pseudo Coriolis interaction, involving the pseudo-spin operators rather than the total angular momentum of the odd particles,[13-15] is expected at large rotational frequencies to align the pseudo spin with the total angular momentum.[14] The pseudo SU(3) scheme is expected to remain valid for SD configurations since the high-j intruder orbitals seem to be well separated from the natural-parity states of the same shell.[2,3,16] The decoupling parameter for a $K=\frac{1}{2}$ rotational band has the value $a=(-1)^N\delta_{\Lambda 0}$ in the normal asymptotic limit and $a=(-1)^{\tilde{N}}\delta_{\tilde{\Lambda}0}$ in the pseudo asymptotic limit.[13,14] Experimental and Nilsson-model-calculated[13,14] decoupling parameters are usually close to the pseudo asymptotic limit.

The lowest SD band in ^{151}Tb $(\pi,r)=(+,-i)$ can be associated with a 6_4 hole in the ^{152}Dy core[3] (see Fig. 2). The excited $\pi=-$ band in ^{151}Tb has identical transitions with the ^{152}Dy core,[4] and is associated with a hole in the $[301]\frac{1}{2}$ Nilsson state, which in the pseudo-spin formalism is $[\tilde{2}\tilde{0}\tilde{0}]\frac{1}{2}$. The $a=1$ associated with this state in the pseudo SU(3) formalism gives identical values of E_γ for ^{151}Tb and the ^{152}Dy core [case (ii)] if the $[\tilde{2}\tilde{0}\tilde{0}]\frac{1}{2}$ hole does not modify the moment of inertia. Indeed the data indicate a similar variation of ≈ 1-2 parts per thousand for $|{}^{152}\mathrm{Dy}\otimes\pi([\tilde{2}\tilde{0}\tilde{0}]\frac{1}{2})^{-1}\rangle$ and the ^{152}Dy core as for $|{}^{152}\mathrm{Dy}\otimes\nu([514]\frac{9}{2})\rangle$ and the ^{152}Dy core discussed above. For the decoupling parameter the condition is not so rigorous. The estimate based on Eq. (1) gives $\Delta E_\gamma(I)/E_\gamma(I)\approx\Delta a/2I$ which yields $\Delta a\approx 0.1$ and $I\approx 50$ and $\Delta E_\gamma\approx 1$ keV. The deduced value of the decoupling parameter for the $[\tilde{2}\tilde{0}\tilde{0}]\frac{1}{2}$ SD band in ^{151}Tb is thus $a=1.0\pm 0.1$.

A similar interpretation can be given for the recently observed[4] excited SD band in ^{150}Gd. The lowest particle-hole excitation corresponds to the promotion of the $[\tilde{2}\tilde{0}\tilde{0}]\frac{1}{2}$ proton to the third $N=6$ Routhian 6_3. The resulting $(-,-i)$ band, $|{}^{151}\mathrm{Tb}_{\mathrm{SD,yrast}}\otimes\pi([\tilde{2}\tilde{0}\tilde{0}]\frac{1}{2})^{-1}\rangle$,

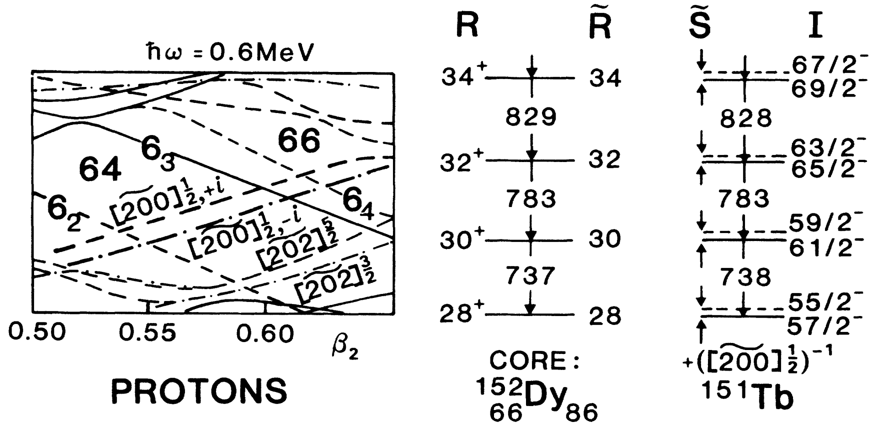

FIG. 2. Predicted single-proton Routhians at 0.6 MeV and the expected deformations of SD states. The experimental ^{152}Dy transitions, and the transitions in the $[\tilde{2}\tilde{0}\tilde{0}]\frac{1}{2}$ SD band in ^{151}Tb are *identical*. A more detailed comparison of these two bands for the complete data range is given in Ref. 4.

is experimentally identical within 2 keV with the lowest SD band in ^{151}Tb which also involves three $N=6$ protons. The excited band in ^{150}Gd can be thus interpreted in terms of the pseudo spin decoupled from the rotational motion of the effective core with total pseudo orbital angular momentum $\tilde{R}=\frac{1}{2},\frac{5}{2},\frac{9}{2},\ldots$.

In the limit of pseudo SU(3) symmetry the $[\tilde{2}\tilde{0}\tilde{0}]\frac{1}{2}$ orbital should be very pure even at high spin. The Coriolis coupling to the closest negative-parity levels, i.e., $[\tilde{2}\tilde{0}\tilde{2}]\frac{3}{2}$ and $[\tilde{2}\tilde{0}\tilde{2}]\frac{5}{2}$, is zero. In realistic calculations[3] some Coriolis coupling is present, but the extracted decoupling parameter is around 0.85. Similar conclusions also can be made within the modified-oscillator particle-rotor model.[17] Reasonably pure examples of the $[301]\frac{1}{2}$ ($[\tilde{2}\tilde{0}\tilde{0}]\frac{1}{2}$) configuration apparently do not occur at normal deformations. Experimentally, the $[301]\frac{1}{2}$ level can be found for N or $Z=39$–43. The sizable deviations from the symmetry limit ($a\approx 0$ and 0.7) in $_{81}$Sr (Refs. 18 and 19) and $^{101}_{139}$Y (Ref. 20), respectively, are attributed[19,21] to triaxiality in these shape-transitional nuclei. Thus, the SD shape seemingly provides the unique possibility to study the normal-parity states that carry only a small amount of angular momentum.

As in the case of ^{151}Tb, the nucleus ^{151}Dy can be viewed as a single-hole system with respect to the ^{152}Dy core. No excited SD bands have been observed in ^{151}Dy and the yrast SD band[9] has been associated with the $(-,+i)$ $|^{152}\mathrm{Dy}\otimes\nu(7_2)^{-1}\rangle$ configuration (see Fig. 1). The lowest positive-parity SD band can be formed in ^{151}Dy by promoting one neutron from the $[\tilde{3}\tilde{1}\tilde{0}]\frac{1}{2}$ ($[411]\frac{1}{2}$) level to the 7_2 Routhian. This $[\tilde{3}\tilde{1}\tilde{0}]\frac{1}{2}$ neutron orbital in ^{151}Dy plays a very similar role to the $[\tilde{2}\tilde{0}\tilde{0}]\frac{1}{2}$ proton level in ^{151}Tb. Both appear just below the SD magic gap and have a particle character, and negative quadrupole moment. However, the decoupling parameter of the $[\tilde{3}\tilde{1}\tilde{0}]\frac{1}{2}$ state is equal to $a=-1$ in the pseudo asymptotic limit (as mentioned above, the decoupling parameter for the $[411]\frac{1}{2}$ proton states in rare-earth nuclei, like ^{177}Lu, is near this value[13]). In this symmetry limit the transition energies in the SD $[\tilde{3}\tilde{1}\tilde{0}]\frac{1}{2}$ band are expected to lie half way between the γ-ray energies of the SD band in ^{152}Dy as indicated schematically in Fig. 1. This band has not yet been observed. Theoretically,[3] the $[\tilde{3}\tilde{1}\tilde{0}]\frac{1}{2}$ Routhian should be slightly disturbed by the Coriolis interaction with the $[\tilde{3}\tilde{1}\tilde{2}]\frac{3}{2}$ orbital lying about 1 MeV below. This coupling is expected to push the $r=+i$ member to slightly higher energy thus increasing the transition energies by a few keV.

The identical γ-ray sequences established[4] for the SD states in the $N=86$ isotones are nearly perfect examples of the $a=+1$ limit. Likewise, the $[514]\frac{9}{2}$ neutron states in ^{153}Dy are equally perfect examples of the $a=0$ limit. The striking similarity between the bands implies that the SD cores are insensitive to the removal (or addition) of certain particles such as a $[301]\frac{1}{2}$ proton (^{151}Tb) or a $[514]\frac{9}{2}$ neutron (^{153}Dy). The common feature of all of these *identical* configurations is a small intrinsically aligned angular momentum. However, the existing models based on the mean-field approach always yield some polarization of the field due to the odd particle (or hole). Moreover, in the macroscopic limit the deviation in moment of inertia would be $|1-(153/152)^{5/3}|\approx 1\%$, that is, about an order of magnitude greater than observed. Understanding the striking precision of these *identical* decay sequences in neighboring isotones and isotopes is, in our opinion, one of the most challenging problems in nuclear structure.

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