Potential Contribution to the $\Delta I = \frac{1}{2}$ Rule in $K \to \pi\pi$

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Final-state interactions in $K \to \pi\pi$ enhance the $\Delta I = \frac{1}{2}$ amplitude and suppress the $\Delta I = \frac{3}{2}$ one. The effect appears to be about an order of magnitude in the ratio of the rates and may be useful in reconciling the outstanding discrepancy between short-distance QCD corrections and experiment.

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The origin of the $\Delta I = \frac{1}{2}$ rule in $K \to \pi\pi$ decays is a long-standing puzzle for the standard model. While the naïve strengths of the $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ pieces of the $\Delta S = 1$ current-current weak interaction favor $\Delta I = \frac{1}{2}$ by a factor of only $\sqrt{2}$, the corresponding experimental amplitudes differ by a large factor, as exemplified by the ratio

$$[\Gamma(K_S^0 \to \pi\pi)/\Gamma(K^+ \to \pi^+\pi^0)]^{1/2} \simeq 25\,, \tag{1}$$

the process in the denominator being pure $\Delta I = \frac{3}{2}$.

This observation, together with the dominance of nonleptonic over leptonic decays, implies a significant enhancement of $\Delta I = \frac{1}{2}$ transitions; a suppression of the $\Delta I = \frac{3}{2}$ rate also appears to contribute to the large ratio in Eq. (1). To a certain extent these effects can be understood in terms of perturbative QCD corrections to the effective weak Hamiltonian H_w. Such corrections are known to increase (decrease) the strengths of the usual $\Delta I = \frac{1}{2}$ ($\Delta I = \frac{3}{2}$) pieces of H_w at scales significantly below M_W.[1] They also induce effective (penguin) operators below the charm quark mass which are pure $\Delta I = \frac{1}{2}$ and whose amplitudes add constructively to those produced by the standard $\Delta I = \frac{1}{2}$ operators.[2] However, perturbatively evaluated Wilson coefficients of H_w give both insufficient suppression of $\Delta I = \frac{3}{2}$ and insufficient enhancement of $\Delta I = \frac{1}{2}$ to account for the observed effect at least for scales at which the perturbative analysis can be trusted.[3-5] This remains so even when one takes into account a plausible increase in the penguin coefficients over the values quoted in Ref. 3 due to the incomplete Glashow-Iliopoulos-Maiani cancellation just above m_c.[6] Attempts to invoke further enhancements in the penguin matrix elements, moreover, tend to produce overly large values of ϵ'/ϵ.[7,8] In the end, there appears to be an outstanding discrepancy of approximately a factor of 2 between theory (i.e., QCD corrections between the W mass and ~ 1 GeV) and experiment in both the enhancement of $\Delta I = \frac{1}{2}$ and the suppression of $\Delta I = \frac{3}{2}$ amplitudes.

In this paper we show that $\pi\pi$ final-state interactions (FSI) may account for a sizable fraction of the missing enhancement and suppression. The essence of our proposed mechanism is that the $\pi\pi$ interactions, which (see Fig. 1) are attractive in $I=0$ (corresponding to the $\Delta I = \frac{1}{2}$ part of H_w) and repulsive in $I=2$ (corresponding to the $\Delta I = \frac{3}{2}$ part), distort the final-state wave functions sufficiently to markedly change the calculated rates. The resulting correction factors of roughly 2 and $\frac{1}{2}$, respectively, in amplitude are analogous to the Fermi factors which correct the logft values deduced in the β^- and β^+ decays, respectively, of high-Z nuclei.

Are such substantial corrections allowed by our present experimental and theoretical understanding of low-energy $\pi\pi$ interactions? We believe so. As we will see, these factors can be deduced, with some model

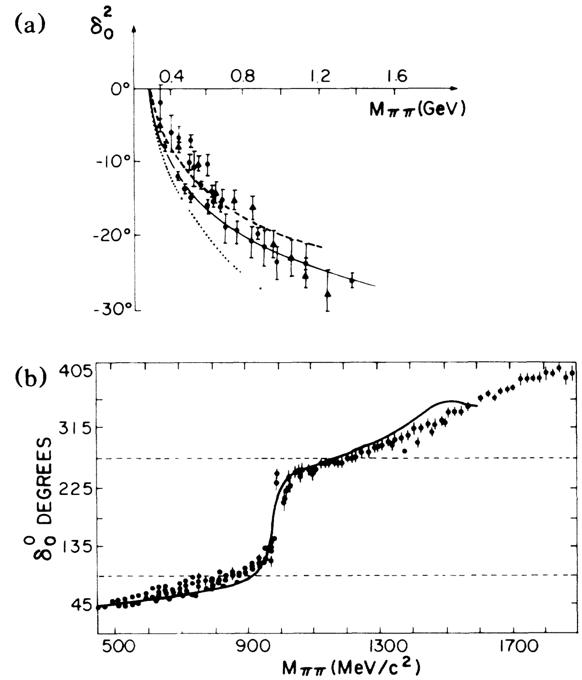

FIG. 1. (a) The experimental (Ref. 16) $I=2$ $\pi\pi$ elastic phase shift, compared to the model of Ref. 10 (solid curve); the dotted curve results from using $m_{\pi\pi} = 2(m_\pi^2 + k^2)^{1/2}$ instead of the nonrelativistic dispersion relation; the dashed curve results from reducing U_2 by 20%. (b) The $I=0$ $\pi\pi$ phase shift from near threshold to ~ 1.5 GeV in the $K\bar{K}$-molecule picture of Ref. 10, compared to the data of Ref. 17.

dependence, directly from the measured low-energy $\pi\pi$ phase shifts, so there can be no conflict with either such experiments or the essentially equivalent predictions of chiral perturbation theory. Current algebra also makes predictions for the relationship between $K\rightarrow\pi\pi\pi$ and $K\rightarrow\pi\pi$ and between $K\rightarrow\pi\pi e^-\bar{\nu}_e$ and $K\rightarrow\pi e^-\bar{\nu}_e$. Calculations indicate that the first of these predictions receives only modest corrections from FSI since their effects tend to cancel in the ratio. (The residual change actually seems to improve agreement with experiment.[9]) The $\pi\pi$ S wave in $K\rightarrow\pi\pi e^-\bar{\nu}_e$, in contrast, would be expected to be enhanced by FSI not present in $K\rightarrow\pi e^-\bar{\nu}_e$, and, indeed, such enhancements may help to improve the predictions of current algebra which are too small by about 50% in amplitude.[9] We thus see no grounds for rejecting such effects on the basis of any of the classic current-algebra tests of $\pi\pi$ interactions. Moreover, there is also evidence for such interactions elsewhere.[10] An example is the low-mass $\pi\pi$ enhancement[11] seen in $\psi\rightarrow\omega\pi\pi$ (Ref. 12). We are aware of only one situation in which one might be concerned that large FSI would ruin existing agreement between theory and experiment. Chiral symmetry, a multipole expansion of the gluon field, and QCD anomalies may be used to predict spectral shapes and normalizations relative to η emission in $\pi\pi$ transitions in heavy quarkonia.[13] These predictions, which neglect FSI, work well for $\psi'\rightarrow\psi\pi\pi$ and $\Upsilon'\rightarrow\Upsilon\pi\pi$. However, (1) the spectral shapes are predicted to be dominated by an amplitude zero near threshold; the resulting spectral shape, modified by FSI, thus remains compatible with that observed experimentally since, in the region where events are predicted, the enhancement varies slowly with energy, (2) the spectral shape observed[14] in $\Upsilon''\rightarrow\Upsilon\pi\pi$ disagrees with the prediction, and (3) the rate prediction for $\psi'\rightarrow\psi\pi\pi$ relative to $\psi'\rightarrow\psi\eta$ is uncertain[13] by at least 50%.

With this reassurance that our present knowledge of low-energy $\pi\pi$ interactions allows a substantial FSI, we proceed to study the effects of the $I=0$ and 2 interactions on $K\rightarrow\pi\pi$. We will describe the low-energy $\pi\pi$ system by a Schrödinger equation; this is certainly legitimate for a sufficiently low $\pi\pi$ invariant mass $m_{\pi\pi}$. The potential in this low-energy approximation represents all higher-mass effects and, as noted above, must be attractive in $I=0$ and repulsive in $I=2$ to explain the observed low-energy phase shifts. The S-wave $\pi\pi$ relative coordinate wave function in $I=0$ ($I=2$) will thus be enhanced (suppressed) at short distance relative to the free $\pi\pi$ wave function. If one treats the decaying kaon as a pointlike source, one then has

$$\frac{\langle(\pi\pi)_{I=0}|H_w^{(1/2)}|K\rangle}{\langle(\pi\pi)_{I=2}|H_w^{(3/2)}|K\rangle}=\left[\frac{d_0}{d_2}\right]^{1/2}\frac{\langle(\pi\pi)_{I=0}^{\rm free}|H_w^{(1/2)}|K\rangle}{\langle(\pi\pi)_{I=2}^{\rm free}|H_w^{(3/2)}|K\rangle}\,, \tag{2}$$

where the superscripts $\frac{1}{2}$ and $\frac{3}{2}$ refer to the change in isospin, and

$$\sqrt{d_I}\equiv\Psi_I(0)/\Psi_I^{\rm free}(0) \tag{3}$$

is the ratio of the true to free $\pi\pi$ relative spatial wave functions at zero separation. We may make a first estimate the magnitude of the FSI effect in Eq. (2) by evaluating the distortions d_I and phase shifts δ_I which arise from square-well potentials having ranges of a_0 and a_2 and strengths $V_0=-U_0$ and $V_2=+U_2$ ($U_0,U_2>0$) for the $I=0$ and 2 channels, respectively. The distortions are in this case given by

$$d_I(k)=\frac{k^2-m_\pi V_I}{k^2-m_\pi V_I\cos^2[a_I(k^2-m_\pi V_I)^{1/2}]} \tag{4}$$

and the phase shifts by

$$\delta_I(k)=\arctan\left[\frac{k}{(k^2-m_\pi V_i)^{1/2}}\tan[a_I(k^2-m_\pi V_I)^{1/2}]\right]-ka_I\,, \tag{5}$$

where $k\equiv|\mathbf{k}_\pi|$ and where $(k^2-m_\pi V_I)^{1/2}\equiv i(m_\pi V_I-k^2)^{1/2}$ for $k^2<m_\pi V_I$. Given the ranges a_I of the $\pi\pi$ forces, one could therefore turn information on the low-energy $\pi\pi$ phase shifts into predictions for the distortion factors d_I. The actual situation is somewhat more complicated than this because two distinct scales can contribute to the effective ranges: one corresponding to possible s-channel resonances (which would be represented by potentials of zero range in the narrow resonance approximation) and a second corresponding to forces generated by quark or meson exchange. (In this paper we will adopt the point of view that t-channel quark exchange dominates meson exchange, although this is not essential to our case.) Of course $I=2$ $\pi\pi$ scattering is resonance-free in the s channel, so that computing the distortion d_2 from the phase shift δ_2 (and an assumed typical range a_2) should be valid. While the situation in $I=0$ is less clear, it is natural to associate the very large nonresonant background terms[15] seen in fits to $I=0$ $\pi\pi$ scattering with quark exchange and to calculate a contribution to d_0 arising from this source analogously in terms of the background phase $\delta_0^{\rm bkg}$ and a_0. We will carry out such a calculation below.

The preceding discussion is meant to emphasize the generality of the effect we are proposing, but one may arrive at the same picture from a more dynamical perspective. In Ref. 10, $I=2$ $\pi\pi$ and $I=0$ coupled-channel ($\pi\pi$, $K\bar{K}$, $\eta\eta,\ldots$) scattering was studied using an explicit dynamical model which incorporates quark exchange. The purpose of the calculation was to elucidate the nature of the scalar resonances $f_0(975)$ and $a_0(980)$, which in the model are found to be "$K\bar{K}$ molecules." Figure 1 shows the model's $I=2$ and $I=0$ $\pi\pi$ phase shifts together with experimental data.[16,17] The quark-exchange components of the $\pi\pi$ interactions (which supply only part of the total $I=0$ phase shift, see below) correspond to local potentials in this model, and can be approximated by the Gaussian forms

$$V_0(r) = -0.59\exp\{-[r/(0.68\ \text{fm})]^2\}\ \text{GeV} \quad (6)$$

and

$$V_2(r) = 0.99\exp\{-[r/(0.61\ \text{fm})]^2\}\ \text{GeV} \quad (7)$$

The rms size associated with these potentials is ≈ 0.8 fm, and choosing this value for the square-well ranges a_0 and a_2, the potentials (6) and (7) correspond to square wells of strengths $U_0 = 0.40$ GeV and $U_2 = 0.80$ GeV, respectively. For small k, for which the nonrelativistic approach is valid, Eq. (5) leads to an enhancement in the ratio of the $\Delta I = \frac{1}{2}$ to $\Delta I = \frac{3}{2}$ rates of

$$\left(\frac{d_0(0)}{d_2(0)}\right)_{\substack{\text{square}\\ \text{well}}} = \frac{\cosh^2[a_2(m_\pi U_2)^{1/2}]}{\cos^2[a_0(m_\pi U_0)^{1/2}]} \simeq 13\,. \quad (8)$$

The corresponding enhancement for the Gaussian forms of the potentials [Eqs. (6) and (7)] is

$$[d_0(0)/d_2(0)]_{\text{Gaussian}} \simeq 10\,. \quad (9)$$

Of course the relevant quantity for the physical process $K \to \pi\pi$ is not the value of this ratio at $k=0$ but instead near $\bar{k} = [m_\pi(m_K - 2m_\pi)]^{1/2}$, where the ratios in Eqs. (8) and (9) are $\simeq 9$ and $\simeq 6$, respectively. In Table I we present the values of $\sqrt{d_I}$ at $k=0$ and $\bar{k}$ for both forms of the potentials.

While the above method is perhaps adequate for the $I=2$ channel, the complexities of the $I=0$ channel require a more realistic treatment: Recall that the d_0 of the first four columns of Table I ignore coupled-channel and resonance effects. The last two columns of Table I, which show $[d_0(k)]^{1/2}$ from the multichannel analysis of Ref. 10 (including $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$, and $^3P_0\ q\bar{q}$ S-wave channels), indicates that the one-channel approximation underestimates the $I=0$ enhancement so that a better estimate would be

$$[d_0(\bar{k})/d_2(\bar{k})] \simeq 19\,. \quad (10)$$

[As one goes from $\pi\pi$ to $\pi\pi + K\bar{K} + \eta\eta + \eta\eta' + \eta'\eta'$ to all channels, the enhancement in the amplitudes rises from 1.5 to 1.7 to 2.2; the additional coupled-channel enhancement is thus mainly due to the broad $f_0(1300)$ resonance. If one assumes that the entire low-energy $I=0$ phase shift at $\bar{k}$ is due to a potential of range 0.8 fm, Eqs. (4) and (5) would also give an enhancement of 2.2.] We should mention that there is no inconsistency between our $\sqrt{d_I}$ and the values obtained in Refs. 9 and 15: The phase integral appearing in the Omnes function of these references is in a once-subtracted form, and thus yields only the energy variation of the d_I and not their absolute magnitudes.

TABLE I. Modifications of the $\pi\pi$ amplitudes in $K \to \pi\pi$ due to final-state interactions.

	Square well		Gaussian		Coupled-channel analysis[a]	
	$k=0$	$k=\bar{k}$	$k=0$	$k=\bar{k}$	$k=0$	$k=\bar{k}$
$[d_0(k)]^{1/2}$	1.7	1.5	1.8	1.5	3.6	2.2
$[d_2(k)]^{1/2}$	0.48	0.51	0.57	0.60	0.48	0.51
$[d_0(k)/d_2(k)]^{1/2}$	3.6	2.9	3.1	2.5	7.0	4.2

[a]Reference 10.

As a test of the sensitivity of our results to our model for the potentials, we first studied the variation of the distortions $d_I(\bar{k})$ with a_I. (We imposed the constraint of constant scattering lengths through an implicit dependence of the potential strength U_I on the range a_I.) We find that $[d_0(\bar{k})]^{1/2}$ ($[d_2(\bar{k})]^{1/2}$) changes by approximately -0.1 ($+0.1$) for each $+0.1$-fm change in a_0 (a_2). Given that the full low-energy dependence of the δ_I on k constrains these ranges to be near 0.8 fm, this source of uncertainty does not appear to be crucial. Another potentially significant source of sensitivity is the dependence on the strengths of the potentials. Their absolute strengths are difficult to predict accurately so that Ref. 10 adjusts U_2 to fit the data of Fig. 1(a). If U_2 is reduced by 20% [so that δ_0^2 lies above the higher-energy data in Fig. 1(a), as shown by the dashed curve in that figure], then $[d_2(\bar{k})]^{1/2}$ changes from 0.51 to 0.58. When the corresponding reduction is made in U_0, the quark-exchange component of the $I=0$ $\pi\pi$ potential, $[d_0(\bar{k})]^{1/2}$, decreases from 2.23 to 2.07. The nominal potentials of Ref. 10 give a satisfactory fit to the data shown in Fig. 1 and produce $I=0$ phase shifts near threshold very similar to those of the global fit of Ref. 15. However, these near-threshold phase shifts are considerably larger than those obtained from analysis of $K \to \pi\pi e\bar{\nu}_e$. Decreasing the U_I by 20% leads to better agreement with these data, while retaining a satisfactory fit to δ_0^2 and only marginally degrading the $I=0$ fit at higher energies. A decrease in $[d_0(\bar{k})/d_2(\bar{k})]^{1/2}$ of 20% from uncertainties in the U_I can therefore not be ruled out in the present approach without more accurate data. (One might be surprised that large changes in the scattering lengths do not lead to large changes in $[d_I(\bar{k})]^{1/2}$. For example, the above-mentioned change in U_0 leads to a 35% decrease in the $I=0$ scattering length, but only a 7% decrease in $[d_0(\bar{k})]^{1/2}$. That they do not may be understood from the Omnes representation in which the $[d_I(\bar{k})]^{1/2}$ are given by a weighted integral over the δ_I at all energies: So long as an overall fit to the phase shifts is maintained, local variations in the fit will have a modest effect. Another way of understanding this is to realize that a given scattering length only determines a family of potentials by giving a relation between U_I and a_I; the $\sqrt{d_I}$ are thus determined by the phase shift at all energies.) One might also be concerned that the assumption of a $\pi\pi$ point source leads to an overestimate of the FSI effects. This approximation is easily removed for a square-well potential. In this case one finds, for example, that the $\Delta I = \frac{1}{2}$ enhancement and the

$\Delta I = \frac{3}{2}$ suppression are each decreased by only $\approx 10\%$ for a source of constant strength over a radius of 0.6 fm. Another source of concern is the use of the Schrödinger equation for $k = \tilde{k}$ where the nonrelativistic approximation is inaccurate. The importance of this approximation should be investigated by solving realistic coupled-channel equations with potentials chosen to imitate the low-energy behavior of those of Ref. 10. Comparison of the dotted and solid curves in Fig. 1(a) suggests that the effects will not be dramatic. Thus, although there are some uncertainties in the predicted magnitude, it seems difficult to avoid the conclusion that $\pi\pi$ final-state interactions make a sizable contribution to the $\Delta I = \frac{1}{2}$ rule.

It should be noted that the possible importance of corrections to $K \to \pi\pi$ due to FSI has been considered in the past. Reference 18 notes the necessity of significant unitarity corrections to the current-algebra relation between $K \to \pi$ and $K \to \pi\pi$ as a result of the strong $I=0$ attraction. Final-state enhancement in the $I=0$ channel has also been discussed in Ref. 19 in the context of a σ-resonance model for the $I=0$ $\pi\pi$ interaction. Reference 9 uses a once-subtracted form of the Omnes function to address the effect of the $I=0$ final-state enhancement on both $K \to \pi\pi$ and $K \to \pi\pi\pi$ and the concomitant corrections to the current-algebra relation between the two, as mentioned above. Closest in spirit to the present work is Ref. 20, where meson rescattering effects are treated to one-loop order in chiral perturbation theory (χPT). This study finds $I=0$ enhancement and $I=2$ suppression factors of approximately 1.5. Presumably the bulk of the effects found in Ref. 21 via the low-energy "meson-evolution" $1/N_c$ corrections to the leading-order-in-N_c fourth-order χPT (Ref. 22) results are also due to FSI. In assessing the results of Refs. 20 and 21, it should be noted that one-loop rescattering corrections for $K \to \pi\pi$ in χPT correspond to rescattering only through the lowest-order tree-level vertex. The results of Ref. 23 on $\pi\pi$ scattering in χPT suggest that such an approximation will be only semiquantitative. Finally we note that lattice calculations, which require an extrapolation of lattice matrix elements from SU(3) symmetric to physical kinematics, must certainly incorporate FSI effects in this extrapolation, at the very least by employing χPT to one-loop order.

In conclusion, we have identified a mechanism contributing to the $\Delta I = \frac{1}{2}$ rule, namely $\pi\pi$ final-state interactions. The resulting $\Delta I = \frac{1}{2}$ enhancement and $\Delta I = \frac{3}{2}$ suppression are substantial and should certainly be taken into account in the attempt to understand this long-standing problem.

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