Charge Fluctuations in Small-Capacitance Junctions

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The current-voltage characteristics of submicron normal-metal tunnel junctions at millikelvin temperatures are observed to exhibit a sharp Coulomb blockade with high-resistance thin-film leads, but to be heavily smeared for low-resistance leads. As the temperature is lowered, the zero-bias differential resistance tends asymptotically to a limit that is greater for junctions with high-resistance leads. Both observations are explained in terms of a model in which quantum fluctuations in the external circuit enhance the low-temperature tunneling rate. The predictions are in reasonable agreement with the data.

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It is predicted theoretically[1-4] and well established experimentally[5-14] that submicron tunneling junctions at low temperatures T can, under appropriate conditions, exhibit charging effects due to the discreteness of the electronic charge. In particular, when the charging energy $E_C = e^2/2C$ associated with the tunneling of a single electron across a capacitance C becomes large compared with k_BT, one may observe suppression of the tunnel current I at voltages $V < e/2C$. As a result, the I-V characteristic at higher voltages is offset by the Coulomb gap, $e/2C$. However, observation of these effects depends strongly on the nature of the environment coupled to the junction. For example, Delsing *et al.*[12] varied the environment by studying two kinds of circuits. In the first, metallic leads were coupled directly to the junction while in the second, linear arrays of submicron junctions were placed in each lead to the junction under study. The Coulomb gap in the first circuit was observed to be greatly smeared out, while in the second, it was much sharper. Geerligs *et al.*[14] studied single junctions and linear arrays of junctions. They found sharp Coulomb gaps in arrays containing a minimum of two junctions, provided that the resistance of the junctions R was much greater than $\hbar/e^2$. For the single junctions the gap was extremely smeared, but at high currents the expected voltage offset was observed. Thus, although it is clear that the environment has a significant effect on the Coulomb blockade, the nature of the effect has remained unexplained until now.

In this Letter, we report measurements on small junctions with thin-film leads of high and low resistance. The Coulomb gap is well defined in the former case for junctions with $R \gg \hbar/e^2$, but very smeared out in the latter, for all values of R. In all junctions studied, the zero-bias differential resistance increases and then flattens out as the temperature is lowered. We propose a simple model in which the Nyquist noise from the external circuit produces charge fluctuations across the junction capacitance. These fluctuations, in turn, enhance the low-voltage tunneling rate, smearing the Coulomb gap. In the zero-temperature limit the noise arises from quantum fluctuations, and our model yields predictions for the I-V characteristic and the zero-bias resistance that are in reasonable agreement with our data at the lowest temperatures.

Small Al-Al-oxide-Al tunnel junctions with areas of typically 0.04 μm^2 were fabricated using standard electron-beam lithography and angled evaporations.

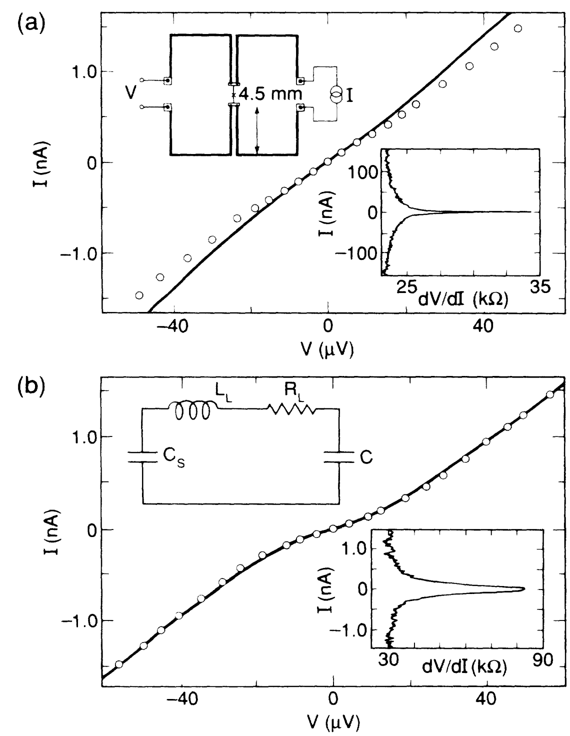

FIG. 1. I-V characteristics (solid lines) for two junctions at 20 mK with (a) $C = 4 \pm 1$ fF and $R = 23$ kΩ and (b) $C = 5 \pm 1$ fF and $R = 28$ kΩ. Dots represent predictions of theory. Inset in each figure is dV/dI vs I; note different current scales. Also inset in (a) is the configuration of junction and leads (not to scale) and in (b) its simplified representation.

Thin-film leads were connected to the junctions in the arrangement shown inset in Fig. 1; for one set of junctions the leads were made of CuAu alloy (25 wt% Cu) with a sheet resistance of about 4 Ω per square, and for the other set the leads were made of NiCr alloy (80 wt% Ni), with a sheet resistance of about 60 Ω per square. Each of these four leads was 2 μm wide and had a total length of 12 mm. The Al lead from each side of the junction to the resistive leads was 30 μm long and 0.2 μm wide. The junctions were mounted on a dilution refrigerator, and all electrical leads to the junctions were filtered above 10 kHz. The Al was driven into the normal state with an external magnetic field.

In Fig. 1 we show the *I-V* characteristics for two junctions, with low- and high-resistance leads, respectively. The tunneling resistance and capacitance of both junctions are within 25% of one another, and both characteristics were taken at the same temperature. There is a striking difference in the low-current behavior of the two junctions; the low-resistance leads clearly give rise to a very smeared Coulomb gap, while the high-resistance leads give a much sharper characteristic. This distinction is emphasized in the plots of the dynamic resistance shown as insets in Fig. 1. Similar differences in behavior were observed in all the junctions we have studied; in Fig. 2, we plot the temperature dependence of the zero-bias dynamic resistance R_0 for five junctions. We see that the low-temperature values of R_0/R are higher for junctions with high-resistance leads than for those with low-resistance leads. Furthermore, for a given lead resistance, the asymptotic value of R_0/R increases with R, in qualitative agreement with the findings of Geerligs *et al.*[14]

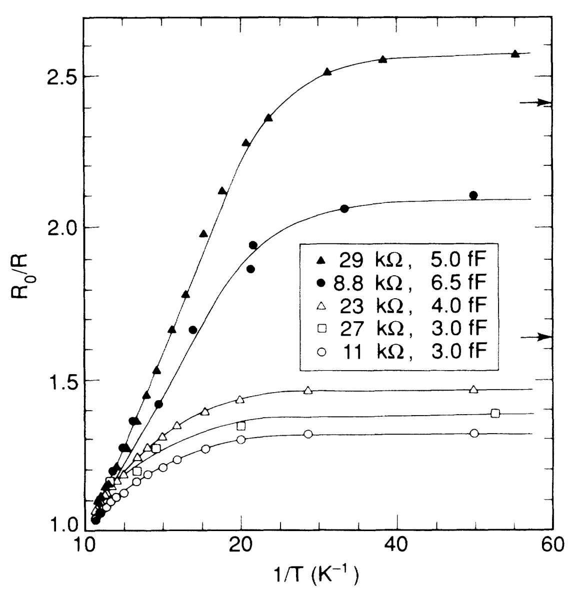

FIG. 2. R_0/R vs $1/T$ for five junctions; the open symbols are for low lead resistance and the solid symbols for high lead resistance. Arrows indicate the predicted values of R_0/R.

We have considered the possibility that the flattening out of R_0/R as the temperature is lowered is due to self-heating or to extraneous noise. Our estimates of hot-electron effects[15] in the leads and in the junction imply that heating is negligible for $V \lesssim e/2C$, even at the lowest temperatures. We have also removed the magnetic field on a low-resistance junction and measured the reduction of the superconducting energy gap due to heating at high currents. These results also imply that heating is negligible in the experiments reported here. We have tested the effects of adding and removing radio-frequency and microwave filters at room temperature, 4.2 K, and 20 mK, and found no effects on the *I-V* characteristics. These observations suggest that external noise is not responsible for the flattening of R_0/R.

To interpret our results, we consider a model circuit in which the junction is connected via leads with inductance L_L and resistance R_L to a line with a stray capacitance $C_s \gg C$. Thus, L_L and R_L represent the combined inductance and resistance of all four leads connected to the junction. The resistance R_L produces a Nyquist voltage noise V_N with a spectral density

$$S_V(\omega) = (\hbar\omega R_L/\pi)\coth(\hbar\omega/2k_BT).$$

Assuming $C_s^{-1} \ll C^{-1}$ and defining $\omega_{LC}^2 \equiv 1/L_LC$, $\omega_{RC} \equiv 1/R_LC$, we write the quantum Langevin equation for the Fourier transform $q(\omega)$ of the fluctuations in charge $q(t)$ on the junction,

$$q(\omega)/C + i\omega R_L q(\omega) - \omega^2 L_L q(\omega) + V_N(\omega) = 0. \quad (1)$$

Solving for $q(\omega)$, we obtain the following mean-square charge fluctuation:

$$\langle q^2(t)\rangle = \int_0^\infty \frac{C^2}{(1-\omega^2/\omega_{LC}^2)^2 + (\omega/\omega_{RC})^2} S_V(\omega)\,d\omega. \quad (2)$$

We remark that, in general, the quantum Langevin equation is likely to be accurate only when any nonlinearity associated with tunneling is negligible so that the energy levels of the circuit are equally spaced, or when the damping is sufficiently large that the quantum levels are smeared out.[16] We believe the latter case to apply to the experiments described here.

In the classical limit of large T, Eq. (2) yields the result of the equipartition theory, $\langle q^2\rangle/2C = k_BT/2$. In the quantum regime of small T, for $\alpha \equiv \omega_{LC}/\omega_{RC} = R_L(C/L_L)^{1/2} < 2$, we find

$$\frac{\langle q^2\rangle_Q}{2C} = \frac{\hbar\omega_{LC}}{4\pi}\frac{1}{(4-\alpha^2)^{1/2}} \times \left[\frac{\pi}{2} - \tan^{-1}\left(\frac{\alpha^2-2}{\alpha(4-\alpha^2)^{1/2}}\right)\right]. \quad (3)$$

In limit where R_L is very small, $\alpha \ll 2$, Eq. (3) reduces to $\langle q^2\rangle/2C = \hbar\omega_{LC}/4$ as expected for a simple harmonic oscillator. On the other hand, in the limit $\alpha > 2$ of interest here, we obtain

$$\frac{\langle q^2\rangle_Q}{2C}=\frac{\hbar\omega_{LC}}{2\pi}\frac{1}{(\alpha^2-4)^{1/2}}\ln\left[\frac{\alpha^2-2+\alpha(\alpha^2-4)^{1/2}}{\alpha^2-2-\alpha(\alpha^2-4)^{1/2}}\right]. \tag{4}$$

In the limit where R_L is very large, $\alpha\gg 2$, Eq. (4) becomes $\langle q^2\rangle/2C=(\hbar\omega_{RC}/\pi)\ln\alpha$. Note that this expression depends on L_L only logarithmically.

A junction with charge Q is driven by these fluctuations to charge $Q+q$ with probability $P(q)$; $P(q)$ is Gaussian distributed with width $\langle q^2\rangle$. These fluctuations in the charge will modify the electron tunneling rate[4] $\Gamma^{\pm}(Q)$ for Q going to $Q\pm e$,

$$\Gamma^{\pm}(Q)=-\frac{e/2\pm Q}{eRC}\left[\exp\left(\frac{\Delta E^{\pm}}{k_BT}\right)-1\right]^{-1}, \tag{5}$$

where $\Delta E^{\pm}=(\pm 2eQ+e^2)/2C$ is the resultant energy change. We replace Eq. (5) with the convolution

$$\langle\Gamma^{\pm}(Q)\rangle=\int_{-\infty}^{\infty}P(q)\Gamma^{\pm}(Q+q)dq\,. \tag{6}$$

Although one can obtain expressions for the temperature-dependent tunneling rate, in this Letter we confine our attention to the limit $T=0$ where

$$\langle\Gamma^{\pm}(Q)\rangle=-\frac{e/2\pm Q}{2eRC}\operatorname{erfc}\left[\frac{e/2\pm Q}{(2\langle q^2\rangle)^{1/2}}\right]+\frac{1}{eRC}\left[\frac{\langle q^2\rangle}{2\pi}\right]^{1/2}\exp\left[-\frac{(e/2\pm Q)^2}{2\langle q^2\rangle}\right]. \tag{7}$$

Thus, even at $T=0$, for large enough fluctuations the Coulomb blockade is heavily smeared out at low Q. On the other hand, for large $Q>0$, we find the simple results $\Gamma^+=0$ and $\Gamma^-=(Q-e/2)/eRC$, at any temperature. The Coulomb blockade is still visible as a voltage shift $e/2C$ for large voltages. This result is consistent with experimental observations.[14] To enable us to make quantitative comparisons with our data, we have carried out Monte Carlo simulations of the charging sequence of a current-biased junction, using Eq. (7) for the tunneling rate. For a given bias current we compute the voltage across the junction as a function of time, and then calculate the average voltage to obtain the I-V characteristic.

The comparison of our data with the predictions obtained from the model circuit is not entirely straightforward. The major difficulty concerns the stray capacitance between the thin-film leads and the nearest ground plane, which is roughly 10 mm away. We estimate this distributed capacitance to be between 1 and 5 fF/(mm length of the leads). We have performed numerical calculations indicating that the direct capacitive coupling between the resistive leads on opposite sides of the junction is an order of magnitude smaller than this figure.[17] In an attempt to develop some feeling for the effect of the capacitance to ground, we performed subsidiary experiments on two junctions where all but the 4.5 mm of the leads closest to the junction were coated with indium. The presence of the indium has no observable effect on the I-V characteristics. Thus, we conclude that, at most, only the first 4.5 mm of each lead contribute to the loading of the junction. We estimate the inductance of each of these leads (L_L in our model) to be 5 nH, while the resistances R_L are 8 and 130 kΩ for the low- and high-resistance cases, respectively. The corresponding distributed capacitance to ground, C_L, is between 5 and 25 fF. This capacitance reduces the impedance of the leads at frequencies above $1/2\pi R_LC_L$; at $10/2\pi R_LC_L$ the resistance is about one-half of that at low frequencies. Above this frequency, which is a few GHz for the high-resistance leads, the impedance scales as $(R_L/\omega C_L)^{1/2}$ and is independent of the length of the leads. However, the high-resistance leads still present a significantly higher impedance than do the low-resistance leads. Thus, to test our model we have ignored the parasitic capacitance, and used values of L_L and R_L corresponding to the first 4.5 mm of the leads. Of course, the relevant length may be shorter, but we have no means of estimating it. We have used values of the junction capacitances C estimated from the voltage offset at high currents. The voltage offsets give capacitances somewhat larger than expected[6] from the estimated geometrical area of the junctions, but not unreasonably so; it is possible that some stray capacitance contributes to this capacitance.

The calculated I-V characteristics are shown as points in Figs. 1(a) and 1(b). The agreement is good for the high lead resistance, and somewhat less good for the low; however, the trends are well predicted. In Fig. 2, we show the predicted zero-temperature values of R_0/R for the high and low lead resistances. In each case, one should compare the prediction with the results from the highest junction resistance, for which the effects of dissipation in the junction should be negligible.[3] The increase in R_0/R when one increases the lead resistance for two similar junctions is clearly demonstrated by the theory. Thus, although our model calculations do not predict the observed I-V characteristics and zero-bias resistances precisely, given the uncertainties in the impedance loading the junction, we feel that the agreement between the predictions and the data is quite satisfactory. In particular, the results show that the flattening of R_0/R as the temperature is lowered arises from the zero-point fluctuations in the external circuit.

Our experiments support the results of Delsing *et al.*[12] and Geerligs *et al.*[14] in demonstrating the critical importance of the high-frequency properties of the circuit loading small junctions. If the external resistance is too small, our results show that noise generated in this resistance results in charge fluctuations on the junction that smear out the Coulomb gap; at low temperatures, this noise arises from quantum fluctuations. Because of the inherent stray capacitance, it is important to insert the high resistance as close as possible to the junction. We note that one could alternatively introduce high inductances in the leads close to the junctions, in which case Eq. (3) would apply. Finally, we expect the fluctuation effects described here to influence observations not only of Coulomb blockade but also of time-correlated processes involving single electrons[13] and pairs.[2]

A more detailed description of this work including calculations at nonzero temperatures will be presented elsewhere.

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