What Lepton Pairs Reveal about Pions in the Nuclear Medium

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We calculate the pion spectrum and its electromagnetic annihilation vertex in nuclear matter taking into account the strong $\pi N \Delta$ interaction. For nuclear densities the pion kinetic energy is approximately compensated by the interaction, but also the free-pion annihilation matrix element is almost canceled by the contribution from the interaction with the medium. Consequently, the enhancement of the dilepton production rate due to the softening of the pion spectrum is strongly reduced.

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The recent experimental observation of electron-positron pair production in collisions of relativistic nuclei[1] may open new possibilities for studying the properties of hot nuclear matter.[2] The advantages of an electromagnetic probe—its immunity to strong interactions while traveling through the collision region—have been stressed many times. There are different mechanisms for producing lepton-antilepton pairs (dileptons) in nuclear collisions.[3] Experimental evidence suggests an enhancement of dielectron production with invariant mass around 300 MeV, and points to pion annihilation as the source. Reference 3 pointed out that the lowering of the pion energy in the nuclear medium would enhance the dilepton rate by increasing the density of initial states, even leading to singular behavior when the pion energy $E(p)$ develops a minimum at finite momentum. The effect was confirmed in a more realistic calculation in Ref. 4, but in both cases the authors assumed the pion electromagnetic-annihilation matrix element was not significantly affected by the presence of nuclear medium. However, since the annihilation matrix element is that of a current interaction, it might also disappear, since the group velocity vanishes at the minimum of the pion dispersion relation $E(p)$. According to the Ward-Takahashi identity this is exactly what happens when the emitted photon has a very long wavelength. We undertake the present calculation to investigate how complete this cancellation is when the photon energy is equal to the sum of the annihilating pion energies and we show that the effect of nuclear matter on the annihilation matrix element, and hence the dilepton production rate, is as important as that of the pion spectrum.

We study pion annihilation in the presence of nuclear medium by consistently taking into account the strong p-wave $\pi N \Delta$ interaction on the pion spectrum as well as on the annihilation matrix element. There are many other important effects neglected in this analysis, notably the short-range Δ-(nucleon)-hole interaction,[5] the πNN interaction, the widths of the Δ and the pion, etc. While some of these effects can be included in a straightforward way, that is out of the scope of this Letter and, furthermore, we believe they will not change the overall picture since our basic conclusions do not depend on the details of the model.

To take into account the strong mixing of the pion with the Δ hole we use the method of linearized equations of motion (random-phase approximation).[6] We have obtained the same results using Feynman diagrams to calculate the correction to the pion self-energy and its electromagnetic vertex. Another possibility would be to consider the photon self-energy, since its imaginary part is related to the dilepton production rate.[7] Our approach then corresponds to attaching two photon lines in all possible ways to the pion-ring diagrams of Ref. 8. We start by constructing an operator $A_+^\dagger(\mathbf{p})$, creating an excitation with momentum $\mathbf{p}$ and the quantum numbers of the positive pion. We use a nonrelativistic approximation and neglect the contributions of the antinucleons and $\bar{\Delta}$, but take into account the antipion (i.e., negative pion). Thus we write

$$\begin{aligned} A_+^\dagger(\mathbf{p}) = {} & \alpha_p a_+^\dagger(\mathbf{p}) + \lambda_p a_-(-\mathbf{p}) \\ & + \sum_{rs} \int \frac{d^3k}{(2\pi)^{3/2}} \frac{\beta_p(\mathbf{k}) \bar{u}_r^\mu(\mathbf{p}+\mathbf{k}) u_s(\mathbf{k})}{[2E_\pi(p) 2E_N(k) 2E_\Delta(p+k)]^{1/2}} \left[\frac{1}{\sqrt{3}} d_{+r}^\dagger(\mathbf{p}+\mathbf{k}) b_{ns}(\mathbf{k}) + d_{++r}^\dagger(\mathbf{p}+\mathbf{k}) b_{ps}(\mathbf{k}) \right] \\ & + \sum_{rs} \int \frac{d^3k}{(2\pi)^{3/2}} \frac{\gamma_p(\mathbf{k}) \bar{u}_r(\mathbf{p}+\mathbf{k}) u_s^\mu(\mathbf{k})}{[2E_\pi(p) 2E_N(p+k) 2E_\Delta(k)]^{1/2}} \left[\frac{1}{\sqrt{3}} b_{pr}^\dagger(\mathbf{p}+\mathbf{k}) d_{0s}(\mathbf{k}) + b_{nr}^\dagger(\mathbf{p}+\mathbf{k}) d_{-s}(\mathbf{k}) \right], \end{aligned} \tag{1}$$

and similarly for the negative pion. The pion annihilation operator is $a_\pm$, that of the Δ is d_{cs} (c denotes the charge, and s the spin), and that for the nucleon is b (n or p as subscript denotes neutron or proton). For the description of the Δ we use the Rarita-Schwinger spinors[9] u^μ. α_p, λ_p, $\beta_p(\mathbf{k})$, and $\gamma_p(\mathbf{k})$ are coefficients determined by the equations of motion and the requirement that $A_\pm$ satisfy boson commutation relations. The $\pi N\Delta$ interaction has been considered by many authors.[9-13] A special form, different from the simple derivative one, has been advocated in Ref. 9. However, because of our nonrelativistic approximation (effectively amounting to keeping the Δ on-mass-shell) we can choose the difference between the two interactions to be zero and use the derivative one. The interaction Hamiltonian density, to first order in the coupling g, is

$$\mathcal{H}_I = -g\,\partial_\mu \pi \bar{\psi}^\mu \psi + \text{h.c.}\,, \tag{2}$$

where isospin indices are dropped and ψ^μ is the Rarita-Schwinger field describing the Δ. Our normalization is such that the coupling of the Δ^{++} is g (the other couplings of the charged pions are then either g or $g/\sqrt{3}$). The equation of motion is

$$[H, A_+^\dagger(\mathbf{p})] = E(p) A_+^\dagger(\mathbf{p})\,, \tag{3}$$

where H is the Hamiltonian. Indeed, if (3) is satisfied, $A_+^\dagger(\mathbf{p})|0\rangle$ is an excited state with energy $E(p)$, since $A_+^\dagger(\mathbf{p})|0\rangle$ is an eigenstate of the Hamiltonian with eigenvalue $E(p)$. Here $|0\rangle$ is the ground state satisfying $H|0\rangle = 0$ and $A_\pm(\mathbf{p})|0\rangle = 0$. The *Ansatz* (1) cannot satisfy (3) exactly, but if after calculating the left-hand side of (3) we replace the fermion bilinear operators $b^\dagger b$ and $d^\dagger d$ by their expectation values, Eq. (3) can be satisfied by choosing the coefficients α, λ, β, and γ as follows:

$$\lambda_p = \alpha_p \frac{E(p) - E_\pi(p)}{E(p) + E_\pi(p)}\,, \tag{4}$$

$$\beta_p(\mathbf{k}) = -ig \frac{2E_\pi(p)}{E(p) + E_\pi(p)} \times \alpha_p \frac{1}{E_\Delta(p+k) - E_N(k) - E(p)}\,, \tag{5}$$

$$\gamma_p(\mathbf{k}) = +ig \frac{2E_\pi(p)}{E(p) + E_\pi(p)} \times \alpha_p \frac{1}{E_\Delta(k) - E_N(p+k) + E(p)}\,. \tag{6}$$

The above coefficients are normalized by the boson commutation relation imposed on the operator A and its Hermitian conjugate.

The energy of the excitation is determined by requiring the existence of a nontrivial solution for the above coefficients and is given as

$$E^2(p) = E_\pi^2(p) - \frac{16}{9} g^2 \int \frac{d^3k}{(2\pi)^3 E_\Delta E_N} \frac{E_\Delta E_N - \mathbf{k}\cdot(\mathbf{p}+\mathbf{k}) + M_\Delta M_N}{(E_\Delta - E_N)^2 - E^2(p)} (E_\Delta - E_N)\langle b^\dagger b(k)\rangle \times \left[\mathbf{p}^2 + \frac{[\mathbf{p}\cdot(\mathbf{p}+\mathbf{k})]^2 + E_\pi^2(\mathbf{p}+\mathbf{k})^2 - 2E_\pi E_\Delta \mathbf{p}\cdot(\mathbf{p}+\mathbf{k})}{M_\Delta^2} \right]. \tag{7}$$

In the above expression $E_\pi \equiv E_\pi(p)$, $E_\Delta \equiv E_\Delta(p+k)$, $E_N \equiv E_N(k)$, and $\langle b^\dagger b(k)\rangle$ is the Fermi-Dirac distribution $f_{\text{FD}}(E_n(k))$, since we assume the nuclear medium to be in thermal equilibrium, as well as a spin-zero and isospin-zero state. Examples of the pion spectrum as given by (7) are shown in Fig. 1 for $T=0$ and different densities close to the nuclear density. We determined the coupling g from the observed total width of the Δ, giving $g = 15.5\ \text{GeV}^{-1}$. The above spectrum is very similar to the widely used one of Ref. 14. Actually, the two coincide if one neglects the residual Migdal interaction and the exponentially falling form factor in Ref. 14, and the Fermi momentum $\mathbf{k}$ and terms of the order m_π/M_Δ (compared to 1) in expression (7).

The electromagnetic-annihilation matrix element for a pair of excitations created by the operators $A_\pm^\dagger$,

$$\mathcal{M}^\mu = [(2\pi)^3 2E_\pi(p) 2E_\pi(p')]^{1/2} \times \langle 0 | J^\mu(0) A_+^\dagger(\mathbf{p}) A_-^\dagger(\mathbf{p}') | 0 \rangle\,, \tag{8}$$

can be written as a sum of three terms

$$\mathcal{M}^\mu \equiv \mathcal{M}_0^\mu + \mathcal{M}_I^\mu + \mathcal{M}_{\text{nc}}^\mu\,. \tag{9}$$

The first represents the contributions which survive in the $g \to 0$ limit and is given as (we are interested only in the spatial components)

$$\mathcal{M}_0^i = (\alpha_p - \lambda_p)(\alpha_{p'} - \lambda_{p'})(p - p')^i\,. \tag{10}$$

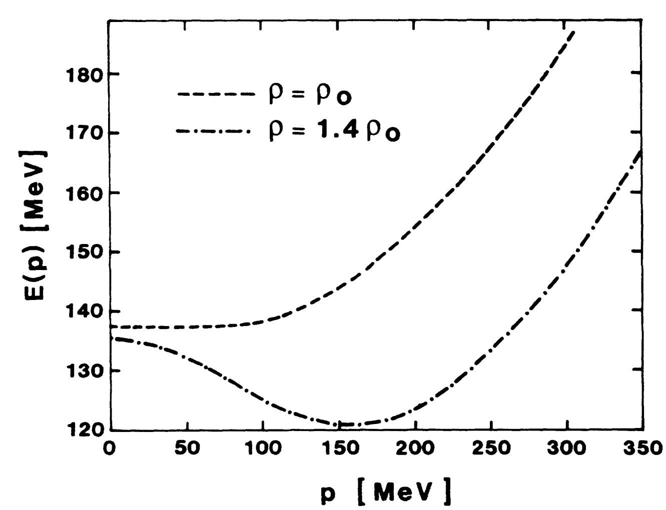

FIG. 1. Pion dispersion relation in nuclear matter at $T=0$ and for densities given in terms of the nuclear density ρ_0.

This is the matrix element used in Refs. 4 and 15, since the factors multiplying the momentum difference correspond to the probability that the pion branch is a pion and not a Δ hole. It is certainly justified to introduce this correction; however, there are other terms of the same order (in the coupling g) which need to be taken into account.

The second term on the right-hand side of (9) comes from the part of the electromagnetic current operator due to the derivative $\pi N\Delta$ interaction (2),

$$J_I^\mu = -ig\pi\bar{\Psi}^\mu\Psi + \text{h.c.} \tag{11}$$

(we dropped the isospin indices), and is necessary to assure the gauge invariance of the Lagrangian. These "contact" terms can be represented by Feynman diagrams given in Fig. 2(a). This contribution can be conveniently written as

$$\mathcal{M}_I^i = -(a_p-\lambda_p)(a_{p'}-\lambda_{p'})[F(p)p^i - F(p')p'^i]\,, \tag{12}$$

with the function $F(p)$ given by

$$\mathbf{p}^2F(p) = \frac{16}{9}g^2\int\frac{d^3k}{(2\pi)^3E_\Delta E_N}\frac{E_\Delta E_N - \mathbf{k}\cdot(\mathbf{p}+\mathbf{k}) + M_\Delta M_N}{(E_\Delta - E_N)^2 - E^2(p)}(E_\Delta - E_N)\langle b^\dagger b(k)\rangle \times\left\{\mathbf{p}^2 + \frac{[\mathbf{p}\cdot(\mathbf{p}+\mathbf{k})]^2 - E_\pi E_\Delta \mathbf{p}\cdot(\mathbf{p}+\mathbf{k})}{M_\Delta^2}\right\}. \tag{13}$$

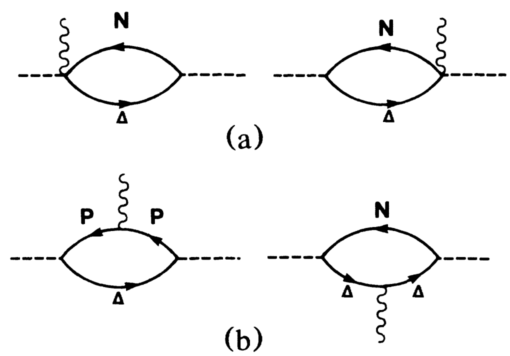

FIG. 2. Diagrammatic representation of the pion annihilation vertex. (a) Diagram corresponding to the "contact" interaction, coming from the part J_I^μ [expression (12)] of the electromagnetic current operator. (b) Diagrams coming from the electromagnetic interaction of the Δ and nucleon hole in the Δ hole produced by the pion.

The meaning of symbols E_π, E_Δ, and E_N is the same as in Eq. (7). Comparing the above expression with (7) we notice that $\mathbf{p}^2F(p)$ is approximately equal to $E_\pi^2(p) - E^2(p)$ (if the pion and nucleon momenta are not very large). This means that when the kinetic energy of the pion is almost compensated by the interaction, $F(p)\approx 1$ and the first and second terms on the right-hand side of (9) almost cancel each other.

Finally, there is a contribution where the photon is emitted from the Δ or the nucleon hole (if charged) with the usual coupling. The corresponding Feynman diagrams are shown in Fig. 2(b). Because of the large Δ and nucleon masses (compared to the pion mass) the current carried by them is suppressed and the third term in (9) is usually much smaller than the first two. It, however, becomes non-negligible for larger pion momenta (starting from around $2m_\pi$), especially if there is a close cancellation of the first two terms. The expression for two arbitrary momenta of the excitations $\mathbf{p}$ and $\mathbf{p}'$ is very long, due to the complicated expression for the propagator of the spin-$\frac{3}{2}$ Δ (a truly nonrelativistic expansion may help) and since we consider here only the relatively simple case of back-to-back annihilation (when $\mathbf{p}+\mathbf{p}'=0$) we give the expression only for that case:

$$\mathcal{M}_{\text{nc}}^i = 2p^i(a_p-\lambda_p)^2\frac{g^2}{9\mathbf{p}^2}\int\frac{d^3k}{(2\pi)^3E_\Delta E_N}\frac{E_\Delta E_N - \mathbf{k}\cdot(\mathbf{p}+\mathbf{k}) + M_\Delta M_N}{(E_\Delta - E_N)^2 - E^2(p)}\langle b^\dagger b(k)\rangle \times\left\{\mathbf{p}^2 + \frac{[\mathbf{p}\cdot(\mathbf{p}+\mathbf{k})]^2 - E_\pi^2(\mathbf{p}+\mathbf{k})^2}{M_\Delta^2}\right\}\left\{\frac{10\mathbf{p}\cdot(\mathbf{p}+\mathbf{k})}{E_\Delta} - \frac{2\mathbf{k}\cdot\mathbf{p}}{E_N}\right\}. \tag{14}$$

Now we calculate the rate for back-to-back dielectron production by pion annihilation in nuclear medium in thermal equilibrium, using the expression from Ref. 3, but also the annihilation matrix element in the medium, instead of the one in free space. The results, compared with the free-space rate, are shown in Fig. 3. A comparison with Ref. 3 shows a striking difference. The effect of the medium on the matrix element is extremely important and greatly reduces the enhancement due to the flattening of the pion spectrum, at least in the case of leptons emitted back to back. However, depending on the density, it is possible to have an enhancement for the invariant mass around $2m_\pi$, but much smaller than previously estimated.[3,15] The rate still diverges at the threshold if the pion spectrum has a minimum at nonzero momentum, the width of the peak being rather small but increasing with increasing density (in the region $\rho > \rho_0$). This happens because the matrix-element cancellation at the minimum of the pion spectrum is not exact and is becoming less pronounced near the threshold at higher densities. Also, around nuclear densities there are invariant masses for which the matrix-element cancellation is exact, leading to a sharp dip in the

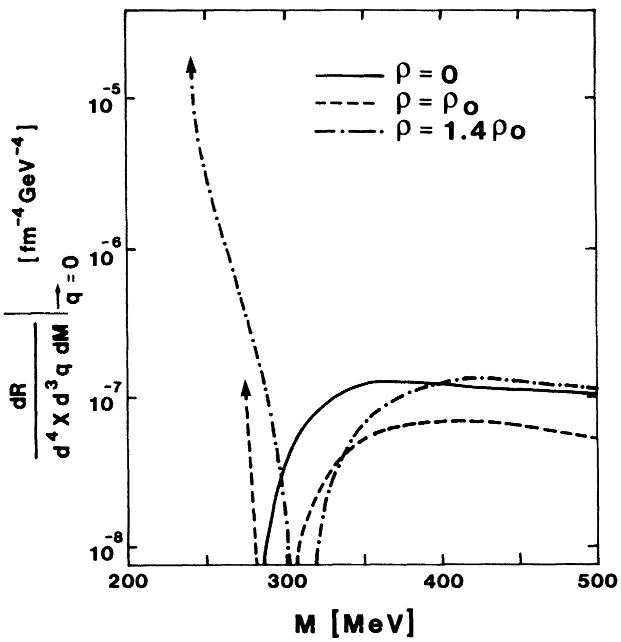

FIG. 3. Rate of dielectron production, per unit volume and time, with total three-momentum $\mathbf{q}=0$ and invariant mass M. The temperature is $T=100$ MeV and the densities are given in terms of the nuclear density ρ_0.

dielectron production rate. This will, however, probably disappear after the widths of the particles are taken into account.

The inclusion of the effects we neglected will change the details of our results. For example, the inclusion of the short-range Δ-nucleon-hole (Migdal) interaction will effectively reduce the strength of the $\pi N \Delta$ interaction, thus requiring higher density to achieve the same flattening of the pion spectrum. However, it will affect the annihilation matrix element in the same way. The importance of the matrix-element correction stems from the fact that it (mainly) comes from a term in the current operator which has the same structure as the interaction Hamiltonian, from which it arises. The close relation between the pion self-energy and electromagnetic-annihilation vertex can be seen also from the Ward-Takahashi identity, which was violated in the previous treatments, but is satisfied in our approach. There are other processes contributing to the dilepton production, notably the annihilation of a pion with Δ hole. However, since the Δ-hole branch is pushed up by the interaction, this process does not contribute significantly in the dilepton-mass range considered in this Letter.

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