New Relation between the Proton Quark Spins and the η' Coupling

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A generalization of the Goldberger-Trieman relation for the singlet channel is obtained. This gives a new relation between the total quark contribution to the proton spin ($\Delta\Sigma$) and the η'-meson coupling constants ($g_{\eta' NN}$ and $f_{\eta'}$). Using these we find $\Delta\Sigma \approx 1$, precisely as in the naive quark model.

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The spin effect discovered two years ago in polarized deep-inelastic scattering by the European Muon Collaboration[1] (EMC) is a very important result, which has attracted much interest from several authors with many theoretical speculations.[2] A great deal of attention has been paid to the mechanism of compensation of quark and gluon spin contributions due to the interaction of the gluons with the virtual photon via the axial anomaly.[3,4] Recently a series of papers have emphasized the relation of this spin effect with chiral dynamics and with the so-called "U(1) problem," whose most relevant question is the large mass of the η' meson. Cheng and Li[5] have found, analyzing the deviation of the Goldberger-Treiman relation from experiment, that the gluon spin contribution is negative, making the situation even more confused. Fritzsch[6] has argued that the chiral dynamics and the Adler-Bardeen relation give nearly a zero quark contribution to the spin of the proton. Veneziano,[7] on the other hand, has stated that the EMC result implies the decoupling of the η' meson from the nucleon. In this paper, we will show that none of these results is quite correct, although they have triggered some new ideas for understanding this fundamental problem of the spin of the proton. We will also give a correct relation between the quark contribution to the proton spin and the η' coupling constants which allows us to estimate this contribution.

In order to see this, let us consider the Adler-Bardeen relation

$$\partial_\nu j_\nu^5 = \partial_\nu K_\nu , \tag{1}$$

where j_ν^5 is the axial-vector singlet quark current

$$j_\nu^5 = \sum_{f=1}^{N_F} \bar{q}_f \gamma_\nu \gamma_5 q_f , \tag{2}$$

N_F being the number of quark flavors, and K_ν is the topological current

$$K_\nu = N_F \frac{\alpha_s}{2\pi} \epsilon_{\nu\mu\sigma\rho} A_\mu^a \left(\partial_\sigma A_\rho^a - \frac{g}{3} f_{abc} A_\sigma^b A_\rho^c \right) , \tag{3}$$

with the main property

$$\partial_\nu K_\nu = N_F \frac{\alpha_s}{8\pi} F^a \tilde{F}^a \equiv Q , \tag{4}$$

where A_μ^a is the gluon field and $F_{\mu\nu}^a$ denotes its strength.

Note that all these expressions are written in the chiral limit, but terms proportional to the quark masses can be added explicitly as in Ref. 5. For nonsymmetric matrix elements between nucleon states of these currents j_ν^5 and K_ν, one has the following general form:

$$\langle p' | j_\nu^5 | p \rangle = \bar{u}(p') [\gamma_\nu \gamma_5 G_1(q^2) + q_\nu \gamma_5 G_2(q^2)] u(p) , \tag{5}$$

$$\langle p' | K_\nu | p \rangle = \bar{u}(p') [\gamma_\nu \gamma_5 \tilde{G}_1(q^2) + q_\nu \gamma_5 \tilde{G}_2(q^2)] u(p) , \tag{6}$$

where $q = p' - p$, $\bar{u}, u$ are the polarized proton wave functions, and the G's are form factors. In the limit $q^2 \to 0$, G_1 and $\tilde{G}_1$ are expressed through the spin content of the proton[4]

$$G_1(0) = \Delta\Sigma - \Delta\tilde{g} , \tag{7}$$

$$\tilde{G}_1(0) = -\Delta\tilde{g} \equiv -\frac{\alpha_s}{2\pi} N_F \Delta g . \tag{8}$$

Here $\Delta\Sigma = \sum_{f=1}^{N_F} \Delta q_f$, where $\Delta q_f, \Delta g$ denote the difference of parton distributions with helicity aligned (+) and antialigned (−) with the proton helicity; e.g., for quark-antiquark of flavor f

$$\Delta q_f = \int_0^1 dx [q_f^+(x) - q_f^-(x) + \bar{q}_f^+(x) - \bar{q}_f^-(x)] ,$$

and similarly for Δg. All quantities depend on a scale (ultraviolet regularization) parameter μ^2.

By substituting Eqs. (5) and (6) into Eq. (1) one gets

$$2MG_1(q^2) + q^2 G_2(q^2) = 2M\tilde{G}_1(q^2) + q^2 \tilde{G}_2(q^2) , \tag{9}$$

and the limit $q^2 \to 0$ depends on whether or not G_2 and $\tilde{G}_2$ have a zero-mass pole in q^2. This is precisely the question related to the U(1) problem.

In Ref. 6, Fritzsch assumed either that there is no zero-mass Nambu-Goldstone boson or that the contribution from the ghost pole is small in the singlet channel and therefore for $q^2 \to 0$ both $q^2 G_2(q^2) \to 0$ and

$q^2\tilde{G}_2(q^2)\approx 0.$[8] As a result he obtained by using Eqs. (7) and (8)

$$\Delta\Sigma\approx 0. \tag{10}$$

However, the statement about the absence of the pole at $q^2=0$ is only true for $\langle p'|j_\nu^5|p\rangle$ because j_ν^5 is a gauge-invariant current. On the other hand, $\langle p'|K_\nu|p\rangle$ generally has a ghost pole, because K_ν is *not* gauge invariant. As a result Eq. (10) has a nonzero right-hand side.

The existence of such a ghost pole is necessary for the propagator

$$\langle 0|T(QQ)|0\rangle = q_\mu q_\nu \langle 0|T(K_\mu K_\nu)|0\rangle = -\lambda^4 \neq 0, \tag{11}$$

to be finite at $q^2=0$ and this is the way the η' meson acquires the mass[9]

$$m_{\eta'}^2 = \lambda^4/f_{\eta'}^2, \tag{12}$$

due to the mixing of the Nambu-Goldstone boson with the ghost. Of course, one must also add the small contribution from the nonzero quark masses which we have disregarded. The physical reason for this ghost[10] is a periodic dependence of the potential energy in gluodynamics (and in QCD) on a collective variable $X=\int d^3x\, K_0(x,t)$. This means that the nature of the pole is totally nonperturbative because in a perturbative approach we work in a local minimum of the potential and we do not feel its structure, as a whole.

The contribution of this ghost pole into $\tilde{G}_2(q^2)$ was calculated in Ref. 7 as an extension of the Goldberger-Treiman relation to the U(1) sector. However, the form factor $\tilde{G}_1$ in Eq. (9) was implicitly neglected in Ref. 7 which leads one to the conclusion that the EMC result, namely, $G_1(0)\approx 0$,[11] would imply a decoupling of the Okubo-Zweig-Itzuka-rule conserving part of the η' meson from the $N\bar{N}$ state. By including this term we obtain instead the relation

$$G_1(0)-\tilde{G}_1(0)\equiv\Delta\Sigma = \frac{\sqrt{N_F}f_{\eta'}}{2M}g_{\eta' NN}, \tag{13}$$

resulting from the following calculation of the ghost-pole residue. In the cross channel by considering the matrix element $\langle 0|\partial_\nu K_\nu|N\bar{N}\rangle$ and saturating it by the η' pole which contributes to G_2 only, one obtains

$$q^2\tilde{G}_2(q^2) = \langle 0|Q|\eta'\rangle \frac{1}{q^2-m_{\eta'}^2}\langle \eta'|N\bar{N}\rangle,$$

the last factor being just the coupling constant $g_{\eta' NN}$. [It is also assumed that there is no direct coupling of the ghost with the nucleon. Such a coupling would lead to an effective contact term $(\bar{N}\gamma_5 N)(\bar{N}\gamma_5 N)$ in NN scattering, which seems not to have been seen.] By using the Lehmann-Symanzik-Zimmerman reduction formula, the first one can be reduced to

$$\langle 0|Q|\eta'\rangle = \int d^4x\, e^{iqx}(\Box + m_{\eta'}^2)\langle 0|T(Q\eta'(x))|0\rangle = \frac{-q^2+m_{\eta'}^2}{m_{\eta'}^2 f_{\eta'}}\langle 0|T(QQ)|0\rangle,$$

where we have made use of PCAC (partial conservation of axial-vector current), i.e., $\eta'(x)=\partial_\nu j_\nu^5/m_{\eta'}^2 f_{\eta'}$, and of the Adler-Bardeen relation [Eq. (1)]. So in the limit $q^2\to 0$ one obtains, using Eq. (11) and the expression for the η' mass Eq. (12),

$$\lim_{q^2\to 0} q^2\tilde{G}_2(q^2) = \frac{\lambda^4\sqrt{N_F}}{m_{\eta'}^2 f_{\eta'}}g_{\eta' NN} = \sqrt{N_F}f_{\eta'}g_{\eta' NN}. \tag{14}$$

Two comments are now in order.

(i) The form factor $\tilde{G}_1$ and the pole contribution $\tilde{G}_2$ are of quite different nature and there is no double counting in taking into account both of them. In principle, $\tilde{G}_1$ could be calculated from perturbation theory, whereas the ghost pole is totally nonperturbative.

(ii) Because of the nonperturbative nature of the right-hand side of Eq. (13) it is independent of any regularization parameter μ^2 and therefore invariant under renormalization. The same has to be true for the left-hand side and it is known[4] that $\Delta\Sigma$ is just the quantity which obeys this property and not the form factor G_1, which is known to be multiplicatively renormalized as implied by Ref. 4. This is why the relation (13) looks more natural than a similar relation in Ref. 7.

The contribution of the ghost pole was also left out in Ref. 5 [see Eqs. (10a)-(10c) of Ref. 5]. Restoring this contribution one has to make the following change:

$$\Delta\tilde{g}\to\Delta\tilde{g}-\frac{\sqrt{N_F}f_{\eta'}}{2M}g_{\eta' NN}. \tag{15}$$

So the negative value obtained for the quantity (15) in Ref. 5 by studying the deviation of the Goldberger-Treiman relation compared to experiment simply means that the second term in Eq. (15) is a bit larger than the first.

The parameters which enter in the right-hand side of the relation (13) are known independently of the EMC experiment. So this relation is important by itself independently of the validity of the EMC result $\Delta\Sigma - \Delta\tilde{g}\approx 0$.

The relation (13) provides, in fact, an estimate of the quark contribution to the proton spin. Actually $f_{\eta'}$ can be obtained from the comparison between the 2γ decay rates of η' and π^0 (with no η-η' mixing since we disregard quark-mass corrections) and one finds

$$f_{\eta'} = 1.26 f_\pi, \tag{16}$$

where $f_\pi = 132$ MeV. The coupling constant of η' to nucleons has been estimated from the one-boson exchange NN potential[12] and was found to be

$$g_{\eta' NN} = 7.5\pm 1.5. \tag{17}$$

Although we know that the reliability of this number is questionable,[13] it is remarkable that when inserting Eqs. (16) and (17) into Eq. (13) we find

$$\Delta\Sigma\approx 1.14\pm 0.2, \tag{18}$$

which means that the proton spin is entirely carried by the quarks just as in the naive quark model. Of course if $\Delta\Sigma \approx \Delta\tilde{g} \approx 1$, Δg is large but it is nearly compensated by the orbital part in the angular momentum sum rule.[14]

Using the constraint from hyperon decay we have[15]

$$\Delta u + \Delta d - 2\Delta s = 0.685 \pm 0.08\,, \tag{19}$$

which combined with Eq. (18) yields a positive contribution for the strange quark

$$\Delta s \approx 0.10 \pm 0.09\,. \tag{20}$$

At first sight this sign seems to contradict the result extracted from νp elastic scattering[16]

$$\Delta s = -0.15 \pm 0.09\,, \tag{21}$$

but if the anomaly contribution is taken into account Eq. (21), in fact, reads[17]

$$\Delta s - \tfrac{1}{3}\Delta\tilde{g} = -0.15 \pm 0.09 \tag{22}$$

and with $\Delta\tilde{g} = 1$ it leads to a positive value of the same order of $+10\%$ with a rather large error.

From its derivation, Eq. (13) is approximate because a continuum spectrum contribution was disregarded. It is exact only at $q^2 = m_{\eta'}^2$, in the same way as a similar expression in Ref. 7 is exact only at $q^2 = 0$. In this sense, Eq. (13) has a less fundamental meaning as the original Goldberger-Treiman relation $\Delta u - \Delta d = \sqrt{2} f_\pi g_{\pi NN}/2M$, but both of them relate a short-distance quark picture of the proton to its long-distance classical picture of a core surrounded by a meson cloud. Remarkably enough, the gluon content somehow dropped out explicitly of the picture but implicitly it remains only in the long-distance η'-meson coupling constants and therefore we might wonder about the role of the long-sought 0 gluonium.

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