Stark-Ladder Resonances in the Propagation of Electromagnetic Waves

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We discuss the existence of the electromagnetic Stark ladder in some optic systems by studying the propagation of light. The transmission coefficient is calculated by means of the transfer-matrix method. It is shown that although the transfer matrix is different from the quantum case it can give, in some particular configurations, a Stark-ladder resonance structure similar to the one observed in quantum systems. The relation between these resonances and a Fabry-Perot-like interference is discussed. Consideration is given to the observability of these ladders in the laboratory.

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As is well known, Stark-ladder resonances[1] (SLR), that is, a series of virtual states whose energy is equally spaced, appeared for the first time in quantum mechanics in the study of a charged particle in the presence of a one-dimensional potential $V(x)$ of the form

$$V(x)=V_p(x)+Eex\,, \tag{1}$$

where $V_p(x)$ is a periodic potential of period p and Eex is the potential of the particle of charge e in the presence of a constant electric field of intensity $-E$. We note that the potential of Eq. (1) has the crucial property

$$V(x+np)=V(x)+Eenp\,, \tag{2}$$

n being an arbitrary integer.

Although the existence of SLR from a theoretical point of view was controversial for some years and their observation in the laboratory was questioned until recently,[2] at present their existence is well established.

In this Letter, we discuss the existence of "electromagnetic SLR" for light traveling in appropriate stratified media, starting with the analogy between the equations that describe the quantum and electromagnetic phenomena. This analogy has mainly been discussed in papers where the Anderson localization for photons, photon band structure, or elastic band structure is studied.[3–8] It has been pointed out in some of these papers that carrying out transport experiments with electrons presents too many difficulties due to the electron-electron and electron-phonon interactions and the scattering caused by the impurities. Furthermore, for the case of SLR, it is necessary to use very strong electric fields which increase the difficulties. This has made the interpretation of the results doubtful in most of the cases. On the other hand, with photons these problems are not present and observation of the SLR appears to require less restrictive experimental setups.

For the electromagnetic case the system we are referring to is a medium with a dielectric function $\epsilon(z)$ which, for a given frequency ω, depends only on z and can be expressed as a sum of a periodic function of period p plus a linear term of slope g. Thus $\epsilon(z)$ satisfies an equation similar to Eq. (2), i.e.,

$$\epsilon(z+np)=\epsilon(z)+gnp\,. \tag{3}$$

For this system the propagation of transverse electromagnetic waves of frequency ω is described by

$$[\partial^2/\partial z^2+\epsilon(z)k^2]F(z)=Q^2F(z)\,, \tag{4}$$

which is obtained from the wave equation after a Fourier transform from the variables x, y, and t to the variables k_x, k_y, and ω.[9] Here $F(z)$ represents any of the components of the electromagnetic field, $Q^2=k_x^2+k_y^2$, and $k=\omega/c$, c being the speed of light. This equation is similar to the Schrödinger equation with Q^2 and $\epsilon(z)$ playing the roles of the energy and the potential, respectively. However, from a mathematical point of view, these two problems are not totally equivalent. The boundary conditions at the interfaces that the wave function and the electromagnetic field must satisfy are different.

Let us discuss briefly a property of Eq. (4) for infinite systems. If we replace z by $z-np$ in (4), we obtain

$$[\partial^2/\partial z^2+\epsilon(z)k^2]F'(z)=(Q^2+gnpk^2)F'(z)\,, \tag{5}$$

where $F'(z)=F(z-np)$. Equation (5) is similar to Eq. (4) except for the term added to Q^2. Therefore, if for a given value Q_0^2 of Q^2 there is a nontrivial solution of (4) then there must be a family of solutions (the Stark ladder) for other values of Q^2 given by $Q_0^2+gnpk^2$. This implies in turn that the distance between any two adjacent solutions of that family (measured in units of k^2) is gp. The same arguments are applicable to the Schrödinger equation with the indicated changes and when the potential is given by Eq. (1).

Although the above reasoning suggests the existence of the SLR, it does not prove they they indeed exist, since it starts from the hypothesis that there exists a solution associated with Q_0. Neither is the existence of a continuum of Q values around Q_0 excluded, in such a way that the Q_0 family could be indistinguishable from other solutions. A family of solutions must be special or different in some sense (for example, a family of resonances) with respect to the neighbor solutions in order to be observable.

We know that in finite quantum systems, where Eq. (1) is satisfied only in a restricted interval of x, such special families of solutions are sometimes present,[10-12] and therefore we expect that it is also true for the electromagnetic case. In what follows we will show that under certain circumstances those families of resonances indeed exist. In particular, we will restrict ourselves to systems where the dielectric function is equal to a real constant for intervals as shown in Fig. 1. As we can see, in each interval of length p we have two sublayers of widths p_1 and p_2 ($p_1+p_2=p$) with different values of $\epsilon(z)$. These values are given by the superposition of a series of steps with increasing height plus a series of barriers with height h.

In order to observe the resonances we have studied the behavior of the transmission coefficient T as a function of Q^2 keeping ω fixed for the case of polarized light with magnetic field parallel to the interfaces. T was calculated by means of the expression[9]

$$T \equiv \frac{\text{transmitted power}}{\text{incident power}} = \frac{\sqrt{\epsilon_T}\,|E_T|^2}{\sqrt{\epsilon_I}\,|E_I|^2}, \tag{6}$$

where $|E_I|$ and $|E_T|$ are the electric-field intensities associated with the incident and transmitted electromagnetic waves, respectively, while ϵ_I and ϵ_T are the dielectric functions of the first and last layers. The fields were calculated by using the well-known transfer-matrix method. However, in order to introduce our notation we discuss briefly the method used here. We have assumed that the left end of our system is at $z=z_0=0$ and we have defined the slab number -1 as the vacuum at $z<0$ with $\epsilon_{-1}=1$ (see Fig. 2). The right end is the layer number N at $z>z_N$. We have assumed also that from the region $z<0$ a plane wave incident on the system with wave vector $\mathbf{k}$ which has a positive z component. Since in each layer the function $\epsilon(z)$ is a constant, the solutions of (4) are also plane waves. We have denoted by $\mathbf{k}_i$ the wave vector of the plane wave traveling to the right in the slab number i with dielectric function ϵ_i as shown in Fig. 2. At the interface $z=z_{i+1}$ the light is partly reflected with wave vector $\mathbf{k}_i'$ and partly transmitted with wave vector $\mathbf{k}_{i+1}$ to the slab $i+1$. All the wave vectors are in the x-z plane.

If k_i and k_i^z denote the magnitude and the z component of $\mathbf{k}_i$, respectively, it can be shown that[9]

$$\mathbf{k}_i=(Q,0,k_i^z), \tag{7a}$$

$$\mathbf{k}_i'=(Q,0,-k_i^z), \tag{7b}$$

$$k_{-1}=\omega/c=k, \tag{8a}$$

$$k_i/\epsilon_i^{1/2}=k_{i-1}/\epsilon_{i-1}^{1/2}=k, \tag{8b}$$

and

$$k_i^z=k(\epsilon_i-q^2)^{1/2}, \tag{8c}$$

with $q=Q/k$. Thus Q is the component parallel to the interfaces of the wave vector $\mathbf{k}_i$ and has the same value in all the layers due to Snell's law. Since $Q=k\sin\theta(\omega/c)\sin\theta$, where θ is the angle of incidence, the experiment can be carried out with monochromatic light by changing θ only. We see that the values of Q are bounded according to the relation $0\le Q\le k$. This is another difference with respect to the quantum case since there

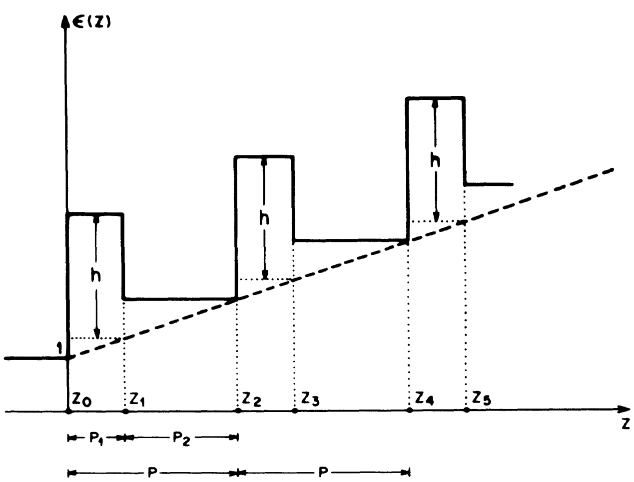

FIG. 1. Plot of $\epsilon(z)$ vs z (solid line). The function $\epsilon(z)$ is a sum of a linear term of slope g (broken line) plus a periodic function of period p. The points z_i are such that $z_{i+2}=z_i+p$ and the value of $\epsilon(z)$ inside the interval $[z_{i-1},z_i]$ is z_ig+1 for i even or z_ig+1+h for i odd.

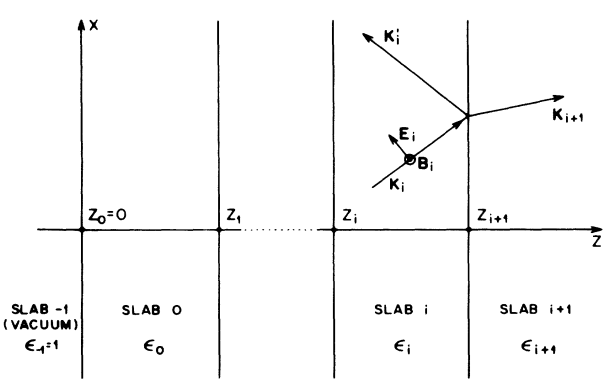

FIG. 2. The multilayered medium with dielectric function as in Fig. 1. In the slab i a traveling ray, with wave vector $\mathbf{k}_i$ in the x-z plane, is reflected and refracted at the interface z_{i+1}. The index i runs from -1 (corresponding to the vacuum at $z<0$) to N (corresponding to the last layer at $z>z_N$).

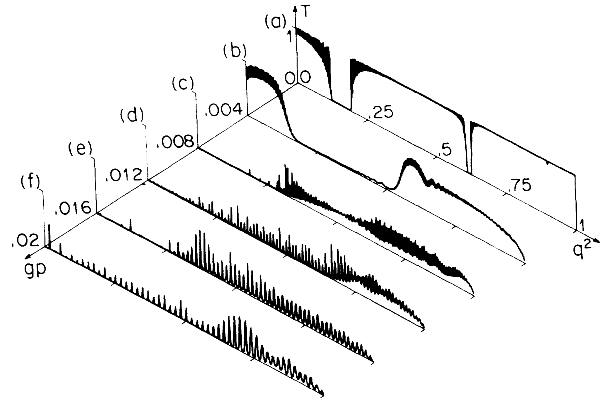

FIG. 3. Plots of the transmission coefficient as a function of q^2 for different values of gp. The values of h and N are 10 and 201, respectively. The curve f of Fig. 4 is also a member of this group corresponding to $gp = 0.1$.

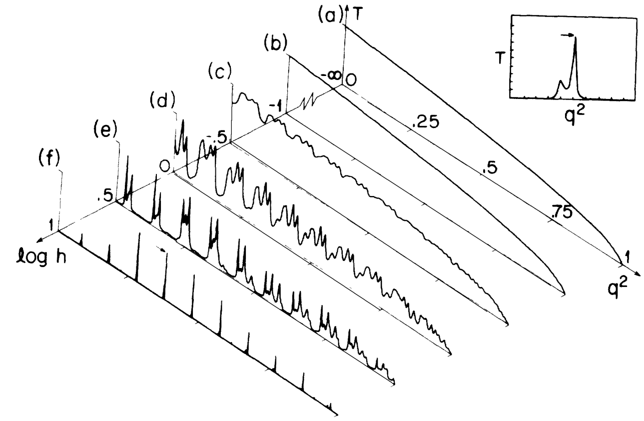

FIG. 4. Plots of the transmission coefficient as a function of q^2 for different values of h on a logarithmic scale. The values of gp and N are 0.1 and 201, respectively. Inset: An amplification of the peak marked with arrow in curve f.

the energy is not restricted.

Since any component of the total electric field in the slab i must satisfy Eq. (4), they are of the form $a_i \exp(ik_i^z z) + b_i \exp(-ik_i^z z)$ with a_i and b_i constants. It can shown that the vectors (a_i, b_i) and (a_{i+1}, b_{i+1}) are related by means of a 2×2 transfer matrix M_i associated with the interface i whose elements $(M_i)^{1,1}$ and $(M_i)^{1,2}$ are given by

$$(M_i)^{1,1/2} = \tfrac{1}{2}(1 \pm \alpha)\exp[\pm i(k_i^z \mp k_{i+1}^z) z_{i+1}] , \quad (9)$$

with $\alpha = \epsilon_i k_{i+1}^z / \epsilon_{i+1} k_i^z$. As mentioned, this matrix is different from the one in the quantum case.[13] However, this is not surprising since the boundary conditions are different.

We have used the product $M_N M_{N-1} \cdots M_2 M_1 M_0$ in order to find the relation between the vectors (a_{-1}, b_{-1}) and (a_N, b_N). Finally, the transmission coefficient was calculated by using that relation.

In Figs. 3-5 we have plotted T as a function of q^2 for different values of the parameters defined in Fig. 1. In all figures $k = 10^7$ m^{-1}, $p_2 = 10^{-6}$ m, and $p_1 = p_2/10$, which correspond to a wavelength of $2\pi \times 10^3$ Å in the optic region and an interval p_2 of the order of two wavelengths. The six curves in Fig. 3 show the evolution of the plot T vs q^2 as a function of gp. As we can see, for $gp = 0$ we obtain the Bloch-theorem prediction, i.e., a band structure due to the periodicity of the system.[5-7] However, if gp is sufficiently increased, the structure evolves to a series of "equally" spaced peaks which we identify as the SLR. When gp is further increased to 0.1, we obtain the curve f of Fig. 4. In this case the SLR are sharper.

The curves in Figs. 4 and 5 show the effect of the parameter h. The scale on the h axis in Fig. 4 is nonlinear in order to show more clearly how the SLR are formed. On the other hand, in Fig. 5 the scale is linear and we have an abrupt change from curve a to curve b. Note that although in all the cases $\epsilon(z)$ satisfies Eq. (3), only for some values of gp and h are the SLR apparent. These two figures show that when $h = 0$ there is no SLR structure. This was also the case for the simpler system with $p_1 = p_2$, $h = 0$. In most of the plots there are too many peaks and it can be laborious to identify a given family of resonances. Even in a "clear" plot such as Fig. 4, curve f, when the interval around each resonance is analyzed with a finer mesh, one can distinguish neighboring resonances (see, for example, the inset in Fig. 4).

From curves f in Figs. 3-5, it is clear that when the SLR are present the distance between peaks increases with gp. In Fig. 4, curve f, the value of gp is double that in Fig. 5, curve f, so that the separation between peaks is "double" in Fig. 4, curve f. However, because of the finite size of the system, the separation of the SLR is not exactly the predicted value gp, a property that has also been observed in finite quantum systems.[12] However, the discrepancy is small. For example, for the case cor-

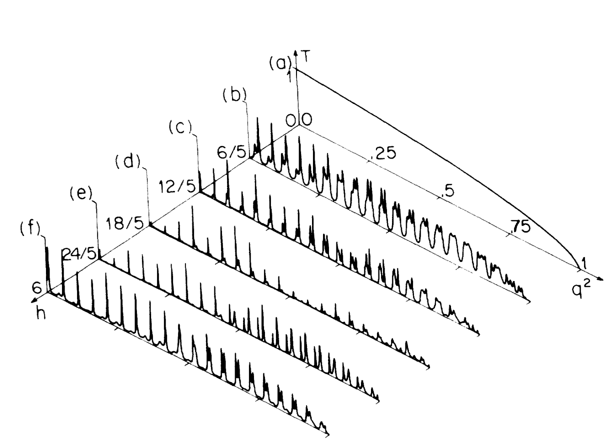

FIG. 5. Same as Fig. 4 but for $gp = 0.05$ and $N = 101$ and a linear scale on the h axis.

responding to Fig. 4, curve f, the average distance between peaks is 0.099027 while $gp=0.1$. In Fig. 3, curve f, the average distance is 0.0195 while $gp=0.02$. This discrepancy is reflected in the fact that there are 51 peaks in Fig. 3, curve f, instead of 50, as expected from the quotient 1/0.02.

The values of the parameters N, g, p, h, and ω needed in order to have clear SLR can be varied in a relatively wide range. In particular, the values of these parameters corresponding to curves f in Figs. 4 and 5 imply that the values of $\epsilon(z)$ must be as large as 21 and 9.5, respectively, with a null imaginary part. At present, values of that order in the optic and infrared regions can be obtained in some semiconductors such as Ge, Si, GaAs, or GaP.[14] By combining some of these materials with ZnS or ZnSe (Ref. 15) in different concentrations one can obtain a wide range of values of $\epsilon(z)$.

We have also done an analysis of the relation between the structure of the plots T vs q^2 and the interference phenomenon. The idea was to consider couples of interfaces as an interferometer of the Fabry-Perot type.[16] It was found that electromagnetic SLR can also be expected from the calculation of the q^2 values which produce destructive interference in the reflection at the interfaces, even in the simpler case when the values of the amplitude transmission and reflection coefficients at each interface are not taken into account. These coefficients are related with the intensities of the electromagnetic waves in each reflection and transmission. They determine in turn the relative importance of the multiple reflections that also were neglected. However, it was found that there is no simple relationship between these values of q^2 and the real positions of the resonances. The process of interference which is the origin of the structure of the plots T vs q^2 may be quite complex.

Actually, the selection of the particular system with its corresponding values of q, associated with distinguishable SLR, can be done after calculations are performed with the transfer-matrix method. This process led us to demonstrate explicitly that the electromagnetic SLR indeed exist.

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