Roton-Roton Scattering in Liquid ^{4}He: A Direct Observation

A. C. Forbes and A. F. G. Wyatt
Department of Physics, University of Exeter, Exeter EX4 4QL, United Kingdom
(Received 19 December 1989)

We have directly observed the effects of roton-roton scattering by crossing two beams of ballistic rotons in liquid ^{4}He at a temperature of ~ 0.1 K. One beam is weak and is used as the probe. Its attenuation is measured as it passes through the stronger scattering beam which is predominantly perpendicular to the probe beam. The beams are pulsed and the dispersion of the roton velocities enables the momenta of the rotons to be resolved by time-of-flight measurements. It appears that the scattering is strongly wave-vector dependent.

PACS numbers: 67.40.−w, 63.20.Hp

The excitation model of liquid ^{4}He envisages a ground state and excitations from this ground state. The excitations, phonons and rotons, are well defined with an inverse lifetime $\Gamma \ll \omega$, where ω is the energy of the excitation. The excitation picture is well established and the dispersion curve of the excitations has been measured by neutron scattering (see Fig. 1). The lifetime of the excitations is limited by scattering between the excitations and so the inverse lifetime is directly proportional to the prevailing density of excitations. Therefore Γ increases with temperature as the ambient excitation density increases. Various methods of measuring the lifetimes confirm this.[1-6] However, these measurements can only be made at high temperatures where the lifetime is short enough to be detectable. At low temperatures the lifetimes are orders of magnitude longer and this can only be seen by time-of-flight measurements.[6]

The scattering measurements to date rely on a thermal distribution of excitations, so the excitation lifetimes which are obtained are necessarily averages over the energies of the excitations and the scattering angles. As it is likely that scattering cross sections are dependent on energy and angle, a more refined type of experiment is desirable. In principle one would like to have two well-defined beams of monoenergetic excitations, let them interact, and monitor the resulting scattering. The experiment we describe here goes some way towards this idea and is the first example of the scattering of beams of excitations in the condensed state.

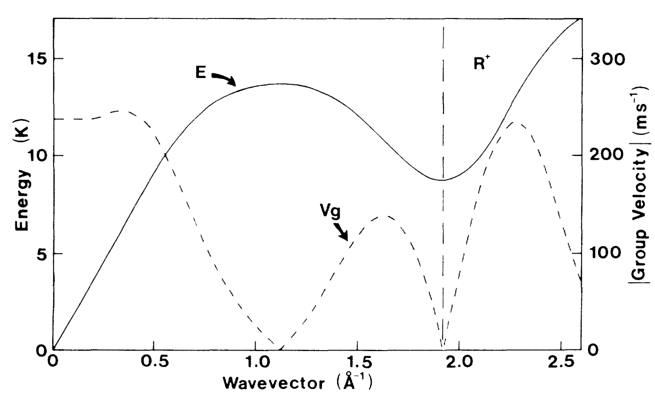

FIG. 1. The dispersion curve for ^{4}He, with the R^+ region indicated. Also shown is the modulus of the group velocity as a function of wave vector.

We inject a probe beam of rotons into liquid ^{4}He at $T \leq 0.1$ K. At this ambient temperature they are ballistic and travel through the liquid ^{4}He without scattering or decay. A second beam of rotons is injected into the path of the probe beam so the two beams cross predominantly at right angles. The probe beam is attenuated as the rotons in it scatter with the rotons in the second beam. We show later that one scattering event is sufficient for a probe roton not to be detected. The second beam is spatially broader and has a higher roton density than the probe beam so that the attenuation of the probe beam can be analyzed in a clear way.

Both roton beams are pulsed and time-of-flight measurements are made. Rotons have a large velocity dispersion, see Fig. 1, and so if they are injected in a short pulse, rotons with different energies arrive at the interaction region at different times. In this way we are able to obtain the scattering rate as a function of roton wave vector.

The experimental arrangement is shown in Fig. 2(a). The probe beam of rotons is injected by a thin metal film which is pulse heated for 10 μs with a power of 2 mW mm^{-2}. This has been shown to give a fully ballistic beam of rotons.[7] The broad beam from the heater h_1 is collimated into a narrow beam by collimator c_1. This beam is incident on the free surface of the liquid He at an angle of incidence of 12° where it quantum evaporates ^{4}He atoms.[8] A bolometer, at an angle of 47° to the normal, detects these atoms. This procedure to detect the rotons is necessary as ballistic rotons cannot be detected to any useful degree of a metal-film bolometer in the liquid.

The scattering rotons are injected above the collimator c_1 by a similar heater h_2. This is pulse heated with a variety of powers and pulse lengths. The rotons from h_2 are emitted over a range of angles but from the geometry the predominant angle between scattering rotons is 90°.

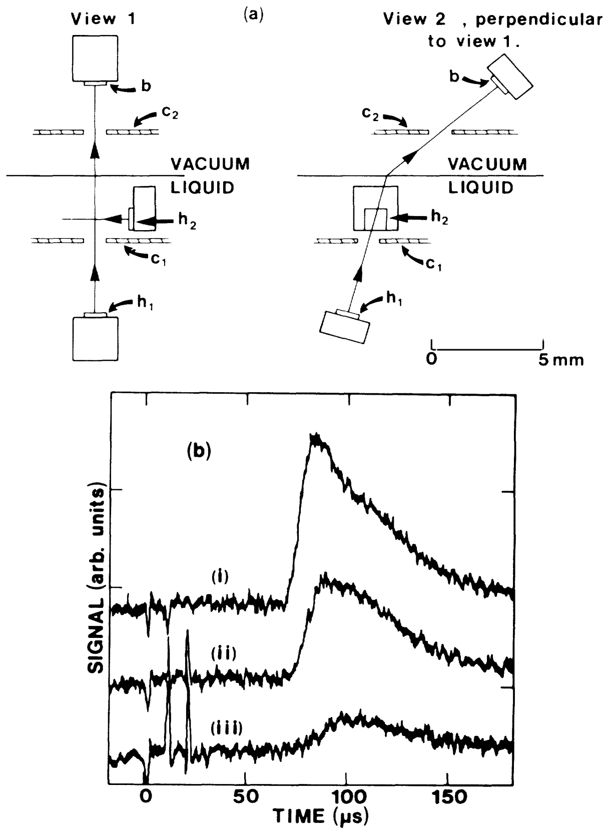

FIG. 2. (a) The experiment shown schematically from two viewpoints perpendicular to each other. The probe roton pulse is generated by h_1, and the scattering pulse by h_2. At the liquid surface the probe rotons evaporate atoms which travel to bolometer b. (b) Typical signal shapes received; the probe-pulse power is constant at 2 mW mm^{-2}. The scattering-heater power is 0, 1.26, and 5.00 mW mm^{-2} in (i), (ii), and (iii), respectively.

The two heaters are pulsed separately with an adjustable time delay between them. Both heaters also inject phonons into the liquid ^{4}He. Phonons above the energy for spontaneous decay, $\omega_c \sim 10$ K, have long mean free paths. The phonons from h_1 do not give any quantum evaporated atoms at the bolometer angle we have chosen. The low-energy phonons from h_1 have too low a density in the interaction region to have any effect. The phonons from h_2 do scatter the probe rotons but as they mostly travel faster than the rotons and as they have little velocity dispersion their effect can be time-resolved out. With a suitable delay between the heater pulses, the probe rotons reach the interaction region after the phonons from the h_2 heater have passed the collimator.

Typical detected signals are shown in Fig. 2(b). The top trace shows the unscattered probe pulse. It is much longer than the heater pulse length of 10 μs because of the roton and atom velocity dispersion. The middle and lowest traces show attenuation caused by the rotons injected from the heater h_2. It is striking that the probe pulse is attenuated more at earlier rather than later times. As the higher momentum rotons give the earlier signal this immediately suggests that they are scattered more strongly. This conclusion is borne out by a more detailed analysis.

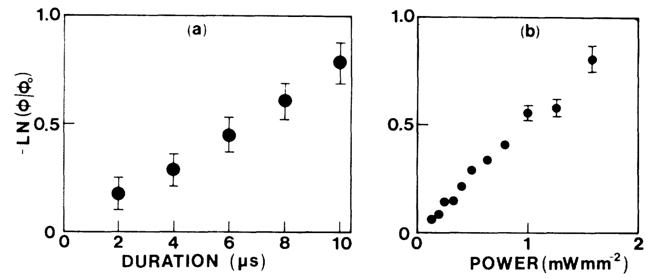

FIG. 3. (a) The linear dependence of the probe-pulse attenuation with scattering-pulse duration. (b) The linear dependence of the probe-pulse attenuation with scattering-pulse power.

We now relate the attenuation of the probe pulse to the density of scattering rotons and the interaction cross section. The probe rotons are scattered by the rotons from h_2, which have a number density $n_2(t)$ at time t, and the flux decreases exponentially as it passes through the interaction region. If the length of this region is D and the scattering cross section is σ, the probe beam is given by

$$-\ln(\phi/\phi_0) = \sigma D n_2(t_s)\,, \qquad (1)$$

where ϕ_0 is the probe-beam flux when there is no scattering and t_s is the time of the scattering in the interaction region. The cross section $\sigma(q_1,q_2)$ will in general depend on both roton wave vectors, so Eq. (1) applies to two beams of monoenergetic rotons. In our measurements we use the time of flight to the interaction region to determine the two roton wave vectors involved.

We test the dependence of Eq. (1) on n_2 in two ways. First, the length of the h_2 pulse is varied keeping the pulse power constant. Because the injected rotons are dispersed over a time much longer than the heater pulse length, the density of rotons, n_2, increases linearly with pulse length. In Fig. 3(a) we show $-\ln(\phi/\phi_0)$ plotted against pulse length, where ϕ is measured at the maximum of the probe-pulse signal. If the same plot is made for other times on the probe signal then straight lines are also obtained but with different gradients. This reflects the different values of σ at different wave vectors. For these measurements the probe pulse is delayed so that it passes through the slower rotons from h_2. This ensures that the time development of the scattering-roton density n_2 is slow compared to the transit time of the probe-roton pulse through the interaction region. So all the probe rotons, of different wave vectors, are scattering from rotons with essentially the same wave vector.

The heater-h_2-pulse power can be varied keeping the pulse length constant and this too varies the scattering

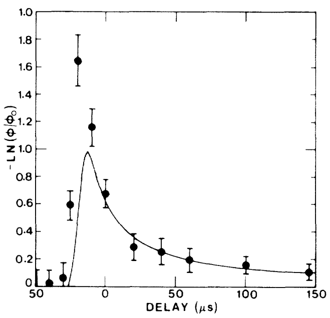

FIG. 4. The measured attenuation of the probe pulse as a function of the delay between the scattering probe and probe pulse. The line shows the calculated roton number density against delay, assuming constant cross section.

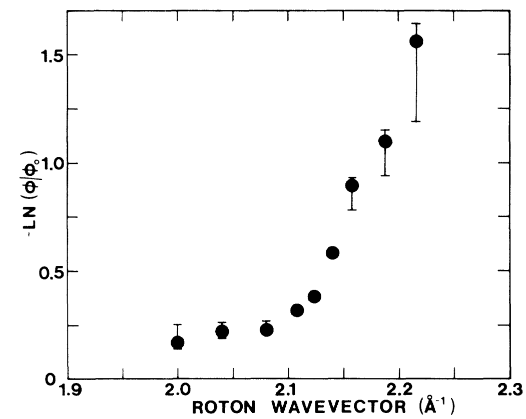

FIG. 5. The measured attenuation of the probe-pulse rotons as a function of their wave vector for a delay of -10 μs. A similar q dependence is found for a delay of 40 μs but the small density of scattering rotons, n_2, and the small cross section give too small an attenuation to be measured for $q_{\rm probe} < 2.1$ Å^{-1}.

density n_2. Previous work has shown that the spectrum or rotons changes slowly with the pulse power and the main effect is to increase the emitted number.[7] In Fig. 3(b) we show $-\ln(\phi/\phi_0)$ as a function of pulse power, with ϕ measured at the time of the maximum probe signal. A linear dependence is found and, again, if ϕ is chosen at different times, straight lines with different slopes are obtained.

Another check on Eq. (1) is to use the temporal development of the density of rotons n_2 injected by heater h_2. The appropriate cross section will also change in time as rotons with different wave vectors pass through the scattering region. The rotons from h_2 travel ballistically and so the density $n_2(t)$ can be varied by varying the time delay between the h_1 and h_2 pulses. The measured attenuation of probe rotons with $q \sim 2.2$ Å^{-1} as a function of delay is shown in Fig. 4. The solid line is calculated assuming that the scattering cross section is a constant. The roton spectrum from h_2 can reasonably be described by the characteristic temperature 1.5 K for a power of 3×10^{-3} W mm^{-2}.[7,9] We also assume that the rotons are emitted isotropically from the heater, but the model is relatively insensitive to this point. We estimate 10^{-2} of the input energy goes into rotons and so with the group velocities from the dispersion curve we find $n_2(t)$ in the interaction region. We take $\sigma = 4\times10^{-20}$ m^2 which makes the theory and experiment agree at long delays. From Fig. 4 it can be seen that there is a good qualitative similarity between the calculation and measurements. Clearly the calculated attenuation is too low in the peak which indicates that the scattering is stronger for the larger-wave-vector rotons from h_2. At the smallest delays shown, i.e., to the left of the peak, some of the attenuation is due to phonons from h_2.

Returning to the wave-vector dependence of the roton scattering, we plot in Fig. 5 the attenuation as a function of probe-roton wave vector, for scattering from slow h_2 rotons ($q \sim 1.95$ Å^{-1}). The probe-roton wave vectors are found directly from the time of the signal $\phi(t)$ and the wave-vector-dependent group velocity.

The strong wave-vector dependence of the scattering is such a dramatic result that we have reexamined the implicit assumptions made in our analysis. First, the scattering measurements are made with the probe density much less than the scattering-roton density (<0.1). This ensures that the scattering-roton density is not altered significantly by the passage of the probe rotons through it. Second, we have made a Monte Carlo simulation of the experiment to see whether it is possible for a roton which is scattered to reach the detector. If we conserve energy and momentum but otherwise allow all possible final states equally then we find that one scattering event is sufficient to remove a roton from the detected flux. This holds for all roton energies.

Other experiments to determine the strength of roton-roton scattering all involve thermal distributions of rotons, with consequent averages over the energies of the incident rotons and their scattering angles. The parameter B in the expression for the inverse roton lifetime $\Gamma = Bn$, where n is the number density of rotons, is calculated by several techniques: Neutron and Raman linewidth gives 2.3×10^{-16} m^{3}s^{-1},[5,7] roton second sound gives 1.6×10^{-16} m^{3}s^{-1},[10] and viscosity data give 6.9×10^{-17} m^{3}s^{-1}.[1,11] Direct comparison of these values with our data is difficult due to a large uncertainty in the absolute roton number density n_2 from the h_2 pulse. The value of n_2 depends on the fraction of the input heat-pulse energy going into roton production. The efficiency for production of high-ω ($\omega > 10$ K) phonons is $\sim 1\%$, and so in the absence of anything better we assume the efficiency for roton production is the same. Performing thermally weighted averages on our values of σ gives $B = 3.5\times10^{-17}$ m^{3}s^{-1} which is not inconsistent with the other measurements.

The probability of roton-roton scattering is proportional to the density of final states. We have calculated the density of final states as a function of the wave vector of the probe roton, for scattering with a roton which is typical for the delay used for the data in Fig. 5 ($q=1.95$ Å^{-1}). We find that the density of states is only weakly dependent on the probe-roton wave vector, increasing by only 30% over the range $2.00 \leq q \leq 2.20$ Å^{-1}. This indicates that it is the matrix element for the scattering process which varies considerably with wave vector.

Finally, we make two observations on our result. The scattering that gives rise to viscosity and neutron linewidth may not be exactly the same as we measure in our scattering experiment. This is because we are sensitive to all scattering processes, including small-angle processes. For even if the roton is only scattered through a small angle it is unlikely to reach the detector and so its contribution to the attenuation is the same as a large-angle scattering event. This is in contrast to the other measurements where small-angle processes contribute much less. The second point is that we know thermal phonons can lead to roton scattering.[7,9] We do not think that there should be any phonons left amongst the slow-moving rotons from heater h_2 at the time of the interaction but as yet we have not eliminated this possibility.

In conclusion, we have developed a new way of studying roton-roton scattering in which rotons of known energy and momentum participate. Beams of ballistic rotons are made to intersect and the attenuation of one of the beams is monitored. Analysis shows that a roton is lost from the beam after one scattering event. We have measured the scattering rate as a function of roton wave vector and find an unexpectedly strong dependence. It appears that this is associated with the matrix element for the scattering process.

We would like to thank Dr. R. Bowley for a useful discussion on the density of states.

[1]D. F. Brewer and D. O. Edwards, Proc. Roy. Soc. London A **251**, 247 (1959).

[2]O. W. Dietrich, E. H. Graf, C. H. Huang, and L. Passell, Phys. Rev. A **5**, 1377 (1972).

[3]A. D. B. Woods and E. C. Svensson, Phys. Rev. Lett. **41**, 974 (1978).

[4]J. A. Tarvin and L. Passell, Phys. Rev. B **19**, 1458 (1979).

[5]F. Mezei, Phys. Rev. Lett. **44**, 1601 (1980).

[6]T. J. Greytak and J. Yan, in *Proceedings of Twelfth International Conference on Low Temperature Physics, Kyoto, 1970,* edited by E. Kanda (Academic, Kyoto, 1971), p. 89.

[7]M. Brown and A. F. G. Wyatt, in Proceedings of the Third International Conference on Phonon Physics and Sixth International Conference on Phonon Scattering in Condensed Matter, Heidelberg, 1989 (World Scientific, Singapore, to be published).

[8]F. R. Hope, M. T. Baird, and A. F. G. Wyatt, Phys. Rev. Lett. **52**, 1528 (1984).

[9]A. F. G. Wyatt, in Proceedings of the Third International Conference of Phonon Physics and Sixth International Conference on Phonon Scattering in Condensed Matter, Heidelberg, 1989 (Ref. 7).

[10]B. Castaing and A. Libchaber, J. Low Temp. Phys. **31**, 887 (1978).

[11]I. M. Khalatnikov, *An Introduction to the Theory of Superfluidity* (Addison-Wesley, London, 1965).