Chiral Schwinger Model in Terms of Chiral Bosonization

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The chiral Schwinger model is reexamined by using chiral bosonization. The Lagrangian is obtained as a gauged Floreanini-Jackiw Lagrangian. We get a bosonic solution which contains one massive free boson and one (free) self-dual field.

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The fermion-boson correspondence is one of the most interesting features of and a powerful tool in two-dimensional quantum field theories. Recently much attention has been paid to so-called chiral bosonization mainly in the context of string theories.[1-5] It is, however, natural to think that chiral bosonization should be important to left-right-asymmetric fermion theories, e.g., anomalous gauge theories in two dimensions.

There are two apparently inequivalent approaches to chiral bosons. Siegel[1] proposed a manifestly covariant Lagrangian of chiral bosons. However, it turns out that the symmetry (a sort of reparametrization invariance) of the classical Lagrangian suffers from the gravitational anomaly at the quantum level.[2] On the other hand, Floreanini and Jackiw[3] proposed an alternative Lagrangian, but there seems to be no definite way to introduce gauge couplings (see Ref. 5) because of the lack of manifest covariance.

Bellucci, Golterman, and Petcher[6] considered the interaction of chiral bosons with Abelian and non-Abelian gauge fields and discussed the chiral Schwinger model[7] as an example. Labastida and Ramallo[8] considered chiral bosons coupled to Abelian gauge fields using the Becchi-Rouet-Stora-Tyutin (BRST) procedure. Both papers are based on the Siegel Lagrangian.

In this Letter, we show how to obtain a gauged Lagrangian for the Floreanini-Jackiw Lagrangian from the conventional bosonic one of the chiral Schwinger model by imposing the chiral constraint $\pi_\phi - \phi' \approx 0$ (see below) in phase space. This Lagrangian contains fewer degrees of freedom than the conventional bosonic one. Furthermore, we quantize the model canonically and solve it. We find an obvious correspondence to the previously obtained solution. The asymptotic fields consist of a massive free boson with the desired mass and a (free) self-dual field (chiral boson).

The details of our analysis will be reported in a separate paper.[9]

Let us consider the following generating functional:[10]

$$Z[A] = \int d\psi\, d\bar{\psi} \exp\left(i\int d^2x\, \mathcal{L}_F\right), \tag{1a}$$

with

$$\mathcal{L}_F = \bar{\psi}\gamma^\mu[i\partial_\mu + e\sqrt{\pi}A_\mu(1-\gamma_5)]\psi = \bar{\psi}_R i\gamma^\mu\partial_\mu\psi_R + \bar{\psi}_L\gamma^\mu(i\partial_\mu + 2e\sqrt{\pi}A_\mu)\psi_L. \tag{1b}$$

Since the right-handed fermion is decoupled, the integration $d\psi_R\, d\bar{\psi}_R$ gives a field-independent constant and may be absorbed in the normalization. The remaining path integral can be performed exactly,[7]

$$Z[A] = \int d\psi_L\, d\bar{\psi}_L \exp\left(i\int d^2x\, \bar{\psi}_L\gamma^\mu(i\partial_\mu + 2e\sqrt{\pi}A_\mu)\psi_L\right) = \exp\left[\frac{ie^2}{2}\int d^2x\, A_\mu\left(a\eta^{\mu\nu} - (\partial^\mu + \tilde{\partial}^\mu)\frac{1}{\Box}(\partial^\nu + \tilde{\partial}^\nu)\right)A_\nu\right], \tag{2}$$

where a is a constant which represents a regularization ambiguity. (We consider only the case $a > 1$ in this Letter.) This generating functional can be written in a local form by introducing an auxiliary scalar field $\phi(x)$,

$$Z[A] = \int d\phi \exp\left(i\int d^2x\, \mathcal{L}_B\right), \tag{3a}$$

with

$$\mathcal{L}_B = \tfrac{1}{2}(\partial_\mu\phi)^2 + e(\eta^{\mu\nu} - \epsilon^{\mu\nu})\partial_\mu\phi A_\nu + \tfrac{1}{2}e^2 a A_\mu A^\mu. \tag{3b}$$

From the above derivation, it is clear that $\mathcal{L}_B$ is equivalent to $\mathcal{L}_F$ in the sense that these two Lagrangians lead to the same $Z[A]$. In the fermionic theory, the whole nontrivial contribution to $Z[A]$ comes from the left-handed fermion. In other words, one can get the same $Z[A]$ even if the right-handed fermion is not included from the beginning. (Let us call this case *minimal.*) On the other hand, $\phi(x)$ in $\mathcal{L}_B$ contains degrees of freedom which do not correspond to the left-handed fermion. To see this, let us consider the $e=0$ case. In this case, the original fermionic theory (2) contains only a free left-handed fermion which depends on only one of the light-cone coordinates $[\psi_L = \psi_L(x^+)]$. On the other hand, the corresponding bosonic theory (3) contains left- and right-moving components. Therefore the Lagrangian $\mathcal{L}_B$ is not suitable for describing the *minimal* fermionic theory. The minimal description can be achieved by considering chiral bosonization.

In order to get the *reduced* Lagrangian, let us consider the Hamiltonian obtained from $\mathcal{L}_B$,[11]

$$\mathcal{H}_B = \tfrac{1}{2}[\pi_\phi - e(A_0 - A_1)]^2 + \tfrac{1}{2}(\phi')^2 - e\phi'(A_0 - A_1) - \tfrac{1}{2}e^2 a A_\mu A^\mu . \quad (4)$$

We impose here the following "chiral constraint,"[12]

$$\Omega(x) \equiv \pi_\phi(x) - \phi'(x) \approx 0 . \quad (5)$$

It turns out that the chiral constraint $\Omega(x)$ is second class:[13]

$$\{\Omega(x), \Omega(y)\} = -2\delta'(x^1 - y^1) . \quad (6)$$

Let us consider the quantum theory which is described by the Hamiltonian (4) subject to the chiral constraint (5). The generating functional is obtained, by noting that $\det\{\Omega,\Omega\}$ is a field-independent constant, as

$$Z_{\rm ch}[A] = \int d\phi\, d\pi_\phi\, \delta(\pi_\phi - \phi')\, |\det\{\Omega,\Omega\}|^{1/2} \exp\left[i\int d^2x(\pi_\phi\dot\phi - \mathcal{H}_B)\right] = \int d\phi \exp\left[i\int d^2x\, \mathcal{L}_{\rm ch}\right] , \quad (7a)$$

with

$$\mathcal{L}_{\rm ch} = \dot\phi\phi' - (\phi')^2 + 2e\phi'(A_0 - A_1) - \tfrac{1}{2}e^2(A_0 - A_1)^2 + \tfrac{1}{2}e^2 a A_\mu A^\mu . \quad (7b)$$

This Lagrangian is a gauged version of the Lagrangian of Floreanini and Jackiw,[3] $\mathcal{L}_0 = \dot\phi\phi' - (\phi')^2$.

By doing the ϕ integration, we can see that the $Z_{\rm ch}[A]$ gives the correct answer (2), $Z_{\rm ch}[A] = Z[A]$. Therefore the $Z_{\rm ch}[A]$ is an alternative local expression of the generating functional $Z[A]$.

It is amusing to note that, although there seems to be no definite way to gauge the apparently noncovariant Lagrangian of Floreanini and Jackiw, we obtained the gauged one (7b) from the *conventional* bosonic Lagrangian (3b) by imposing the chiral constraint (5) in phase space.

In the following, we canonically quantize the bosonized chiral Schwinger model described by the Lagrangian[14] $\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{\rm ch}$.

The system turns out to be a constrained system with two primary constraints, $\Omega_1 \equiv \pi^0 \equiv \partial\mathcal{L}/\partial\dot A_0 \approx 0$ and $\Omega_2 \equiv \pi_\phi - \phi' \approx 0$.[13] The total Hamiltonian is written as

$$H_T = \int dx^1 (\mathcal{H} + u_i \Omega_i) , \quad (8a)$$

$$\mathcal{H} = \pi^\mu \dot A_\mu + \pi_\phi\dot\phi - \mathcal{L} \approx \tfrac{1}{2}(\pi^1)^2 + \pi^1\partial_1 A_0 + (\phi')^2 - 2e\phi'(A_0 - A_1) + \tfrac{1}{2}e^2(A_0 - A_1)^2 - \tfrac{1}{2}e^2 a A_\mu A^\mu , \quad (8b)$$

with $\pi^1 = \partial\mathcal{L}/\partial\dot A_1$. The u_i $(i=1,2)$ are Lagrange-multiplier fields.

Consistency of Ω_1 under the time evolution, $\dot\Omega_1 \approx 0$, gives the *Gauss-law* constraint, $\Omega_3 \equiv \partial_1\pi^1 + 2e\phi' + e^2[(a-1) \times A_0 + A_1] \approx 0$, while $\dot\Omega_2 \approx 0$ and $\dot\Omega_3 \approx 0$ determine u_2 and u_1, respectively. We have no further constraints.

It is easily shown that all these constraints $\Omega_i \approx 0$ $(i=1,2,3)$ are second class. Thus we can define the Dirac bracket[13] as usual,

$$\{A(x),B(y)\}^* = \{A(x),B(y)\} - \int dz\, dw \{A(x),\Omega_i(z)\} C_{ij}(z,w) \{\Omega_j(w),B(y)\} , \quad (9a)$$

where $C_{ij}(x,y)$ is defined by

$$\int dy\, C_{ij}(x,y)\{\Omega_j(y),\Omega_k(z)\} = \delta_{ik}\delta(x-y) . \quad (9b)$$

In our case, one may get the following explicit expression:

$$C_{ij}(x,y) = \frac{1}{e^2(a-1)} \begin{pmatrix} 0 & e\delta(x-y) & \delta(x-y) \\ -e\delta(x-y) & -e^2(a-1)\epsilon(x-y)/4 & 0 \\ -\delta(x-y) & 0 & 0 \end{pmatrix} , \quad (10)$$

where $\epsilon(x)$ is the sign function, $\epsilon(x) = +1$ for $x > 0$ and $\epsilon(x) = -1$ for $x < 0$.

By means of the Dirac bracket, we can eliminate redundant variables consistently and quantize the system, replacing $\{,\}^*$ by $-i[,]_{\rm ET}$. Under an appropriate boundary condition,[5] $\Omega_2(\infty) = -\Omega_2(-\infty)$, we can choose ϕ, A_1, π^1 as independent variables with the nonvanishing commutators

$$[\phi(x),\phi(y)]_{\rm ET} = -i\epsilon(x^1 - y^1)/4 , \quad (11a)$$

$$[A_1(x),\pi^1(y)]_{\rm ET} = i\delta(x^1 - y^1) , \quad (11b)$$

and the Hamiltonian

$$H=\int dx^1\left\{\frac{1}{2}(\pi^1)^2+\frac{1}{2e^2(a-1)}(\partial_1\pi^1)^2+\frac{a+1}{a-1}(\phi')^2+\frac{e^2a^2}{2(a-1)}(A_1)^2+\frac{2}{e(a-1)}\phi'\partial_1\pi^1+\frac{2ea}{a-1}\phi'A_1+\frac{1}{a-1}A_1\partial_1\pi^1\right\}. \tag{12}$$

The remaining task of deriving equations of motion and solving them is straightforward. Here we present only the solution. In spite of the superficial differences, the solution is the same as the one obtained in Ref. 7,

$$\phi=\sigma-h\,, \tag{13a}$$

$$A_\mu=-(1/ea)\{\partial_\mu\phi+(a-1)\tilde\partial_\mu\phi-a\tilde\partial_\mu h\}\,, \tag{13b}$$

except for the fact that the h field is now a self-dual field, $(\partial_0-\partial_1)h=0$. This field satisfies the commutation relations $[h(x),h(y)]_{\rm ET}=-i\epsilon(x^1-y^1)/4$ and $[h(x),\dot h(y)]_{\rm ET}=(i/2)\delta(x^1-y^1)$. The field $\sigma(x)$ is a massive free scalar with the correct mass $m^2=e^2a^2/(a-1)$ and the commutation relations $[\sigma(x),\sigma(y)]_{\rm ET}=0$ and $[\sigma(x),\dot\sigma(y)]_{\rm ET}=i\delta(x^1-y^1)/(a-1)$.

This *minimal* description of the chiral Schwinger model has very important consequences. (i) The massive scalar field $\sigma(x)$ is made up only of the gauge field and the left-handed fermion. The right-handed fermion plays no role in making the "meson." (ii) The right-handed fermion, if we include it, should continue to be free. Therefore the chiral anomaly should not influence the current conservation of the (free) right-handed fermion current. Note that several authors[15] considered an "anomalous" right-handed fermion current. We think it unnatural. (iii) The self-dual field $h(x)$ corresponds to the pole $k^2=0$ of the A_μ propagator.[7] As suggested in Ref. 7, it may be natural to think that the field $h(x)$ represents an "unconfined" fermion because a self-dual field has a completely local description in terms of fermionic variables.

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[1] W. Siegel, Nucl. Phys. **B238**, 307 (1984).

[2] C. Imbimbo and A. Schwimmer, Phys. Lett. B **193**, 455 (1987); J. M. F. Labastida and M. Pernici, Nucl. Phys. **B297**, 557 (1988); B. McClain and Y-S. Wu, University of Utah report (to be published).

[3] R. Floreanini and R. Jackiw, Phys. Rev. Lett. **59**, 1873 (1987).

[4] M. E. V. Costa and H. O. Girotti, Phys. Rev. Lett. **60**, 1771 (1988); M. Bernstein and J. Sonnenschein, *ibid.* **60**, 1772 (1988); H. O. Girotti *et al.*, *ibid.* **60**, 1913 (1988); L. D. Faddeev and R. Jackiw, *ibid.* **60**, 1692 (1988).

[5] J. Sonnenschein, Nucl. Phys. **B309**, 752 (1988).

[6] S. Bellucci, M. F. L. Golterman, and D. N. Petcher, Nucl. Phys. **B326**, 307 (1989).

[7] R. Jackiw and R. Rajaraman, Phys. Rev. Lett. **54**, 1219 (1985); **54**, 2060(E) (1985); H. O. Girotti, H. J. Rothe, and K. D. Rothe, Phys. Rev. D **33**, 514 (1986).

[8] J. M. F. Labastida and A. V. Ramallo, Phys. Lett B **222**, 231 (1989).

[9] K. Harada (to be published).

[10] Notation: $\gamma^0=\sigma^1$, $\gamma^1=-i\sigma^2$, $\gamma_5=\gamma^0\gamma^1=\sigma^3$, $\eta_{\mu\nu}={\rm diag}(+,-)$, $\epsilon^{01}=-\epsilon_{01}=+1$, $\gamma_5\psi_{L,R}=\mp\psi_{L,R}$, $\tilde\partial_\mu=\epsilon_{\mu\nu}\partial^\nu$, $\dot\phi=\partial_0\phi$, $\phi'=\partial_1\phi$.

[11] For bosonic operator solutions, see K. Harada and I. Tsutsui, Z. Phys. C **39**, 137 (1988).

[12] Note that the chiral constraint (5) is used in the free case by Bernstein and Sonnenschein in Ref. 4 and is applied to an interacting case in F. Bastianelli and P. van Nieuwenhuizen, Phys. Lett. B **217**, 98 (1989).

[13] P. A. M. Dirac, *Lectures on Quantum Mechanics* (Yeshiva Univ. Press, New York, 1964).

[14] Although this Lagrangian does not have manifest covariance, it is shown that the suitably defined Poincaré generators satisfy the Poincaré algebra (see Ref. 9).

[15] See, for example, S. Miyake and K. Shizuya, Phys. Rev. D **36**, 3781 (1987).