# Test of the Gravitational Redshift Effect at Saturn

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The results of a test of the gravitational redshift effect at Saturn are reported. Measurements of the redshift were obtained with the Voyager 1 spacecraft during its encounter with Saturn in 1980. Because the spacecraft was equipped with an ultrastable crystal oscillator (USO), it is possible to test the redshift effect at an interesting level of accuracy. Assuming that radiation in the Saturn magnetosphere has had a negligible effect on the USO, we verify the prediction of general relativity to an accuracy of 1%. This limit could be of interest for constraining possible alternative theories of gravity.

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By analyzing radio-frequency measurements generated by the Deep Space Network (DSN), we have been able to test accurately the gravitational redshift effect at Saturn with the radio system of the Voyager 1 spacecraft. The two Voyager spacecraft are the first probes to the outer planets to carry an ultrastable crystal oscillator (USO), thereby providing a precise reference frequency for one-way radio transmissions from the spacecraft to the Earth.[1] As a part of the Voyager radio-science investigations involving the USO,[2] measurements of the gravitational redshift effect were performed during the Saturn encounters by both spacecraft. The effect is apparent as a "dip" in the frequency of the one-way radio signal transmitted by the spacecraft as it moves in and out of the gravitational field of the planet. Both spacecraft passed close enough to Saturn that a redshift of several hertz was predicted for the 2.3-GHz downlink frequency. Possible radiation effects on the USO were apparent in the Voyager 2 data. However, the Voyager 1 USO experienced prehardening at Jupiter. In this case, radiation effects were not directly apparent at Saturn.

For a planetary flyby the relativistic frequency shift of the transmitted frequency $f_0$ can be described by

$$f = f_0 \left[ 1 + \sum_k \frac{\Delta U_k}{c^2} + \frac{1}{2} \frac{\Delta v^2}{c^2} \right] , \tag{1}$$

where $U_k$ denotes the Newtonian potential of the $k$th body [defined positively according to Eq. (3) below], $v^2$ is velocity squared, and $\Delta$ denotes that these quantities for the spacecraft are to be subtracted from those for the receiver at the Earth. In terms of the dominant potentials, we have

$$\sum_k \Delta U_k = -\alpha U_p^{(s)} + \Delta U_\odot + U_\oplus^{(r)} , \tag{2}$$

where the first term is the potential of the planet at the position of the spacecraft, the second term is the difference in the potential of the Sun between the spacecraft and the receiver, and the last term is the potential of the Earth at the receiver. Not listed in Eq. (2) are smaller potential terms due to the other bodies in the solar system. For a highly oblate planet, such as Saturn, it is necessary to include the contribution from the mass quadrupole moment in the expression for the Newtonian potential of the planet, which is given by

$$U_p = \frac{GM}{r} \left[ 1 - J_2 \frac{R^2}{r^2} \left( \frac{3\cos^2\theta - 1}{2} \right) \right] , \tag{3}$$

where $M$ is the mass, $R$ is the radius, and $J_2$ is the dimensionless quadrupole moment parameter; the quantities $(r,\theta)$ denote the planet-centered spherical coordinates of the field point. For Saturn, $GM = 3.79 \times 10^7$ km$^3$/s$^2$, $R = 60\,330$ km at the equator, and $J_2 = 1.63 \times 10^{-2}$.[3]

We have inserted a dimensionless parameter $\alpha$ in Eq. (2) in order to test the gravitational redshift of signals transmitted by the spacecraft. While other parametrizations are possible, we have chosen a conventional parametrization which is of interest for the Voyager flyby.[4] This parameter can be interpreted to provide a measure of a possible breakdown of the Einstein equivalence principle (EEP).[5] Violations of the EEP can occur for nonmetric couplings between gravity and the laws of nongravitational physics, such as the equations of electromagnetism. The nature of the coupling could depend upon the detailed composition of a massive body, which is why it is of interest to test the redshift in a gravitational field other than that of the Earth. In the nonsymmetric gravitational theory (NGT) of Moffat, for example, the redshift depends upon the total number of fermions in a body. Recently Moffat has considered a possible coupling to cosmions.[6] Because the number of cosmions in Saturn could differ substantially from the number in the Earth, a test of the redshift at Saturn could yield a result which differs from Earth-based tests (see Ref. 5 for a review of previous redshift tests). In the NGT the redshift can arise both through the metric and by a nonmetric coupling to the action of electromagnetism.[7] The nonmetric coupling can be shown to make the redshift *both* clock dependent *and* composition dependent.[8] Although a minimal coupling to the nonsymmetric metric is not significantly limited by the Voyager USO data (see below), other possible couplings might be constrained.[9]

The Voyager USO was manufactured by Frequency Electronics Incorporated. A 19.13-MHz output frequency is generated by a dual oven-controlled, *AT*-cut quartz-crystal ($SiO_2$) resonator. Lead and other dense materials were used around the crystal resonator to provide shielding against radiation, while the entire USO assembly was in turn aluminum shielded. The resulting frequency stability for applied radiation is 2 parts in $10^{12}$ per rad (silicon) for electron or proton bombardment. This amount of shielding provided adequate short-term frequency stability for the other radio-science experiments, the primary concern, but was not sufficient to protect the USO completely for the redshift test, as discussed further below. For external magnetic-field variations of less than 0.1 G, the frequency stability is better than 5 parts in $10^{12}$. The 19.13-MHz output frequency is multiplied by a factor of 120 to provide the 2.3-GHz transmitted downlink frequency. This signal was transmitted simultaneously at a frequency which was multiplied upwards by a factor of $\frac{11}{3}$ to provide for the calibration of intervening plasma (see below). However, all frequency shifts discussed below are referred to the 2.3-GHz frequency band.

Voyager 1 made its closest approach to Jupiter on 5 March 1979, with a Jupiter periapsis of $r=3.5\times10^5$ km (4.9 Jupiter radii). During the encounter, a large, systematic frequency shift of $-216$ Hz was observed in the one-way downlink, which can be attributed to radiation effects on the USO as the spacecraft passed through the radiation environment of the Jupiter magnetosphere. In addition, Voyager 1 passed directly through the Io flux tube, which contributed to the total radiation exposure during the flyby. It was possible to verify that the size of the observed frequency shift was consistent with estimates of the total radiation exposure which the USO could have received (amounting to several thousand rads at energies above 1 MeV). A large frequency shift was not observed with Voyager 2, however, which had a larger Jupiter periapsis of $r=7.2\times10^5$ km (10.1 Jupiter radii) on 9 July 1979.

This situation was reversed at Saturn, with Voyager 2 passing slightly closer to the planet than Voyager 1. The Voyager 1 periapsis occurred on 12 November 1980 with $r=1.8\times10^5$ km (2.9 Saturn radii), while the Voyager 2 periapsis was on 26 August 1981 with $r=1.6\times10^5$ km (2.7 Saturn radii). A large step in frequency of $+0.4$ Hz was observed in the one-way downlink from Voyager 2 during the encounter. The size of the shift is consistent with the levels of radiation measured at Saturn, which were much smaller than at Jupiter. Radiation effects were not directly apparent in the Voyager 1 data, which is consistent with the reduced radiation levels at Saturn and possible prehardening of its USO at Jupiter.[10] An estimate of the total radiation exposure at Saturn implies a reduction by as much as a factor of $10^4$ of the dosage received at Jupiter. Furthermore, laboratory tests of quartz-crystal resonators have demonstrated that large dosages of radiation can reduce the influence of further radiation exposure.[11]

The one-way downlink from Voyager 1 was received at the DSN stations located at Goldstone, California, near Madrid, Spain, and near Canberra, Australia. One-way Doppler data were generated at all three stations at times which depended upon spacecraft visibility. Horizon elevation angles were generally greater than 15°. Only one-way data received at 64-m-diam antennas were used for the redshift analysis, because the ground receivers at these antennas are referenced to hydrogen-maser frequency standards. Descriptions of the ground radio system have been presented elsewhere (see Ref. 1). Closed-loop data were used solely, in which the ground receiver is locked phase coherently to the downlink and cycles of phase are counted during a specified time interval in order to provide a measurement of the frequency shift between transmission and reception of the radio signal. A count time of 60 s was used for the redshift analysis. Two-way Doppler, three-way Doppler, two-way range, and Saturn optical data were also acquired during the flyby, which provided for an accurate determination of the orbit of the spacecraft (see Ref. 3 for a complete description). Ten passes of one-way data, which contained a total of 262 1-min-averaged frequency measurements, were included in the analysis. No pass lasted longer than 1 h, with one phase having a duration as short as 7 min. This particular batching was determined by the need to optimize the acquisition of other data during the encounter. The one-way data cover a region between roughly $(0.3$ and $1.5)\times10^6$ km in Saturn range, or between 5 and 25 Saturn radii. Over this time period, the spacecraft orbited from $+2°$ on ingress to within $-40°$ at periapsis in Saturn latitude, and then back up to $+20°$ on egress.

Modeling of all the radiometric and optical data was accomplished using the JPL orbit-determination program (ODP) (see Ref. 12 for an overview). In the ODP, the frequency shift of the received radio signal is computed by differencing the range to the spacecraft at the beginning and the end of the phase-count interval and dividing by the count time. All computations are performed in an isotropic coordinate frame which is centered on the solar system barycenter. Relativistic corrections to the frequency shift can enter the computation through the computed range and the time transformation from spacecraft time to local station time. The gravitational redshift enters the computation through this time transformation.

Shown in the top half of Fig. 1 is the expected Saturn gravitational redshift due to the potential of Eq. (3)

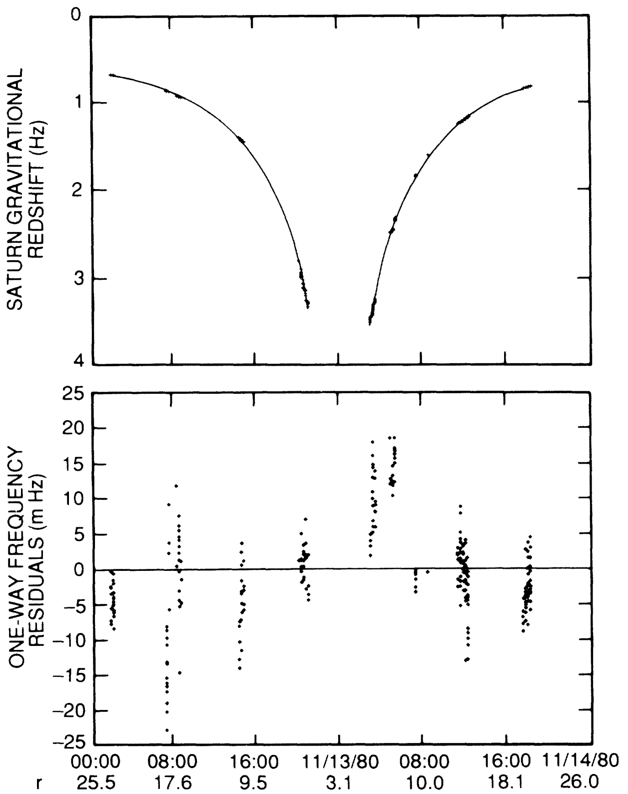

FIG. 1. Top: The Saturn gravitational redshift in frequency (relative to the nominal 2.3-GHz downlink frequency) expected during the Voyager 1 flyby. Bottom: The resulting frequency residuals, where the date and time axis begins on 12 November 1980 and $r$ denotes the radial distance of Voyager 1 from the center of Saturn measured in Saturn radii ($R_S = 6.033\times 10^4$ km).

alone. While it is possible to fit the relativity parameters $\beta$ and $\gamma$, there are currently no provisions in the JPL ODP to fit the redshift parameter $\alpha$. The general relativistic prediction for the redshift has been assumed to be correct. A least-squares fit was performed with the ODP to all of the navigational data over a two-day period centered on the time of Saturn periapsis (see Ref. 3 for details). Shown in the bottom half of Fig. 1 are the resulting one-way Doppler residuals, with an rms of 7.3 mHz. Two parameters representing a bias and a linear drift in time of the frequency of the USO were estimated. The bias accounts for an unknown offset in the frequency at the epoch of the one-way data, while the term linear in time accounts for the effect of aging of the USO. In order to fit $\alpha$, we have used Eq. (3) to compute the redshift due to the gravitational potential of Saturn alone. We then fitted this model to the residuals of Fig. 1. Corrections to the bias and drift parameters were included in the fit in order to account for correlations between these parameters and $\alpha$. The other parameters in the ODP fit were not expected to correlate as significantly with $\alpha$. They are also constrained by the two-way data and optical data. There resulted a value of $\alpha = 0.9956 \pm 0.0004$, where 0.0004 is the formal uncertainty. The fit reduced the rms of the residuals to 6.0 mHz. The formal uncertainty is unrealistic, however, because of non-Gaussian errors that could remain in the one-way data.

Propagation errors are possible due to the passage of the radio signal through interplanetary plasma, the ionosphere, and the troposphere.[13,14] However, the calibrations which were applied to the one-way data should have rendered these errors negligible. Ionospheric and interplanetary plasma effects in the Doppler data are frequency dependent, where the plasma can be characterized as having an index of refraction with a $1/f^2$ dependence. The main effect on the Doppler data is a variation in frequency, at a level of $\pm 20$ mHz for the ionosphere, as the path of the radio signal moves through spatial variations in the plasma density due to the motion of the spacecraft or the rotation of the Earth. It was possible to calibrate the one-way data for this effect by differencing the simultaneous 2.3- and 8.4-GHz signals transmitted by the spacecraft [see Ref. 15, Eq. (1)]. This technique is valid provided that the signals travel along identical paths. Errors could occur because of bending of the paths by different amounts, but this is a higher-order contribution which can be shown to be negligible for the frequencies involved. The effect of the troposphere is independent of frequency over our frequency band. An accurate model of the troposphere which accounts for both the wet and dry components is used in the ODP. Incorporated into the model are monthly mean refractivity profiles derived from extensive studies of both ground weather data and atmospheric data collected with balloon radiosondes at or near each DSN station. The estimated accuracy of the model is better than 1 mHz for elevation angles above 15°. Random frequency fluctuations due to tropospheric irregularities are also expected to be negligible.[16,17]

Even in the absence of known perturbations to the USO, such as radiation, random frequency fluctuations can occur. These fluctuations have also been analyzed in one-way data taken away from the encounter during regularly scheduled tests of the USO to determine its health and performance. The level and nature of these fluctuations varies with integration time, which has been determined from the behavior of the Allan variance and phase power spectrum (e.g., see Fig. 9.11 of Ref. 14). At 60-s integration times the USO exhibits flicker frequency noise at a level of 1 part in $10^{12}$ in fractional frequency fluctuations, or 2.3 mHz at the downlink frequency. More importantly, the USO downlink frequency has been observed to random walk by 10 mHz on the average over a time scale of 1 h. This uncertainty is expected to grow with time at a rate of $\tau^{1/2}$. We have modeled a systematic frequency shift which was bounded by this time dependence but strongly correlated with the redshift. It was found to cause not more than a 1% change in the fitted value of $\alpha$. We therefore adopt $\pm 0.01$ as a realistic estimate of the uncertainty in the fit to $\alpha$. It can be seen that the fitted value of $\alpha=0.9956$ is consistent with general relativity at this level of uncertainty.[18] A rigorous weighting that accounts for the non-Gaussian character of the USO frequency fluctuations is being investigated. While this could yield a smaller uncertainty in $\alpha$, a significant reduction in the quoted uncertainty is not expected.

We believe that these results are limited primarily by the possibility of radiation effects on the USO that might have masked a more significant deviation of $\alpha$ from 1. Whether a more accurate determination can be derived from the Voyager 2 flybys of Uranus and Neptune is being investigated. It might also be possible in the near future to test the redshift effect further at Jupiter during the planned Galileo mission. The Galileo spacecraft will perform several successive, highly eccentric orbits of Jupiter that will take the spacecraft in and out of the magnetosphere from $150R_J$ to within $10R_J$ of the planet. Because repeated radiation hardening of the Galileo USO could occur, it might be possible to exploit the full frequency stability of the USO for the later orbits. Furthermore, the Jupiter redshift could be tested repeatedly, which might allow possible radiation effects to be separated from the redshift measurement.

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[1] G. L. Tyler, Proc. IEEE **75**, 1404 (1987).

[2] G. L. Tyler, V. R. Eshleman, J. D. Anderson, G. S. Levy, G. F. Lindal, G. E. Wood, and T. A. Croft, Science **212**, 201 (1981); **215**, 553 (1982).

[3] J. K. Campbell and J. D. Anderson, Astron. J. **97**, 1485 (1989).

[4] Parametrization of the other potentials in Eq. (2) can be neglected because only the potential of Saturn changed significantly during the flyby.

[5] C. M. Will, Phys. Rep. **113**, 345 (1984).

[6] J. W. Moffat, Phys. Rev. D **39**, 474 (1989); **40**, 683(E) (1989).

[7] C. M. Will, Phys. Rev. Lett. **62**, 369 (1989); R. B. Mann, J. H. Palmer, and J. W. Moffat, Phys. Rev. Lett. **62**, 2765 (1989).

[8] T. P. Krisher (to be published).

[9] This is a question that is currently under investigation.

[10] In addition to radiation, the USO could have been affected by the magnetic field of Saturn. The dipole magnetic-field strength is given roughly by $B_S \approx 0.2(r/R_S)^{-3}$ G for $r$ in Saturn radii. However, for $r > 2.9R_S$, the effect on the USO frequency is estimated to be less than 1 mHz.

[11] J. R. Norton, J. M. Cloeren, and J. J. Suter, in *Proceedings of the Thirty-Eighth Annual Frequency Control Symposium, Philadelphia, 1984* (IEEE, New York, 1984), p. 63.

[12] J. D. Anderson, in *Experimental Gravitation,* International School of Physics "Enrico Fermi," Course LVI, edited by B. Bertotti (Academic, New York, 1974).

[13] M. P. M. Hall, *Effects of the Troposphere on Radio Communications* (IEEE, New York, 1979).

[14] A. R. Thompson, J. M. Moran, and G. W. Swenson, Jr., *Interferometry and Synthesis in Radio Astronomy* (Wiley, New York, 1986).

[15] J. K. Campbell and S. P. Synnott, Astron. J. **90**, 364 (1985).

[16] J. W. Armstrong, R. Woo, and F. B. Estabrook, Astrophys. J. **230**, 570 (1979).

[17] R. N. Truehaft and G. E. Lanyi, Radio Sci. **22**, 251 (1987).

[18] For a minimal coupling in the NGT, it can be shown that $\alpha=1$ and that there is an extra composition-dependent frequency shift given by $\Delta f/f=\frac{3}{2}(l/r)^4$ for the case of a crystal oscillator. Fits to the $l$ parameter of Moffat did not provide an upper bound of better than $l=350$ km for Saturn, which, as was expected, is not a significant limit. A significant test of the NGT could be provided by a close flyby of Jupiter [see T. P. Krisher, Phys. Rev. D **40**, 1372 (1989)].