Plasma-Based Adiabatic Focuser

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(Received 30 October 1989)

Theoretical analysis is made of an intense relativistic electron beam moving through a plasma of increasing density, but density always less than that of the beam (underdense). Analysis is made of the beam radiation energy loss and it is noted that the focuser is insensitive to the beam energy spread due to radiation loss. Furthermore, because of the scaling behavior in the nonclassical regimes, the radiation limit on lenses (the Oide limit) can be exceeded.

PACS numbers: 41.80.Ee, 29.25.Fb, 41.70.+t, 52.40.Mj

To avoid increasing energy loss through synchrotron radiation in storage rings, it is generally agreed that future high-energy e^+e^- colliders will necessarily be linear.[1] To compensate for the much lower collision rates in linear colliders, one is forced to collide much tighter beams. For example, in the design of a TeV collider (TLC) by Palmer,[2] the beam sizes at the interaction point are as miniscule as $\sigma_x = 190$ nm, $\sigma_y = 1$ nm. For multi-TeV colliders in the far future, the beam size is expected to be even smaller. This demanding requirement on the beam size imposes stringent constraints on the stability and tolerance in the final focusing beam optics system. Furthermore, it was recently demonstrated by one of us[3] that the chromatic effect due to the synchrotron radiation triggered at the final focusing lens imposes a strong limitation on the minimal possible beam size.

In this paper, we present a different concept of beam focusing called *adiabatic focusing,* which promises to evade the synchrotron-radiation limit set by Oide. This is achieved by implementing a beam optics system where the focusing gradient is continuously and slowly increased along the direction of beam propagation, such that the β function decreases linearly along the lens. In such a focusing system, beam particles with different energies would always oscillate within a definite envelope and eventually be focused down to within the designated size. The problem of chromatic aberration associated with conventional discrete focusing lenses can thus be alleviated.

The insensitivity of this focusing scheme to the particle energy does not imply that the system is entirely free from the constraint due to synchrotron radiation. For high-energy physics purposes, the focused beams should not suffer from signification energy degradation. But as will be shown, the corresponding limitation on the attainable beam size is mild so long as the focusing is strong enough that the synchrotron radiation enters into the nonclassical regime.

In general, in a focusing (or defocusing) environment a particle with coordinate y satisfies the equation of motion

$$d^2y/ds^2 + K(s)y = 0\,, \tag{1}$$

and the well-known solution is[4]

$$y(s) = \beta^{1/2}(s)\cos[\psi(s)+\phi]\,, \tag{2}$$

where

$$d\beta/ds = -2\alpha(s),\quad \psi(s) = \int^s ds'/\beta(s')\,.$$

(We restrict our analysis to one dimension and it will be appropriate to either round beams or flat beams.)

In adiabatic focusing, we demand that the change in β, occurring in a length given by β, is small compared to β. For the sake of simplicity, we shall assume that $d\beta/ds = \text{const}$. Hence we take

$$\beta(s) = \beta_0 - 2\alpha_0 s\,, \tag{3}$$

where α_0 is the initial condition and a constant of the system that characterizes the amount of adiabaticity.

Since $\alpha(s) = \alpha_0 = \text{const}$, we have $d\alpha/ds = 0$, and the focusing strength along the channel varies as

$$K(s) = \frac{1+\alpha_0^2}{\beta^2} = \frac{1+\alpha_0^2}{(\beta_0 - 2\alpha_0 s)^2}\,. \tag{4}$$

Notice that the focusing strength scales inverse quadratically with $\beta(s)$.

For a particle with less energy than the design energy E_0, i.e., $E = (1-\delta)E_0$, it is possible to show that its amplitude of oscillation never exceeds that of the reference particle.[5] If one chooses the design energy of the focuser at the maximum energy of the incoming beam, the entire beam is expected to be focused. This *achromatic* nature of the focuser will hold true for a particle which emits radiation while traversing the focuser and is the very basis of the adiabatic-focuser concept.

The rate of energy loss of a relativistic electron due to synchrotron radiation is well known.[6] In order to perform simple analytic calculations, it is convenient to approximate the exact formula by the following expressions in the *classical*, the *transition*, and the *quantum* regimes[7] (see Fig. 1):

$$\frac{d\gamma}{ds} = -\frac{2}{3}\frac{\alpha}{\bar{\lambda}_c} \times \begin{cases} \Upsilon^2, & \Upsilon \lesssim 0.2\,, \\ 0.2\Upsilon, & 0.2 \lesssim \Upsilon \lesssim 22\,, \\ 0.556\Upsilon^{2/3}, & 22 \lesssim \Upsilon\,, \end{cases} \tag{5}$$

where γ is the Lorentz factor of the electron, α is the fine-structure constant, and $2\pi\bar{\lambda}_c$ is the Compton wavelength. We see that the energy loss is uniquely determined by the parameter Υ, which is Lorentz invariant and defined as $\Upsilon \equiv \gamma B/B_c$, where $B_c = m^2c^3/e\hbar \simeq 4.4 \times 10^{13}$ G is the Schwinger critical field.

Since the external magnetic field induces a bending of the electron trajectory, Υ can also be expressed in terms of the instantaneous radius of curvature ρ of the particle,

$$\Upsilon = \gamma^2\bar{\lambda}_c/\rho = \gamma^2/m\rho\,. \tag{6}$$

In the above equation and for the rest of the paper we adopt the convention of natural units, i.e., $c = \hbar = 1$.

Since $1/\rho = K(s)y$, and the beam size $\sigma = \langle y^2\rangle^{1/2} = \langle\beta\epsilon\rangle^{1/2}$, where ϵ is the emittance of the beam, we can express the above equation as a function of s explicitly,

$$\Upsilon(s) = \bar{\lambda}_c\sqrt{\epsilon}(1+\alpha_0^2)\frac{\gamma^2(s)}{[\beta_0 - 2\alpha_0 s]^{3/2}}\,, \tag{7}$$

where $1/m$ has been replaced by $\bar{\lambda}_c$. Notice that one essential character of synchrotron radiation is that the actual emittance, not the normalized emittance ($\epsilon_n = \gamma\epsilon$), is conserved, to an accuracy of the order $\mathcal{O}(1/\gamma)$, by the radiation process. Thus, the energy loss as a function of the distance of travel becomes

$$\frac{d\gamma(s)}{ds} = -\frac{2}{3}\frac{\alpha}{\bar{\lambda}_c} \times \begin{cases} \bar{\lambda}_c^2\epsilon(1+\alpha_0^2)^2\dfrac{\gamma^4(s)}{[\beta_0-2\alpha_0 s]^3}, & \Upsilon(s) \lesssim 0.2\,, \\ \frac{1}{5}\bar{\lambda}_c\sqrt{\epsilon}(1+\alpha_0^2)\dfrac{\gamma^2(s)}{[\beta_0-2\alpha_0 s]^{3/2}}, & 0.2 \lesssim \Upsilon(s) \lesssim 22\,, \\ 0.556[\bar{\lambda}_c\sqrt{\epsilon}(1+\alpha_0^2)]^{2/3}\dfrac{\gamma^{4/3}(s)}{\beta_0-2\alpha_0 s}, & 22 \lesssim \Upsilon(s)\,. \end{cases} \tag{8}$$

It is now straightforward to evaluate the energy loss in any regime and in going from one regime to another. For example, the fractional energy loss, $\delta = [\gamma_0 - \gamma(s)]/\gamma_0$, in the classical regime is

$$\delta_c(s) = \frac{1}{6}\alpha\bar{\lambda}_c\gamma_0^3\epsilon\frac{(1+\alpha_0^2)^2}{\alpha_0}\left[\frac{1}{\beta^2(s)} - \frac{1}{\beta_0^2}\right]. \tag{9}$$

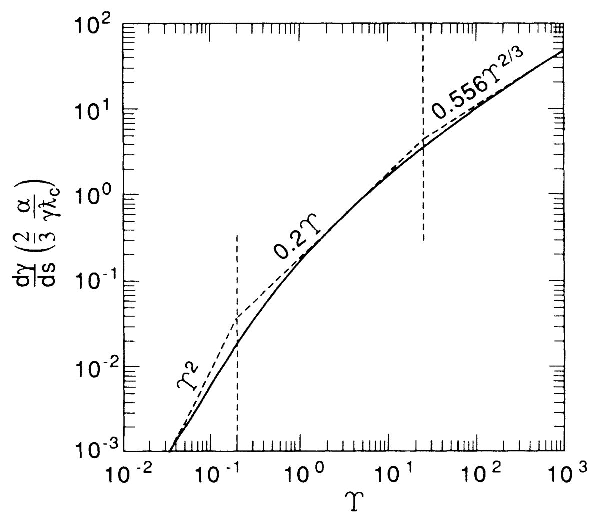

FIG. 1. The rate of synchrotron-radiation loss, in units of $2\alpha/3\gamma\bar{\lambda}_c$, as a function of the dimensionless parameter Υ. The solid curve is from the exact expression, while the dashed lines are from our approximate formulas.

In the transition regime, we have

$$\delta_t(s) = \frac{2}{15}\alpha\gamma_0\sqrt{\epsilon}\frac{1+\alpha_0^2}{\alpha_0}\left[\frac{1}{[\beta(s)]^{1/2}} - \frac{1}{(\beta_0)^{1/2}}\right], \tag{10}$$

while in the quantum regime

$$\delta_q(s) = \frac{0.556}{3}\alpha\left[\frac{\gamma_0\epsilon(1+\alpha_0^2)^2}{\bar{\lambda}_c\alpha_0^3}\right]^{1/3}\ln\left[\frac{\beta_0}{\beta(s)}\right]. \tag{11}$$

We now look for the optimal value of α_0 for attaining a desired β function with minimum energy loss. From Eqs. (9)-(11) we see that the dependence of energy loss on α_0 is different in the three regimes. By imposing $d\delta/d\alpha_0 = 0$ on the three equations, we find the optimum α_0 to be

$$\alpha_0 = \begin{cases} 1/\sqrt{3} \text{ (classical)}\,, \\ 1 \text{ (transition)}\,, \\ \sqrt{3} \text{ (quantum)}\,. \end{cases} \tag{12}$$

It should, in principle, be possible to set up an adiabatic focuser where the increase of its focusing strength varies in accordance with the three different optimum values given above. But the focuser may be experimentally more convenient if α_0 is fixed throughout the system. If a focuser covers all three regimes of radiation, an obvious compromise would be $\alpha_0 = 1$. Alternatively, since the radiation loss occurs primarily near the end of an adiabatic focuser, a choice of α_0 according to the final regime is most advisable.

As mentioned at the beginning, in a conventional focusing of charged-particle beams by discrete magnets, there exists a fundamental limit on the minimal attainable beam size due to the unavoidable synchrotron radiation that the beam suffers during the passage through the final quadrupole. The fact that this occurs at the last focusing element, and that the radiation is stochastic in character, renders the induced aberration uncorrectable. This limit on beam size at the focus can be expressed as

$$\sigma \gtrsim 3.4\times 10^{-4}\epsilon_n^{5/7} \tag{13}$$

in the vertical dimension for flat beams.

The situation is different in the case of a continuous focusing environment such as the adiabatic focuser. Off-momentum particles in this case would still be focused down adiabatically. However, the adiabatic focuser is not free from constraints.

Insensitive to the chromatic effect as it is, a beam would be useless if a substantial amount of energy were lost. The ultimate limitation is certainly that the fractional energy loss be much less than unity. In the classical regime, this means that

$$\beta \gg \{\tfrac{1}{6}[(1+\alpha_0^2)^2/\alpha_0]\alpha\lambda\!\!\!^-_c\gamma_0^3\epsilon\}^{1/2}. \tag{14}$$

Therefore, if the focuser is a purely classical one, then the beam size is limited as

$$\sigma_c = (\beta\epsilon)^{1/2} \gg \left[\frac{1}{6}\frac{(1+\alpha_0^2)^2}{\alpha_0}r_e\epsilon_n^3\right]^{1/4} \text{ (classical)}, \tag{15}$$

where $r_e = \alpha\lambda\!\!\!^-_c$, and the normalized emittance $\epsilon_n = \gamma_0\epsilon$ has been restored. If we take $\alpha_0 = 1$, this gives $\sigma_y = 0.5$ nm for $\epsilon_n = 2.5\times 10^{-8}$ m, which is numerically very close to the Oide limit with discrete focusing.

In the quantum regime, the same constraint leads to the condition

$$\sigma_q \gg \sigma_0 \exp\left\{-3\left[\frac{\alpha_0^3}{(1+\alpha_0^2)^2}\frac{\lambda\!\!\!^-_c}{\alpha^3\epsilon_n}\right]^{1/3}\right\} \text{ (quantum)}. \tag{16}$$

In order that the beam penetrates down to the quantum regime, there is, however, a requirement on the initial normalized emittance. For the beam to penetrate through the classical regime we find that

$$\epsilon_n \ll (5^4 6\lambda\!\!\!^-_c/\alpha)\alpha_0^3/(1+\alpha_0^2)^2 \text{ (classical)}. \tag{17}$$

Taking $\alpha_0 = 1/\sqrt{3}$, we find $\epsilon_n \ll 3.76\times 10^{10}\lambda\!\!\!^-_c = 0.014$ m. In order to reach the quantum regime the beam must penetrate through the transition regime. We find

$$\epsilon_n \ll \frac{15^3}{2^3 22}\frac{\lambda\!\!\!^-_c}{\alpha^3}\frac{\alpha_0^3}{(1+\alpha_0^2)^2} \text{ (transition)}. \tag{18}$$

For $\alpha_0 = 1$, this condition requires that $\epsilon_n \ll 4.7\times 10^{-6}$ m, in order to enter the quantum regime. When this condition on the emittance is satisfied, we obtain

$$\sigma_q \gg \left[\tfrac{1}{22}\lambda\!\!\!^-_c\epsilon_n^2(1+\alpha_0^2)\right]^{1/3} \times\exp\left[-3\left(\frac{\alpha_0^3}{(1+\alpha_0^2)^2}\frac{\lambda\!\!\!^-_c}{\alpha^3\epsilon_n}\right)^{1/3}\right]. \tag{19}$$

Notice that the limits on the emittance in Eqs. (17) and (18) depend only on fundamental physical parameters and the adiabaticity of the system. In both equations, the dependence on α_0 has a maximum value at $\alpha_0 = \sqrt{3}$. We thus call the quantity

$$\epsilon_c \equiv \frac{3^{3/2}15^3}{2^3 4^2 22}\frac{\lambda\!\!\!^-_c}{\alpha^3} = 6.17\times 10^{-6}\text{ m} \tag{20}$$

the *critical emittance*. For an emittance $\epsilon_n = \epsilon_c/10$, we find that $\sigma_q \gg 2.68\times 10^{-9}$ m, which is much smaller than the Oide limit.

One essential issue for an optical element is to estimate the sensitivity of the element to the less than ideal initial condition caused by errors of other optical elements upstream. Since our consideration here on the adiabatic focuser is about its linear optics, one expects the sensitivity to be essentially the same as that from the linear analysis of the conventional optics and analysis shows that such is, in fact, the case.[5]

Another degradation is the effects due to the nonlinear force in the focuser. To elucidate the issue, we consider a sextupolelike nonlinear force which increases adiabatically as a fixed proportion of the linear force. The equation of motion is now

$$d^2y/ds^2 + K(s)y = K(s)(\mu/\sigma)y^2, \tag{21}$$

where $\sigma = \sqrt{\beta\epsilon}$, $K(s)$ is given in Eq. (4), and the dimensionless parameter μ characterizes the degree of nonlinearity of the force. From particle tracking in the phase space of such a Hamiltonian system, and from the direct particle-in-cell computer simulations, we find that a nonlinearity as large as $\mu = 0.12$ is still tolerable with no significant loss of beam particles.[5]

We have generated, and checked with numerical simulations, three examples of an adiabatic focuser. The first is a proof-of-principle case using the beam in the SLAC End Station. The second involves the use of a focuser on the SLAC Linear Collider (SLC), and the third is a focuser on a TLC being considered at SLAC. Parameters of the beam, the focuser, and the expected performance are displayed in Table I. In the first two cases, round beams, i.e., $\sigma_y = \sigma_x$, are assumed, whereas in the third case for the TLC, the beam is assumed to be flat ($\sigma_y \ll \sigma_x$). The variation in plasma density, ramping from the initial value, n_0, to the final value, n^*, is over a length L.

The most likely way to realize the concept is to employ an underdense plasma column with graded density. The device has a number of advantageous properties creating, for example, a significantly improved luminosity for a

TABLE I. Three examples of an adiabatic focuser.

	SLAC End Station	SLC	TLC
Initial beam properties			
E_0 (GeV)	15	50	500
ϵ_n (m)	1×10^{-4}	3×10^{-5}	1×10^{-8}
σ_0 (μm)	20	3	5×10^{-3}
β_0 (cm)	12	3	0.25
Focuser properties			
α_0	5×10^{-2}	$1/\sqrt{3}$	$\sqrt{3}$
L (cm)	119	2.6	0.07
n_0 (cm^{-3})	1.2×10^{14}	8.4×10^{15}	1.8×10^{19}
n^* (cm^{-3})	1.2×10^{18}	8.4×10^{19}	1.8×10^{23}
Final beam properties			
δ	Negligible	3%	1%
σ^* (μm)	2	0.3	0.5×10^{-3}

collider, but requires a plasma with very high density near the interaction point. This plasma will cause scattering of the beam and hence emittance blowup. The effect has been analyzed by Montague and Schnell[8] and it can be verified that the growth of emittance is negligibly small in all three examples which we discussed above. In addition, the plasma will create background events.[9,10] We have not analyzed the effect of these events on the design of a detector.

Of the three examples given, the first two cases require densities of materials in the gaseous state, whereas for the TLC case it would be in the liquid, or even solid, state. For focusers in the gaseous state, a smooth increase of density (and therefore focusing strength) should be possible simply using differential pumping. Thus one should be able to realize the first example (a proof of principle) and the second example which is interesting in its own right. The third example is presented as almost a challenge to the community, for it raises many yet unanswered plasma physics and fabrication questions, but shows the rich return if a high-density adiabatic focuser can be realized.

This work was supported by Department of Energy Contracts No. DE-AC03-76SF00515, No. DE-AC03-76SF00098, and No. W-7405-ENG-48.

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