Dark Matter, Time-Varying G, and a Dilaton Field

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We demonstrate the observational viability of having an exactly massless dilaton field couple with gravitational strength to most matter in the Universe. This is done by constructing a generalized Jordan-Brans-Dicke model in which the scalar couples with different strengths to visible and to conjectured "dark" matter. In this model, improved $\dot{G}$ measurements may provide nontrivial bounds on the coupling constants of the dilaton to matter.

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Kaluza-Klein and superstring theories naturally give rise to "dilaton fields," i.e., to neutral scalar fields whose background values determine the strength of some of the coupling constants of the effective four-dimensional theory. The experimental consequences of a dilaton field are of two sorts: (i) modification of large-scale gravitational phenomena (due to the admixture of a scalar to the usual tensor interaction), and (ii) violation of the equivalence principle (through the dilaton-induced space-time dependence of the locally measured effective coupling constants, notably Newton's gravitational constant). Both types of effects are severely constrained by present experiments. This is why it is often hoped that the dilatons will somehow become sufficiently massive for cutting off all experimental deviations at length (or time) scales greater than its Compton length. However, it is not obvious that one or more massless scalars do not survive, coupled to mater with gravitational strength.

The purpose of this Letter is to prove the compatibility between present experimental constraints and the existence of an exactly massless dilaton coupled with most matter in the Universe with a strength comparable to gravity. We shall prove this possibility by exhibiting a simple model theory in which a scalar field is coupled more strongly to dark matter than to visible matter. The spirit of our present analysis will be phenomenological, and we leave to future work the task of investigating how such a theory might be derived from a more fundamental theory. Another motivation for introducing our model is to construct a field-theory framework justifying the usual phenomenological analysis of the cosmological variation of Newton's constant consisting of replacing it in the equations of motion by a function of time, $G(t)$: See our result (26) which depends on a new parameter which is independent of the usual Jordan-Brans-Dicke parameter ω_V which measures the coupling of $G(x)$ to the local matter distribution.

The model theory that we shall analyze is a generalization of the simplest scalar-tensor theory, namely the Jordan-Brans-Dicke theory.[1,2] The usual presentation of this theory is given by the following action S (ignoring boundary terms and using signature $-+++$, and $c=1$):

$$S=\int\left(\tilde{\phi}\tilde{R}-\frac{\omega}{\tilde{\phi}}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\phi}\,\partial_\nu\tilde{\phi}\right)\tilde{g}^{1/2}d^4x+S_m[\psi,\tilde{g}_{\mu\nu}]\,, \qquad (1)$$

where $\tilde{g}_{\mu\nu}$ is the space-time metric (in what we shall call the Jordan conformal frame), $\tilde{R}$ its curvature scalar, and ω is a dimensionless coupling constant. (Note that $\tilde{\phi}$ as defined here is $1/16\pi$ times ϕ as defined in Ref. 2.) The last term in (1) denotes the action of the matter, which is a functional of some matter variables, collectively denoted by ψ, and of the metric $\tilde{g}_{\mu\nu}$. The functional $S_m[\psi,\tilde{g}]$ should reduce to the corresponding Minkowski matter action, $S_m[\psi,\eta]$, when $\tilde{g}_{\mu\nu}\to\eta_{\mu\nu}$, and, therefore, should contain, besides ψ, $\tilde{g}_{\mu\nu}$, and their derivatives, only some *constant* parameters: masses, coupling constants, etc.

The dimensionless coupling constant of the Jordan-Brans-Dicke theory is severely constrained by present observations to be $\omega^{-1}<0.004$ (2σ limit).[3] In order to construct a generalization of this theory which can allow stronger couplings of the scalar field let us perform a Weyl scaling of the metric $\tilde{g}_{\mu\nu}$ to transform the action (1) to a more useful form.[4] We define a new metric $g_{\mu\nu}$, which defines what we will call the Einstein conformal frame, and a new scalar field σ by

$$g_{\mu\nu}=(16\pi\mathcal{G}\tilde{\phi})\tilde{g}_{\mu\nu}=(2\kappa^2\tilde{\phi})\tilde{g}_{\mu\nu}\,, \qquad (2)$$

$$\sigma=-(\omega+\tfrac{3}{2})^{1/2}\ln(2\kappa^2\tilde{\phi})\,, \qquad (3)$$

where $\mathcal{G}$ is a constant with the same dimensions as

Newton's constant and $\kappa^2=8\pi\mathcal{G}$, to obtain (modulo boundary terms) the equivalent action

$$S=\int\left[\frac{R}{2\kappa^2}-\frac{g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma}{2\kappa^2}\right]g^{1/2}d^4x+S_m[\psi,e^{2\beta\sigma}g_{\mu\nu}]\,, \tag{4}$$

where R denotes the curvature scalar of $g_{\mu\nu}$ and where

$$\beta=\frac{1}{2(\omega+\frac{3}{2})^{1/2}}\,,\quad \tilde{\phi}=\frac{1}{2\kappa^2}e^{-2\beta\sigma}.$$

In the simple case where the matter is phenomenologically represented as a set of point particles of mass $\tilde{m}$ and worldline $x^\mu=z^\mu(\lambda)$ (with g-proper time $ds=[-g_{\mu\nu}(z)dz^\mu dz^\nu]^{1/2}$), the matter action reads

$$S_m[z,e^{2\beta\sigma}g_{\mu\nu}]=-\sum\int\tilde{m}e^{\beta\sigma}ds\,,$$

which may be recast in terms of a space-time–dependent Einstein-frame mass, $m=\tilde{m}e^{\beta\sigma}$, where $\tilde{m}$ is the constant Jordan-frame mass.

It is important to realize that by means of suitable Weyl rescalings and redefinitions of the scalar field any reasonable scalar-tensor theory has a kinetic term given by the first two terms in (4). What distinguishes different scalar-tensor theories is the coupling to matter and any possible mass or self-interaction terms. Besides its "dilaton" nature (shifting σ by a constant entails a multiplicative change of the scale of the gravitational couplings), the matter-scalar coupling chosen by Brans and Dicke has two important features.

(1) All matter couplings are metric; e.g., σ appears in the matter Lagrangian only in the combination $e^{2\beta\sigma}g_{\mu\nu}$.

(2) All material systems couple to the same metric, i.e., are "freely falling" in this universal metric.

These features are motivated by the very precise experiments which support the "weak equivalence principle" and the "Einstein equivalence principle."[3] They are certainly very plausible assumptions for all conventional, visible matter that we have knowledge of at present. On the other hand, motivated, say, by the desire to reconcile a theoretically preferred spatially flat cosmological model with the observed luminous matter density, we shall assume here the existence of invisible matter in the Universe. It is then not at all obvious that the invisible matter need couple to σ with the same strength as the visible matter.

The model.— We propose here to investigate a new class of scalar-tensor models in which the visible matter couples to the metric $e^{2\beta_V\sigma}g_{\mu\nu}$, while the invisible one couples to the different, but conformally related, metric $e^{2\beta_I\sigma}g_{\mu\nu}$. In other words, we shall investigate the consequences of Lagrangians of the type

$$S=\int\left[\frac{R}{2\kappa^2}-\frac{g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma}{2\kappa^2}\right]g^{1/2}d^4x+S_V[\psi_V,e^{2\beta_V\sigma}g_{\mu\nu}]+S_I[\psi_I,e^{2\beta_I\sigma}g_{\mu\nu}]\,, \tag{5}$$

where now the basic dimensionless coupling constants, β_i with $i=I,V$, can be positive or negative (actually only the relative sign matters). Then the corresponding ω_i's are defined by $\omega_i+\frac{3}{2}\equiv(4\beta_i^2)^{-1}$ (so that $\omega_i>-\frac{3}{2}$). Note that the value $\omega=-1$ (which some authors claim to arise naturally from superstring theory[5]) corresponds to $\beta=\pm 1/\sqrt{2}$. In the following, we shall investigate to what extent observations give us constraints on β_I and β_V.

In the Einstein conformal frame $g_{\mu\nu}$ the field equations derived from the Lagrangian (5) are

$$G_{\mu\nu}=\sigma_{,\mu}\sigma_{,\nu}-\tfrac{1}{2}g_{\mu\nu}(\nabla\sigma)^2+\kappa^2(T^V_{\mu\nu}+T^I_{\mu\nu})\,, \tag{6}$$

$$\Box\sigma=-\kappa^2(\beta_V T^V+\beta_I T^I)\,, \tag{7}$$

where $G_{\mu\nu}$ is the Einstein tensor of the metric $g_{\mu\nu}$, ∇_μ its Levi-Civita connection (with $\Box\equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$), and

$$T_i^{\mu\nu}\equiv 2g^{-1/2}\delta S_i[\psi_i,e^{2\beta_i\sigma}g_{\mu\nu}]/\delta g_{\mu\nu} \tag{8}$$

the stress-energy tensor of the i type of matter ($i=V,I$) in the Einstein frame (all indices are moved by the g metric and T^i denotes the g trace of $T^i_{\mu\nu}$). The invariance of $S_i[\psi_i,e^{2\beta_i\sigma}g_{\mu\nu}]$ under coordinate transformations leads, when the ψ_i equations of motion are satisfied, to

$$\nabla^\lambda T^i_{\lambda\mu}=\beta_i T^i\nabla_\mu\sigma\quad\text{(no summation on } i\text{)}\,. \tag{9}$$

The corresponding equations in the (visible) Jordan frame are given in the Appendix. In the approximation of a smooth distribution of dark matter over, e.g., solar-system or binary-pulsar scales, these equations show that locally our model reduces to a usual Jordan-Brans-Dicke theory with $\omega=\omega_V$, except for the possibly different time dependence of the spatially asymptotic value of the scalar field. To determine the latter, which is influenced by the large-scale matter distribution, and depends on both β_V and β_I, let us consider the cosmological solutions of our model.

Perfect-fluid distributions, i.e., $T_i^{\mu\nu}=(\rho_i+p_i)u_i^\mu u_i^\nu+p_i g^{\mu\nu}$, with $g_{\mu\nu}u_i^\mu u_i^\nu=-1$, can act as sources of a Robertson-Walker metric (in the Einstein frame)

$$ds^2\equiv g_{\mu\nu}dx^\mu dx^\nu=-dt^2+a^2(t)dl^2\,, \tag{10}$$

with

$$dl^2=\frac{dr^2}{1-kr^2}+r^2(d\theta^2+\sin^2\theta d\varphi^2)\,, \tag{11}$$

if $u_V^\mu=u_I^\mu$, and if all physical quantities depend only on time. Then the field equations (6) and (7) give (with an overdot $\equiv d/dt$)

$$3\frac{\dot{a}^2+k}{a^2}=\kappa^2\sum_i\rho_i+\tfrac{1}{2}\dot{\sigma}^2\,, \tag{12}$$

$$-3\frac{\ddot{a}}{a}=\tfrac{1}{2}\kappa^2\sum_i(\rho_i+3p_i)+\dot{\sigma}^2\,, \tag{13}$$

$$-a^{-3}(a^3\dot{\sigma})^{\cdot}=\kappa^2\sum_i\beta_i(\rho_i-3p_i)\,. \tag{14}$$

We will now assume that the Universe is dynamically dominated by some kind of invisible matter, so that we can neglect ρ_V and p_V compared to ρ_I and p_I on very large distance scales. For the moment, we shall also assume that the pressure is zero. Introducing the dynamical variables $H=\dot{a}/a$, $y=\dot{\sigma}$, and adopting units in which $\kappa^2=1$, we obtain a constrained dynamical system out of (12)-(14). The constraint can be eliminated by defining $F=ka^{-2}H^{-2}$, obtaining

$$\begin{aligned}\dot{y}&=-3Hy-3\beta_I(F+1)H^2+\tfrac{1}{2}\beta_I y^2,\\ \dot{H}&=-\tfrac{3}{2}H^2-\tfrac{1}{4}y^2-\tfrac{1}{2}FH^2,\\ \dot{F}&=(F+F^2)H+\frac{1}{2}\frac{y^2F}{H}.\end{aligned} \tag{15}$$

$F=0$ ($k=0$) is an invariant plane in phase space. In view of the existence of convincing arguments (e.g., inflation) for $k=0$ (or more exactly, for F being exponentially small) we shall limit ourselves to this subset of solutions. (A similar but not identical treatment of Jordan-Brans-Dicke cosmology as a dynamical system has recently been given by Romero *et al.*[6])

The simple dynamical system (15) is written in the Einstein frame, which is, however, not directly accessible to observation (one would need, e.g., a system of two black holes, for which $T^I_{\mu\nu}=0$, and $\sigma=\text{const}$,[7] to make up a clock ticking the Einstein time). Observations are rather made with objects made of visible matter (this includes atomic clocks as well as the binary pulsar), coupled to, and therefore, measuring only the "visible metric,"

$$d\tilde{s}^2=e^{2\beta_V\sigma}ds^2. \tag{16}$$

In this "Jordan conformal frame" (or "atomic frame") the Robertson-Walker metric (10) becomes

$$d\tilde{s}^2=e^{2\beta_V\sigma}[-dt^2+a^2(t)dl^2]=-d\tilde{t}^2+\tilde{a}^2(\tilde{t})dl^2, \tag{17}$$

where

$$\tilde{a}=e^{\beta_V\sigma}a,\quad d\tilde{t}=e^{\beta_V\sigma}dt \tag{18}$$

are the cosmological variables directly accessible to observation. Generalizing the system (15) to a γ-law equation of state, $p_I=(\gamma-1)\rho_I$, and rewriting it in terms of $\tilde{a}$ and $\tilde{t}$, with the definitions $\tilde{y}=d\sigma/d\tilde{t}$, $\tilde{H}=(d\tilde{a}/d\tilde{t})/\tilde{a}$, $r=4-3\gamma$, one obtains ($k=0$)

$$\begin{aligned}\frac{dy}{d\tilde{t}}&=-3r\beta_I\tilde{H}^2+(6r\beta_I\beta_V-3)\tilde{H}\tilde{y}+(2\beta_V-3r\beta_I\beta_V^2+\tfrac{1}{2}r\beta_I)\tilde{y}^2,\\ \frac{dH}{d\tilde{t}}&=(-\tfrac{3}{2}\gamma-3r\beta_I\beta_V)\tilde{H}^2+(3\gamma\beta_V-4\beta_V+6r\beta_I\beta_V^2)\tilde{H}\tilde{y}+(\tfrac{1}{4}\gamma-\tfrac{1}{2}-\tfrac{3}{2}\gamma\beta_V^2+3\beta_V^2-3r\beta_I\beta_V^3+\tfrac{1}{2}r\beta_V\beta_I)\tilde{y}^2.\end{aligned} \tag{19}$$

A study of its phase space shows that there are two repellers and one attractor, which happen to be straight lines through the origin, corresponding to power-law solutions. The two repellers are

$$\begin{aligned}\tilde{H}&=\frac{\beta_V\pm(\frac{1}{6})^{1/2}}{\beta_V\pm3(\frac{1}{6})^{1/2}}\tilde{t}^{-1},\\ \tilde{y}&=\frac{1}{\beta_V\pm(\frac{1}{6})^{1/2}}\tilde{H},\end{aligned} \tag{20}$$

and the attractor is

$$\tilde{H}=\frac{2-\gamma-2r\beta_I\beta_V}{3\gamma-\frac{3}{2}\gamma^2+r^2\beta_I^2-2r\beta_I\beta_V}\tilde{t}^{-1}, \tag{21}$$

$$\tilde{y}=\frac{r\beta_I}{r\beta_I\beta_V-1+\frac{1}{2}\gamma}\tilde{H}. \tag{22}$$

These correspond to the results in Ref. 8 for the special case $\beta_V=\beta_I=\beta$, $\gamma=1$.

As shown, e.g., in Ref. 3 the value of the observable gravitational constant on solar-system scales is given in terms of the cosmological value of the scalar field, $\sigma_c(t)$, by

$$\tilde{G}=\frac{2\omega_V+4}{2\omega_V+3}\frac{1}{16\pi\tilde{\phi}_c}=\mathcal{G}(1+2\beta_V^2)e^{2\beta_V\sigma_c}. \tag{23}$$

The reason for the factor $1+2\beta_V^2$ in (23) is that in Jordan-Brans-Dicke theory the inverse-square-law attraction between two static masses is due not only to graviton exchange as in conventional general relativity but also to the exchange of σ quanta which gives an additional attraction. Equation (23) implies for the observable (Jordan-time) variation of Newton's constant

$$\frac{dG}{d\tilde{t}}\Big/\tilde{G}=2\beta_V\tilde{y}. \tag{24}$$

If we assume that the world is now in a state very close (by angle) to the attractor, this becomes

$$\frac{dG}{d\tilde{t}}\Big/\tilde{G}=\frac{2r\beta_V\beta_I}{r\beta_V\beta_I-1+\frac{1}{2}\gamma}\tilde{H}, \tag{25}$$

or, if $p_I=0$ ($\gamma=1=r$),

$$\frac{dG}{d\tilde{t}}\Big/\tilde{G}=-\frac{4\beta_V\beta_I}{1-2\beta_V\beta_I}\tilde{H}=-\frac{4\beta_V\beta_I}{\frac{3}{2}+\beta_I^2-2\beta_I\beta_V}\tilde{t}^{-1}, \tag{26}$$

which generalizes the usual Jordan-Brans-Dicke result.[8] For highly relativistic matter ($\gamma=\frac{4}{3}$) we find $dG/d\tilde{t}=0$, which was to be expected from the fact that σ couples to the trace of the energy-momentum tensor only.

Observational constraints.— We know of three independent constraints on the dimensionless coupling constants β_V and β_I of our model.

(i) From radar time-delay measurements, $|\beta_V|$ is independently constrained to be smaller than 0.032 ($\omega_V > 250$, 2σ limit of Ref. 9).

(ii) A firm lower limit on $\tilde{H}_0\tilde{t}_0$, where $\tilde{H}_0$ is the Hubble constant and $\tilde{t}_0$ is the age of the Universe, is 0.4 (corresponding to the 2σ lower limits $\tilde{H}_0^{\min}=48$ km/s Mpc, $\tilde{t}_0^{\min}=7.8$ Gyr quoted in Ref. 10). For the above small values of β_V Eq. (21), with $\gamma=1=r$, then constrains $|\beta_I|$ to be smaller than 1.0 nearly independently of β_V. This allows, for instance, the "string value" $\omega_I=-1$ ($|\beta_I|=1/\sqrt{2}$). The combination of these first two constraints allows $|(dG/d\tilde{t})_0/\tilde{G}_0|$, according to Eq. (26), to be as large as 6.6×10^{-22} yr^{-1} for the consistently extreme values $|\beta_I|^{\max}=1$, $\tilde{t}_0^{\min}=7.8$ Gyr. (Note that the sign of $dG/d\tilde{t}$ is opposite to the relative sign of β_V and β_I.) This upper limit is within the reach of $\dot{G}$ measurements.

(iii) Solar-system estimates of $\dot{G}/G$ are based essentially on the Viking-lander data. Taken at their 2σ limits, they give (in units of 10^{-12} yr^{-1}) $-6<(dG/d\tilde{t})_0/\tilde{G}_0<10$ according to Hellings *et al.*,[11] and $-22<(dG/d\tilde{t})_0/\tilde{G}_0<18$ according to an analysis of the Center for Astrophysics group quoted in Ref. 12. The (independent) binary-pulsar measurements[13] give (2σ level) $-14<(dG/d\tilde{t})_0/\tilde{G}_0<38$. As for the limits based on primordial nuclosynthesis[14] let us emphasize, on the one hand, that they strongly depend on the many simplifying assumptions that enter the standard big-bang model, and, on the other hand, that they do not restrict the present value of $\dot{G}/G$ but rather the average $|\tilde{G}_{\rm now}-\tilde{G}_{\rm nucleo}|/\tilde{G}_{\rm now}(t_{\rm now}-t_{\rm nucleo})$ (below a level $\sim 20\times 10^{-12}$ yr^{-1}).

Therefore present observational evidence is compatible with the existence of an exactly massless scalar field coupled with gravitational strength ($|\beta_I|\sim 1$) to dark matter, and a possible test of the existence of such a scalar could come from an improvement in the precision of $\dot{G}$ experiments.

Appendix.— When transforming to the Jordan frame, $\tilde{g}_{\mu\nu}=\exp(2\beta_V\sigma)g_{\mu\nu}$ (adapted to the visible matter), the action (5) reads

$$S=\int\left[\tilde{\phi}\tilde{R}-\frac{\omega_V}{\tilde{\phi}}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\phi}\,\partial_\nu\tilde{\phi}\right]\tilde{g}^{1/2}d^4x$$

$$+S_V[\psi_V,\tilde{g}_{\mu\nu}]+S_I[\psi_I,(2\kappa^2\tilde{\phi})^{(\beta_V-\beta_I)/\beta_V}\tilde{g}_{\mu\nu}]\,,$$

and leads to Jordan-frame field equations of the form

$$\tilde{G}_{\mu\nu}=\frac{1}{2\tilde{\phi}}(\tilde{T}^V_{\mu\nu}+\tilde{T}^I_{\mu\nu})+\frac{\omega_V}{\tilde{\phi}^2}[\tilde{\phi}_{,\mu}\tilde{\phi}_{,\nu}-\tfrac{1}{2}\tilde{g}_{\mu\nu}(\tilde{\nabla}\tilde{\phi})^2]$$

$$+\frac{1}{\tilde{\phi}}(\tilde{\nabla}_\mu\tilde{\nabla}_\nu\tilde{\phi}-\tilde{g}_{\mu\nu}\tilde{\Box}\tilde{\phi})\,,$$

$$\tilde{\Box}\tilde{\phi}=\beta_V^2\left[\tilde{T}^V+\frac{\beta_I}{\beta_V}\tilde{T}^I\right],$$

where $\omega_V+\frac{3}{2}\equiv(4\beta_V^2)^{-1}$, all metric operations are performed with $\tilde{g}_{\mu\nu}$, and $\tilde{T}^{\mu\nu}_i\equiv 2\tilde{g}^{-1/2}\delta S_i[\psi_i,\tilde{\phi},\tilde{g}_{\mu\nu}]/\delta\tilde{g}_{\mu\nu}$ denotes the stress-energy tensor of the i type of matter in the Jordan frame. The latter satisfy (from the invariance of S_V and S_I under coordinate transformations) the conservation laws

$$\tilde{\nabla}_\lambda\tilde{T}^{\lambda\mu}_V=0\,,$$

$$\tilde{\nabla}_\lambda\tilde{T}^{\lambda\mu}_I=\frac{1}{2}\frac{\beta_V-\beta_I}{\beta_V}\tilde{T}_I\tilde{\nabla}^\mu\ln\tilde{\phi}\,,$$

and are related to their Einstein-frame counterparts by $T^{\mu\nu}_i=e^{6\beta_V\sigma}\tilde{T}^{\mu\nu}_i$. Hence the density, pressure, and four-velocity (for any type of matter) in the Einstein conformal frame are related to those in the Jordan (atomic) frame by $\rho=e^{4\beta_V\sigma}\tilde{\rho}$, $p=e^{4\beta_V\sigma}\tilde{p}$, $u^\mu=e^{\beta_V\sigma}\tilde{u}^\mu$.

[1]P. Jordan, Nature (London) **164**, 637 (1949); *Schwerkraft und Weltall* (Vieweg, Braunschweig, 1955); Z. Phys. **157**, 112 (1959).

[2]C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).

[3]C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge Univ. Press, Cambridge, 1981).

[4]W. Pauli, quoted in Sec. 28 of Jordan's book, Ref. 1; M. Fierz, Helv. Phys. Acta **29**, 128 (1956); R. H. Dicke, Phys. Rev. **125**, 2163 (1962).

[5]J. Scherk and J. H. Schwarz, Nucl. Phys. **B81**, 118 (1974); K. I. Maeda, Mod. Phys. Lett. A **3**, 243 (1988).

[6]C. Romero *et al.*, Centro Brasileiro de Pesquisas Físicas Report No. CBPF-NF-045/88 (to be published).

[7]S. W. Hawking, Commun. Math. Phys. **25**, 167 (1972).

[8]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).

[9]R. D. Reasenberg *et al.*, Astrophys. J. Lett. Ed. **234**, L219 (1979).

[10]W. A. Fowler, Q. J. Roy. Astron. Soc. **28**, 87 (1987).

[11]R. W. Hellings *et al.*, Phys. Rev. Lett. **51**, 1609 (1983).

[12]J. D. Anderson *et al.*, Adv. Space Res. **9**, (9)71 (1989).

[13]T. Damour, G. W. Gibbons, and J. H. Taylor, Phys. Rev. Lett. **61**, 1151 (1988); J. H. Taylor and J. M. Weisberg, Astrophys. J. **345**, 434 (1989).

[14]J. D. Barrow, Mon. Not. Roy. Astron. Soc. **184**, 677 (1978); J. Yang *et al.*, Astrophys. J. **227**, 697 (1979); T. Rothman and R. Matzner, Astrophys. J. **257**, 450 (1982).