Fermi-Liquid Parameters and Superconducting Instabilities of a Generalized t-J Model

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We investigate the Fermi-liquid parameters and the superconducting instabilities of the t-J model using a systematic large-N expansion. We compare our results with the predictions of the Brinkman-Rice picture. The leading superconducting instability is in the d-wave channel and has $\cos K_x - \cos K_y$ symmetry and increases with doping.

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There has been a recent surge of interest in the strong correlation problem motivated by the discovery of the heavy-electron systems[1] and the high-temperature superconductors.[2] Since there are no exact solutions of strongly interacting fermion Hamiltonians in dimensions higher than 1, a number of approaches, such as variational methods, exact diagonalization of small systems, and Monte Carlo simulations, have all been applied to this problem. It is believed that when the number of particles is small the ground state is metallic, while at half filling, when the number of particles per site equals 1, the ground state is a magnetic insulator. The theoretical challenge is to describe the strongly correlated metallic state, the insulating state, and the transition between the two as a function of doping.

A great deal of insight into the strongly correlated metallic phase is due to Brinkman and Rice.[3] They described the strongly correlated metallic phase as a Fermi liquid with a magnetic susceptibility and an effective mass inversely proportional to the proximity to half filling. In this Letter we investigate this regime using a large-N expansion applied to a generalization of the large-U limit of the Hubbard model Hamiltonian, usually referred to as the t-J model. Using a systematic approach, which is not perturbative in the coupling constants, we study the behavior of the Fermi-liquid parameters in the metallic phase and its superconducting instabilities. Differences and similarities with the Brinkman-Rice picture are emphasized.

The Hamiltonian that we will study is given by

$$H=-\frac{t_0}{N}\sum_{ij\sigma}C_{i\sigma}^{\dagger}C_{j\sigma}+\frac{J}{N}\sum_{ij\sigma\sigma'}C_{i\sigma}^{\dagger}C_{i\sigma'}C_{j\sigma'}^{\dagger}C_{j\sigma} \tag{1a}$$

subject to the system of local constraints

$$\sum_{\sigma}C_{i\sigma}^{\dagger}C_{i\sigma}\le q_0N\,. \tag{1b}$$

In the t-J model, $q_0=1/N$. However, as is customary in the heavy-fermion literature,[4] the constraint (1b) will be relaxed and the parameter q_0 in Eq. (2) will be taken to be of the order of 1 to carry out a controlled $1/N$ expansion. For simplicity we will study a square lattice in two dimensions and ij in (1a) run over nearest neighbors. Mean-field theories for this model were studied by several authors,[5,6] and a large-N expansion at half filling was formulated by Affleck and Marston.[7,8] They also proposed an extension of their model away from half filling. This extension[7,8] is always in the weak-correlation regime since the on-site repulsion is scaled as $1/N$. This should be contrasted with our model, which describes the strongly correlated situation. The $J=0$ limit of this model was considered in a previous publication.[9]

The partition function of the model is defined by the functional integral

$$Z=\int d\Delta^{\dagger}d\Delta\, df^{\dagger}df\, db^{\dagger}db\, d\lambda \exp\left(-\int_0^{\beta}a\right), \tag{2a}$$

$$a=\sum_{i\sigma}\left[f_{i\sigma}^{\dagger}\left(\frac{\partial}{\partial\tau}-\mu\right)f_{i\sigma}+b_i^{\dagger}\frac{\partial}{\partial\tau}b_i+\frac{N}{J}\sum_{\eta}|\Delta_{i,i+\eta}|^2 + i\lambda_i(f_{i\sigma}^{\dagger}f_{i\sigma}+b_i^{\dagger}b_i-q_0N)\right]+H\,, \tag{2b}$$

$$H=-\sum_{i\eta}f_{i\sigma}^{\dagger}f_{i+\eta\sigma}\left(\Delta_{i,i+\eta}+\frac{t_0}{N}b_{i+\eta}^{\dagger}b_i\right)+\text{c.c.} \tag{2c}$$

b_i is a slave boson[4,10] field introduced in the decomposition of the electron operator $C_{i\sigma}^{\dagger}=f_{i\sigma}^{\dagger}b_i$ to convert the inequality constraint (1b) into a holonomic constraint enforced by a Lagrange multiplier λ_i which is time independent since the Hamiltonian commutes with the constraint. When $N=2$ and $q_0=0.5$, Eq. (2) represents the partition function of the original t-J model. $\Delta_{i,i+\eta}$ is a complex link variable used to decouple the exchange term in the Hamiltonian (1a). The index η runs over x,y and N_s is the number of lattice sites. μ is a chemical potential determined from Eq. (3) so as to have δ holes per site:

$$\sum_{i\sigma}\langle f_{i\sigma}^{\dagger}f_{i\sigma}\rangle=N_sNq_0(1-\delta)\,. \tag{3a}$$

In the large-N limit, the partition function may be calculated by a saddle-point approximation in the fields $\Delta_{i,i+\eta}$, λ_i, and b_i. In this paper we will restrict ourselves to a saddle point in which the variables are uniform in space. We have shown that this solution is stable for $\delta t > \alpha J$, with α a number smaller than 1 which depends on the doping. The region where this saddle point is stable is shown in Fig. 1. For $J=0$ and a small δ one expects an instability towards ferromagnetism in the Hubbard model (Nagaoka theorem). This instability is suppressed in the large-N limit considered in this paper. The uniform saddle-point solution is formally identical to the resonating-valence-bond (RVB) uniform phase, discovered by Baskaran, Zou, and Anderson[5] which has been recently investigated by Ioffe and Larkin.[11] It is important to emphasize, however, that in the large-N approach, the uniform saddle point, whenever it is stable, describes a Fermi liquid with strong antiferromagnetic correlations. The ground state of the model has the same Fermi surface as the corresponding noninteracting system defined by ignoring the constraint and setting J equal to zero in Eq. (1). In this paper we will investigate the behavior of the Fermi-liquid parameters as a function of doping and the superconducting instabilities of the Fermi-liquid state.

The saddle-point equations are given by

$$\begin{aligned} \lambda &= \frac{4t_0}{N_s}\sum_k \cos K_x f(E_k)\,, \\ \Delta &= \frac{J}{N_s}\sum_k \cos K_x f(E_k)\,, \\ b^2 &= Nq_0\delta\,. \end{aligned} \tag{3b}$$

The summations run over the Brillouin zone. The quasiparticle energies at $N=\infty$ are given by

$$E_k = -2\left(\Delta+\frac{b^2t_0}{N}\right)(\cos K_x+\cos K_y)+\lambda-\mu\,. \tag{4}$$

The effective interaction between the quasiparticles is given, to leading order in $1/N$, by the Gaussian fluctuations of the Bose fields around the saddle point. Integrating out the fermions and expanding to quadratic order in the Bose fields we find the effective action

$$\delta L = \tfrac{1}{2}NX^i(q,\Omega)[B_{ij}+\pi_{ij}(q,\Omega)]X^j(-q,-\Omega)\,, \tag{5}$$

where we introduced the notation $\mathbf{X}_i=(r_i,\lambda_i,r_i^x,r_i^y,A_i^x,A_i^y)$ for the fluctuating Bose fields. Here we work in the radial gauge,[12] that is $\lambda_i=\lambda+\dot\theta_i$ and $b_i=b(1+r_i)\times e^{i\theta_i}$, $\Delta_i^\eta=\Delta(1+iA_i^\eta+r_i^\eta)$ with $\eta=x,y$ and r_i, A_i^η, r_i^η real fields. The inverse propagator of the Bose fields is defined in terms of the matrix

$$\tfrac{1}{2}B_{ij}=\begin{pmatrix} \dfrac{b^2\lambda_0}{N}\displaystyle\sum_\delta \sin^2\left(\frac{q\delta}{2}\right) & i\dfrac{b^2}{N} \\ i\dfrac{b^2}{N} & 0\end{pmatrix}\oplus\left(\frac{\Delta^2}{J}\right)1_{ij}\,, \tag{6}$$

where 1_{ij} is a 4×4 identity matrix, $K_+=K+q/2$, $K_-=K-q/2$, and the polarization bubbles

$$\pi_{ij}=\sum_k \frac{f(E_{K_+})-f(E_{K_-})}{E_{K_+}-E_{K_-}-i\Omega}\Lambda^i(K_+K_-)\Lambda^j(K_-K_+) \tag{7}$$

are defined in terms of the vertices

$$\begin{aligned} \Lambda^r(K_+,K_-) &= -\frac{4t_0b^2}{N}\sum_\eta \cos\left(\frac{q\eta}{2}\right)\cos(K\eta)\,, \\ \Lambda^\lambda(K_+,K_-) &= i\,, \\ \Lambda^{A^\eta}(K_+,K_-) &= 2\Delta\sin(K\eta)\,, \\ \Lambda^{r^\eta}(K_+,K_-) &= -2\Delta\cos(K\eta)\,. \end{aligned} \tag{8}$$

The fields λ and r are the usual slave boson fields which mediate a hard-core repulsion between the quasiparticles. Their physical origin is the single-occupancy constraint. The field A^η becomes a gauge field in the continuum limit. It is decoupled from the other fields and its propagator has mass of order $t_0\delta$. At half filling it describes an overdamped spin-singlet collective mode, analogous to the resonon mode in the short-range RVB.[13] Away from half filling it mixes with charge excitations and acquires a mass.

We now compute physical quantities to leading order in $1/N$. The optical conductivity is obtained by extracting the q_x^2/ω term of the density-density correlation function. The corresponding diagrams are shown in Fig. 2. The result is

$$\sigma(\omega)=2\delta Nq_0t_0(\Delta/J)\delta(\omega)\,. \tag{9}$$

In this calculation, the propagator of the fields A^x and A^y screen the current fluctuations, renormalizing the optical mass from its bare value [Fig. 2(a)] t_0 to $t_0\delta$. In a strongly correlated electron system, the conductivity scales as the number of holes as one would expect. This result is consistent with the optical sum rule for the t-J model which can be derived following the work of Ref.

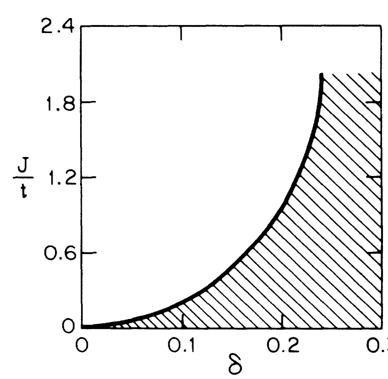

FIG. 1. Domain of stability of the Fermi-liquid phase (shaded area).

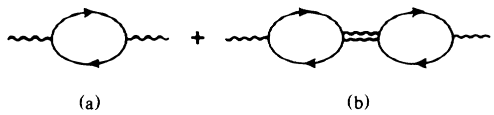

FIG. 2. Density-density correlation function diagrams used to compute the conductivity. The wavy line denotes the gauge-field propagator while the fermion lines are electron quasiparticle propagators.

14 on the Hubbard model.

This behavior should be contrasted with the behavior of the static compressibility which is the $\Omega=0$, $q\rightarrow 0$ limit of the density-density correlation function. In this limit one finds

$$\frac{dn}{d\mu}=\frac{N\rho}{1+4t_0\rho|\epsilon_0|-J\epsilon_0^2\rho}. \tag{10}$$

Here ρ is the renormalized quasiparticle density of states per spin,

$$\rho=\frac{\rho_0}{2(\Delta+\delta q_0 t_0)}$$

and

$$\rho_0=\sum_k \delta(\cos K_x+\cos K_y+\epsilon_0),\quad \epsilon_0\equiv\frac{\mu}{2(\delta q_0 t_0+\Delta)}.$$

Still different renormalizations are found in the density of states which appears in the low-temperature specific heat and the spin susceptibility;

$$C_v=\frac{\pi^2 T\rho_0 N}{6(\Delta+\delta q_0 t_0)},\quad \chi=\frac{\mu_B^2\rho_0 N}{2(\Delta+\delta q_0 t_0)}. \tag{11}$$

T is the temperature and μ_B is the electron magnetic moment. Equations (9), (10), and (11) illustrate the basic ideas of Fermi-liquid theory in the context of an exactly soluble model. Different physical quantities acquire different renormalizations. The magnetic susceptibility and the specific heat are of order $1/J$ close to half filling because there is a finite density of magnetic excitations which contribute to these quantities. The optical conductivity and the compressibility, on the other hand, probe the charge degrees of freedom and are renormalized by different Landau parameters, F_s^1 and F_s^0, respectively.

We now turn to the superconducting instabilities of this model. They are of order $1/N$ and the BCS weak coupling is justified in the framework of the $1/N$ expansion. We write the effective interaction between electrons on the Fermi surface as

$$\Gamma(K,-K|K',-K')=-\Lambda^i(K',K)D_{ij}(K'-K)\Lambda^j(-K',-K)+\Lambda^i(-K',K)D_{ij}(K'+K)\Lambda^j(K',-K) \tag{12}$$

with $D_{ij}(q)=(B+\pi)_{ij}^{-1}(q,\Omega=0)$ and determine whether there is a superconducting instability by calculating the sign and the magnitude of the following coupling constants:

$$c_i=\frac{\int(ds/|v_k|)\int(ds'/|v_k'|)g_i(K)\Gamma(K,-K|K',-K')g_i(K')}{\int(ds/|v_k|)g_i(K)^2}, \tag{13}$$

TABLE I. Dimensionless superconducting coupling constants $c_i/2$, with $\cos K_x-\cos K_y$ symmetry, $\sin K_x\sin K_y$ symmetry, and $\cos K_x+\cos K_y$ symmetry.

J \ δ	0.15	0.3	0.5	0.7
		$\cos K_x-\cos K_y$		
0.001	6.99×10^{-3}	-5.26×10^{-3}	-9.50×10^{-3}	-1.92×10^{-3}
0.005	-1.27×10^{-3}	-6.93×10^{-3}	-9.97×10^{-3}	-2.05×10^{-3}
0.05	-0.114	-2.80×10^{-2}	-1.50×10^{-2}	-2.96×10^{-3}
0.1	-0.294	-5.28×10^{-2}	-2.06×10^{-2}	-3.96×10^{-3}
J \ δ	0.15	0.3	0.5	0.7
		$\sin K_x\sin K_y$		
0.001	-2.72×10^{-2}	-2.46×10^{-3}	3.10×10^{-3}	9.09×10^{-4}
0.05	-1.80×10^{-2}	2.38×10^{-3}	4.56×10^{-3}	1.10×10^{-3}
0.1	-2.06×10^{-2}	6.56×10^{-3}	6.03×10^{-3}	1.29×10^{-3}

J \ δ	0.15	0.3	0.5
		$\cos K_x+\cos K_y$	
0.05	0.647	0.384	0.286
0.1	0.675	0.390	0.289

and $g_i(K)$ are cubic harmonics with different symmetries. We considered $g_2(K)=\cos K_x - \cos K_y$, $g_1(K)=\sin K_x \sin K_y$, and $g_0(K)=\cos K_x + \cos K_y$. The values of these coupling constants, for $q_0=0.5$ and for several filling factors, are tabulated in Table I. For very small values of J the d-wave channel with $\sin K_x \sin K_y$ symmetry is attractive close to half filling. However, this channel becomes repulsive for moderate values of J. This situation is reversed in the d-wave channel with $\cos K_x - \cos K_y$ symmetry which is very attractive close to half filling. The origin of this attraction can be understood as the residual interaction between the quasiparticles generated by the original superexchange interaction between the spins. This is a screened version of Anderson's superexchange mechanism,[15] which now induces pairing between dressed quasiparticles.

It is known that for a dilute system of particles interacting with a hard-core repulsion the dominant superconducting instability is in the p-wave channel.[9,16] We found that this attraction disappears rapidly as J is increased.

In summary, we analyzed in detail the Fermi-liquid phase of a model of strongly correlated fermions. The Fermi-liquid renormalizations are not as simple as in the Brinkman-Rice theory. The magnetic effects, embodied in the mean-field parameter Δ, decrease the effective mass and the magnetic susceptibility relative to the Brinkman-Rice value. A similar renormalization was conjectured by Anderson.[17] On the other hand, all these effects cancel out in the optical conductivity which scales with the number of holes, as predicted by the Brinkman-Rice theory. The Fermi-liquid phase is stable up to very small values of the doping. This is because the Fermi-liquid state contains strong antiferromagnetic correlations and takes significant advantage of the magnetic exchange energy. While nothing is rigorously known about the convergence of the large-N expansion to the physical value of $N=2$, we believe this expansion gives a qualitatively correct picture when the doping is not too small so that the Fermi-liquid phase is a good starting point. It is then interesting to set $N=2$ and insert the relevant parameters for the copper-oxide systems $t_0 \sim 1$ eV and $j \sim 0.1$ eV in our formulas. The Fermi liquid is stable for $\delta \geq 0.1$. The leading superconducting instability, which occurs in the d-wave ($g2$) channel, increases with doping. For large doping the superconducting couplings in Table I are much smaller than 1, so we have a weakly coupled superconductor. Close to half filling the characteristic energies of the Fermi liquid $\delta t_0 q_0 + \Delta$, with $\Delta \sim 0.2J$, are smaller than the electronic energies t_0 and J appearing in the Hamiltonian. The coupling constants are close to unity indicating that the pairing energies are comparable with the Fermi energy. This limit is then closer to a strongly coupled superconductor.

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