Direct versus Sequential Fragmentation of Neutron-Rich Nuclei

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The dissociation of neutron-rich nuclei from secondary beams incident on several targets can be explained within two distinct models: (a) The weakly bound neutrons form clusters near the nuclear surface, and (b) all protons can vibrate against all neutrons in a soft mode. We show that the momentum widths of the projectile fragments, as well as the total cross sections for the dissociation, are consistent with both hypotheses. Consequently, the interpretation of almost all recent experimental studies with secondary radioactive beams is ambiguous.

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Recently, a Japanese group[1] has intensively investigated and measured the interaction cross sections of secondary radioactive beams at the LBL Bevalac. Such experiments have also been performed at GANIL with intermediate-energy beams.[2,3] We shall concentrate here on the most relevant aspects of the data and study especially the fragmentation of ^{11}Li. The fragmentation of other neutron-rich nuclei, such as ^{14}Be, should follow the same scheme. Among the several distinctive features of the experimental results, an intriguing one is related to the momentum distribution of the ^{9}Li fragments originated from the reaction $^{11}\text{Li}+\text{target}\rightarrow {}^{9}\text{Li}+X$. These fragments originate from peripheral reactions and give information about the nuclear matter distribution near the surface of the ^{11}Li isotope. The perpendicular momentum distribution of the ^{9}Li fragments shows a "two-peak" structure,[1] with a narrow peak on top of a wider one. The widths of Gaussian fits to these peaks are given by $\sigma_{\text{wide}}=95\pm 12$ MeV/c for the wider peak, and $\sigma_{\text{narrow}}=23\pm 5$ MeV/c for the narrower one. Such structure has also been found in the reaction $^{14}\text{Be}+\text{target}\rightarrow {}^{12}\text{Be}+X$. In the case of ^{11}Li it is known that the separation energy of the last two neutrons is $S_{2n}=0.19\pm 0.10$ MeV, while the separation energy of only one nucleon is as much as $S_{1n}=0.96\pm 0.1$ MeV.

Hansen and Jonson[4] have argued that it is the strength of the neutron pairing which is responsible for the differences in the separation energies of ^{11}Li and of other neutron-rich nuclei. This pairing makes the bond between the two loosely bound neutrons much stronger than the respective bonds between each of them and the ^{9}Li core. That is, the ^{11}Li is much like a cluster nucleus with a dineutron system bound to the ^{9}Li core. It is the aim of this paper to show that both the widths of the momentum distributions and the total cross sections can be explained by assuming a simple clusterlike structure for ^{11}Li as a dineutron bound to a ^{9}Li core. But we also show that analogous results can be obtained by considering the excitation of a soft vibration of the protons against the neutrons in ^{11}Li. The presently available data do not unambiguously distinguish between the two models.

Because of the small energy necessary to remove the neutron pair, the reaction process is of peripheral nature. The fragmentation is then originated by the nuclear field when the tails of the nucleonic distributions just touch each other, or by the Coulomb field even when the nuclei pass several tens of fm far from each other. The scattering angle θ is therefore very small, and the momentum transfer in the reaction Δp is related to energy transfer by

$$\Delta p = p_f\cos\theta - p_i \simeq \frac{E^*}{v}, \tag{1}$$

where v is the projectile velocity. Since the energy E^* transferred in peripheral processes are typically of order of a few MeV, it cannot be absorbed by a single nucleon. The nucleon would carry a momentum $\sim(2mE^*)^{1/2}$, which is appreciably larger than that of Eq. (1) for $v\sim c$. However, such energy could be absorbed by a nucleon pair, or a pair of clusters, which can have high kinetic energy and small total momentum, when the nucleons move approximately with opposite directions. The relation (1) can also be satisfied if collective excitations, like vibrational modes, are excited.

Let us assume that the energy E^* deposited in the nucleus with mass number A leads to its fragmentation into two pieces which fly apart with opposite momenta with the same magnitude p. If one of the fragments has mass number a, the following relations holds

$$E^* - \epsilon = \frac{p^2}{2(A-a)m_N} + \frac{p^2}{2am_N}, \tag{2}$$

where m_N is the nucleon mass and ϵ is the binding energy between the two clusters. The momentum widths of the fragments is obtained, after an average of (2), as

$$\langle p^2\rangle = 2m_N\langle K\rangle\frac{a(A-a)}{A}, \tag{3}$$

where $\langle K\rangle = \langle E^*\rangle - \langle\epsilon\rangle$ is the average kinetic energy of the fragments.

This formula is very much like the one obtained by Goldhaber[5] for the momentum width of a fragment of mass number a in the fragmentation of a nucleus of mass number A. No wonder, because both approaches rely on momentum and energy conservation. Goldhaber assumes that the momentum width results from an average of the net momentum obtained by adding the individual momenta of the nucleons inside the fragment at the exact moment it flies off the nucleus. This procedure relates $\langle p^2\rangle$ to the Fermi momentum P_F of nucleus A. The final result (which assumes $\langle E^*\rangle \sim 0$) is Eq. (2) with $2m_N\langle K\rangle$ replaced by $P_F^2/5$.

Since the transferred energy depends on the specification of the target, as well as on the beam energy, then by means of a variation of these parameters the measurement of $\langle p^2\rangle$ yields precious information about $\langle\epsilon\rangle$. In the case of $^{11}\mathrm{Li}\rightarrow{}^9\mathrm{Li}+(2n)$, the narrow peak with width $\langle p^2\rangle^{1/2}=23\pm5$ MeV, gives $\langle K\rangle=0.17\pm0.08$ MeV, while for the wide peak with width $\langle p^2\rangle^{1/2}=95\pm12$ MeV/c one obtains $\langle K\rangle=2.9\pm0.8$ MeV. Since the binding energy ϵ of any pair of neutrons in ^{11}Li cannot be larger than some MeV (one could imagine that at least one of the neutrons come from the inner part of ^{11}Li, where it is more tightly bound), the above results show that the energy E^* transferred in the process cannot be larger than some MeV, too. This means that the dissociation is very soft and occurs at very large impact parameters, probing the tail of the nuclear matter distribution in ^{11}Li. The average kinetic energy $\langle K\rangle$ associated with the narrow peak is of the same magnitude as the binding energy of the loosely bound neutrons. Then, it may give information about the correlation distance between the dineutron system and the ^{9}Li core, within the clusterlike hypothesis. On the other hand, the wider peak reveals that a more tightly bound neutron is taken out of ^{11}Li. An analysis of the dissociation cross section as a function of the relative final momentum of the fragments confirm the above hypothesis, as we show next.

Assuming that the ^{11}Li possess a binary cluster structure (dineutron $+{}^6$Li), one can make simple estimates of the cross sections for its dissociation. Using a deuteronlike wave function for the pair of clusters and a strong absorption model, simple expressions were obtained in Ref. 6. The nuclear contribution to the differential cross section, in the limit that $q\rightarrow0$, is obtained as

$$\frac{d\sigma_N}{dq}\simeq R_T\frac{q^2}{(\eta^2+q^2)^2}\,, \tag{4}$$

where q is the relative momentum of the clusters after the dissociation, R_T is the target radius, and $\eta=(2\mu\epsilon)^{1/2}/\hbar$, with μ equal to the reduced mass of the clusters.

The Coulomb contribution to the differential cross section (taking only the $E1$-multipole contribution) in the same limit, is given by

$$\frac{d\sigma_C}{dq}=\frac{128}{3}Z_T^2\alpha\left(\frac{c}{v}\right)^2\left(\frac{Z_1A_2-A_1Z_2}{A}\right)^2\frac{\eta q^4}{(\eta^2+q^2)^4}\times\left[\ln\left(\frac{\gamma v}{\delta w R}\right)-\frac{v^2}{2c^2}\right], \tag{5}$$

where $\gamma=(1-v^2/c^2)^{-1/2}$ is the relativistic Lorentz factor, $\delta=0.891$, and $\hbar\omega=\hbar^2(\eta^2+q^2)/2\mu$. A_i (Z_i) refers to the mass (charge) number of cluster i ($A=A_1+A_2$) and $R=R_T+R_P$.

The above expressions reveal that the spread in q^2 is of order of $\langle q^2\rangle\cong\eta^2$. This means that the relative kinetic energy of the clusters after the dissociation is on the average of the same value as their binding energies. This is indeed what we obtained above for $\langle K\rangle$ associated with the narrow momentum component. Therefore, the narrow momentum component can be interpreted as originating from the removal of two neutrons weakly bound in ^{11}Li. The root-mean-square radius for ^{11}Li, supposed to be a deuteronlike system, is $\langle r^2\rangle^{1/2}=1/\sqrt{2}\eta\sim5.8$ fm. The experimental value[1] for the rms radius of the ^{9}Li core is about 2.5 fm. Therefore, the dineutron system forms a neutron *halo* around the ^{9}Li core.

As has been pointed out by Tanihata[7] the amount of kinetic energy associated with the broad momentum width (~3 MeV) is related to the binding energy of neutrons in the ^{9}Li core. As in the case of $^9\mathrm{Li}+(2n)$ described above, a pair of neutrons in the ^{9}Li core can also absorb the transferred energy in the reaction with their final relative momentum and energy obeying Eq. (1). In this case the decay constant η in Eqs. (4) and (5) can be related to the average binding energy of neutrons in the ^{9}Li core as $\eta=(m_N\epsilon_c)^{1/2}/\hbar$. Taking $\epsilon_c\sim3$ MeV, this yields a rms radius of about 2.65 fm, which agrees very well with the rms radius of ^{9}Li.

Neutrons coming out of the ^{9}Li core can also have their origin in the collective excitation of it. The most effective way of creating such excitations is by means of the Coulomb interaction. It gives the same "kick" to all Z protons inside ^{9}Li, leading to their collective motion. For collisions with impact parameter b, this kick leads to an energy transfer which can easily be calculated as[8] $\Delta E_1=2Z(Z_Te^2)^2/m_Nb^2v^2$, where Z_T is the target charge. But the protons are not free and they pull the neutrons together. This leads to a movement of the whole nucleus, and the Coulomb recoil that one obtains by assuming that the nucleus with mass number A is a rigid body is $\Delta E_2=2(ZZ_Te^2)^2/Am_Nb^2v^2$. The difference between these energies goes to the vibration of the Z protons against the N neutrons, and is

$$E^*=\Delta E_1-\Delta E_2=2\frac{NZ}{A}\frac{(Z_Te^2)^2}{m_Nb^2v^2}\,. \tag{6}$$

If we assume that only the protons and neutrons in the ^{9}Li participate in these vibrations ($N=6$, $Z=3$), and for

^{11}Li beams (0.8 GeV/nucleon) incident on Pb, one finds $E^* = 0.26$ MeV in a collision with $b = 15$ fm. This energy is far below the excitation energy of giant dipole resonances (GDR) in normal nuclei, which means that the excitation cross section of a giant dipole mode in the ^{9}Li core is small.

Indeed, assuming that this dipole resonance excited on the ^{9}Li core can be accounted for in the same way as a normal giant dipole resonance positioned at E_{GR}, and using the Thomas-Reich-Kuhn sum rule, one finds for the total Coulomb cross section

$$\sigma_{GR} = \frac{2}{\pi} Z_T^2 \alpha \left(\frac{c}{v}\right)^2 \frac{S}{E_{GR}\,(\mathrm{MeV})} \times \left[\xi K_0 K_1 - \frac{v^2 \xi^2}{2c^2}(K_1^2 - K_0^2)\right] \mathrm{mb} \tag{7a}$$

with the sum rule S,

$$S = 60 \frac{NZ}{A}, \tag{7b}$$

where all modified Bessel functions K_n are functions of $\xi = E_{GR} R/\gamma\hbar v$, and N, Z, and A refers to the neutron, charge, and mass number of the ^{9}Li core (6, 3, and 9, respectively). Assuming that the resonance lies in the energy range $E_{GR} = 10\text{–}20$ MeV, and for beams with 0.8 GeV/nucleon incident on Pb, one finds $\sigma_{GR} \simeq 50\text{–}400$ mb.

One could think about other vibrations modes in ^{11}Li, like all protons vibrating against all neutrons, or a ^{9}Li core vibrating against the dineutron system (such type of motion has been recently studied by Suzuki, Ikeda, and Sato[9]). For the former case ($N = 8$, $Z = 3$, and $A = 11$) we find $E^* = 0.29$ MeV, while for the latter case one makes the substitution of Z by $Z^2/(A-2)$ in the equation for ΔE_1 and obtains $E^* = 0.02$ MeV. From these values one sees that it is very unprobable that the latter vibration mode could be excited. It is much more reasonable to think that another possible way for the ^{11}Li to absorb energy is by the excitation of vibrations of all protons against all neutrons in it. Because of the existence of the neutron halo, one might think that the protons move almost freely inside the ^{11}Li and that the excitation of such dipole vibrations will occur at very small energies (soft dipole mode).

Recently, Kobayashi *et al.*[10] have measured the total cross section for the dissociation of ^{11}Li [into ^{9}Li$+(2n)$] incident on several targets (Pb, Cu, and C) with 0.8-GeV/nucleon beams. We shall refer to their particular result for Pb targets which has the advantage of having a large Z, and induces a large Coulomb cross section. They obtained the value $\sigma_C = 1.31 \pm 0.13$ b. In the ^{9}Li$+(2n)$ cluster model, the total cross section for direct Coulomb dissociation is obtained by an integration of (5) which results in

$$\sigma_{CD} = \frac{4\pi}{3} Z_T^2 \alpha^2 \left(\frac{c}{v}\right)^2 \left(\frac{Z_1 A_2 - A_1 Z_2}{A}\right)^2 \times \frac{1}{\eta^2}\left[\ln\left(\frac{\gamma\hbar v}{\delta\epsilon R}\right) - \frac{v^2}{2c^2}\right]. \tag{8}$$

For the reaction cited above it gives $\sigma_{CD} \simeq 1.44^{+2.9}_{-0.7}$ b, where the uncertainties are due to the error in the binding energy.

The nuclear contribution to the direct breakup cannot be obtained by an integration of (4) because it was based on the impulse approximation, neglecting the interference with an eclipse term. Including such effect the cross section is well described by the Glauber formula[11]

$$\sigma_{ND} = \frac{\pi}{6}\left(2\ln 2 - \tfrac{1}{2}\right)\frac{R_T}{\eta}. \tag{9}$$

In addition to this (diffractional) dissociation, one has to account for the absorption of the $(2n)$ system by the target (stripping). The cross section for this process was obtained for the deuteron by Serber.[12] For other cluster-like $[a + (A - a)]$ nuclei one has

$$\sigma_{NS} = \frac{\pi}{2}\frac{a}{A}\frac{R_T}{\eta}. \tag{10}$$

For the reaction ^{11}Li$+$target$\rightarrow$ ^{9}Li$+X$ one obtains $\sigma_{ND} = 270^{+100}_{-53}$ mb and $\sigma_{NS} = 165^{+63}_{-32}$ mb, respectively. One then sees that the Coulomb dissociation accounts for the main part of the measured cross section, although the nuclear contribution is not negligible. At this point we observe that the Coulomb-nuclear interference in these reactions may be neglected for the following reason. The nuclear contribution to the total cross section can at most come from those impact parameters (from b_{min} to b_{max}) for which the neutron halo of ^{11}Li touches the nuclear matter distribution of Pb. The contribution of the Coulomb field to the total cross section from this interval of impact parameters is, percentually, given by

$$\Delta = \frac{\ln(b_{max}/b_{min})}{\ln(\gamma\hbar v/\delta\epsilon b_{min})}. \tag{11}$$

Using typical values of $b_{min} \simeq 10$ fm and $b_{max} \simeq 13$ fm, one finds $\Delta \simeq 5\%$. This means that only about 5% of the Coulomb contribution should interfere with the nuclear contribution. The reason is that, although the fragmentation induced by the Coulomb interaction may be small in a single collision, the interval of impact parameters contributing to the total cross section is very large, up to some hundreds of fm. Therefore, we can write $\sigma_{total} \simeq \sigma_N + \sigma_C$. Adding the Coulomb dissociation, the nuclear diffraction dissociation, and the stripping cross sections one can reproduce quite well the experimental value of Kobayashi *et al.*[10] for the total cross sections for two-neutron removal from secondary beams of ^{11}Li incident on Pb.

If we now restrict our study to the Coulomb contribution to the dissociation, which is the dominant part of the cross sections, we find that the excitation of giant resonances as described above can also lead to great values of the cross sections. In fact, if we assume that the energy of excitation E_{GR} of a soft vibration mode in ^{11}Li is of the order of 1 MeV, and that the contribution of this soft mode to the sum rule S is of about 10%, we find (using $N=8$, $Z=3$, and $A=11$) $\sigma_{GR} \simeq 1.3$ b. Because of its low binding energy, one of the main channels for the decay of this resonance must be the emission of the two neutrons. This indicates that the excitation of this soft dipole mode is another possible mechanism to explain the narrow momentum component in the data for $^{11}\mathrm{Li} \rightarrow {}^{9}\mathrm{Li} + X$, as well as the total cross section for the fragmentation.

From the currently available data it does not seem to be possible to know if the fragmentation $^{11}\mathrm{Li} \rightarrow {}^{9}\mathrm{Li} + X$ in secondary-beam reactions proceed via the direct breakup of a two-cluster system or by the excitation of a soft dipole mode. But note that the two mechanisms assume very distinct structures for ^{11}Li. The direct breakup supposes that the protons are tightly bound to the neutrons in the ^{9}Li core, while the excitation of the soft mode assumes that the protons move almost freely against a neutronic background. Since the Coulomb kick to the protons does not depend in either hypothesis, only one of the two mechanisms could be responsible for the measured cross sections. Because of the large errors in the knowledge of the binding energy of two neutrons in ^{11}Li, and also due to lack of information about the energy location as well as of the strength of the photonuclear cross section for ^{11}Li at the energies involved, precise theoretical calculations based on either of these models are not conclusive, and the agreement with the experimental data is not unique. Certainly, more experimental results and theoretical discussions are needed in order to determine which of the nuclear models are adequate.

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[1] I. Tanihata *et al.*, Phys. Lett. **160B**, 380 (1985); Phys. Rev. Lett. **55**, 2676 (1985); Phys. Lett. B **206**, 592 (1988).

[2] W. Mittig *et al.*, Phys. Rev. Lett. **59**, 1889 (1987).

[3] M. G. Saint-Laurent *et al.*, Z. Phys. A **332**, 457 (1989).

[4] P. G. Hansen and B. Jonson, Europhys. Lett. **4**, 409 (1987).

[5] A. S. Goldhaber, Phys. Lett. **53B**, 306 (1974).

[6] C. A. Bertulani and G. Baur, Nucl. Phys. **A480**, 615 (1988); G. Baur, in *Proceedings of the International Symposium on Heavy Ion Physics and Nuclear Astrophysical Problems, Tokyo, Japan, 1988,* edited by S. Kubono, M. Ishihara, and T. Nomura (World Scientific, Singapore, 1989), p. 225.

[7] I. Tanihata, Nucl. Phys. **A478**, 795c (1988).

[8] G. Baur and C. A. Bertulani, Phys. Rev. C **35**, 836 (1987).

[9] Y. Suzuki, K. Ikeda, and H. Sato, Niigata University report, 1989 (to be published).

[10] T. Kobayashi *et al.*, Phys. Lett. B **232**, 51 (1989).

[11] R. Glauber, Phys. Rev. **99**, 1515 (1955).

[12] R. Serber, Phys. Rev. **72**, 1008 (1947).