Radiative Neutrino Decay in a Medium

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(Received 27 October 1989; revised manuscript received 3 January 1990)

We calculate the rate of decay of a heavy neutrino into a lighter neutrino and a photon in a background that contain electrons but no μ's or τ's. Unlike the same decay in the vacuum, the rate of this process is not suppressed by the Glashow-Iliopoulos-Maiani mechanism. Some physical implications of the process are discussed.

PACS numbers: 13.35.+s, 13.40.Hq, 14.60.Gh, 98.80.Cq

The subject of unusual properties of neutrinos in material media has attracted a lot of attention recently. Wolfenstein[1] showed that the masses and mixings of neutrinos may be significantly affected by the presence of a medium. Mikheyev and Smirnov[2] realized that this fact could provide a solution to the solar-neutrino problem. More recently, it has been demonstrated[3] that the electromagnetic properties of neutrinos can even be qualitatively different within a medium compared to the properties in the vacuum. For example, a Majorana neutrino may have electric and magnetic dipole moments in a medium, whereas in the vacuum these quantities must vanish. As discussed in Ref. 3, the reason behind this qualitative difference lies in the transformation properties of the medium under the discrete symmetries *C*, *P*, *T*, and combinations thereof. In particular, the medium can introduce *CP* and *CPT* asymmetries in the effective particle interaction, which would be absent in the vacuum that is supposed to be *CPT* symmetric. As a complement to this analysis, the matter-induced electromagnetic form factors have been calculated in a recent paper by the present authors.[4]

In this Letter, we want to discuss the effects that a background medium inflicts on another interesting process involving neutrinos, viz., the radiative decay of a heavier neutrino to a lighter one:

$$\nu(k) \rightarrow \nu'(k') + \gamma(q) , \tag{1}$$

where in parentheses we have written the four-momenta of the particles. Our motivation is the following. On the one hand, this process has been studied for the possibility of detecting the cosmic neutrino background by observing the photons coming out of the process.[5] Recently, Fukugita[6] has raised the possibility that a suitable lifetime for the process in Eq. (1) might be crucial in explaining the deviation[7] of the cosmic microwave spectrum from a pure blackbody distribution. On the other hand, it is known that in the vacuum the rate for this process is suppressed by the Glashow-Iliopoulos-Maiani (GIM) mechanism. However, a medium that contains electrons but not muons or taons is not flavor symmetric. Therefore, in such a medium, the background-dependent part of this amplitude is not GIM suppressed. As a result, the decay rate could be much larger than that in the vacuum. In light of these questions, we calculate below the lifetime of the process in a thermal medium, which is essential for discussing the evolution of neutrinos in the early Universe. Besides, the present calculation will also provide insight into neutrino processes in stellar objects.

We begin the calculation by assuming that the dispersion relations of the particles are not significantly affected by the medium, so that we can use the relations valid in the vacuum. In order to be self-consistent, we should then take the wave-function renormalization factors associated with all external lines to equal unity. In effect, these assumptions imply that we consider only the transverse polarization states of the photon. In addition, we assume that the mass of ν' vanishes, or at least is so small compared to m, the mass of ν, that it can be neglected.

The matrix element of the process in Eq. (1) can now be written as

$$i\mathcal{M}' = -i\bar{u}(k')\Gamma'_\alpha u(k) e^{\alpha *}(q) , \tag{2}$$

where $u(k)$ denotes the Dirac spinor with momentum k and e^α denotes the polarization vector of the photon. We have used the primes on $\mathcal{M}$ and Γ_α to denote that we will consider the background-dependent part only. Following the notation used in Ref. 4, Γ'_α can be written as

$$\Gamma'_\alpha = U^*_{e\nu} U_{e\nu'} \mathcal{T}_{\alpha\beta} \gamma^\beta L , \tag{3}$$

where U is the lepton mixing matrix and L is the projection operator for left-handed fermions. We assume that the background contains electrons but not muons or

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taons, nor any W bosons for that matter. As discussed in Ref. 4, the matrix $\mathcal{T}_{\alpha\beta}$ satisfies the relations

$$q^\alpha \mathcal{T}_{\alpha\beta}=0=q^\beta \mathcal{T}_{\alpha\beta}\,, \tag{4}$$

where the first equality follows from gauge invariance and the second one, although not valid in general, is nevertheless obeyed by the expression obtained for $\mathcal{T}_{\alpha\beta}$ to the lowest order in $1/M_W^2$. In addition, to this order, $\mathcal{T}_{\alpha\beta}$ is a function of $q=k-k'$ only, and not of k and k' separately. Hence, the most general form of $\mathcal{T}_{\alpha\beta}$ is given by

$$\mathcal{T}_{\alpha\beta}=\mathcal{T}_T R_{\alpha\beta}+T_L Q_{\alpha\beta}+\mathcal{T}_P P_{\alpha\beta}\,, \tag{5}$$

where

$$\begin{aligned} R_{\alpha\beta}&=g_{\alpha\beta}-q_\alpha q_\beta/q^2-Q_{\alpha\beta}\,,\\ Q_{\alpha\beta}&=\tilde{v}_\alpha\tilde{v}_\beta/\tilde{v}^2\,,\\ P_{\alpha\beta}&=(i/Q)\epsilon_{\alpha\beta\gamma\delta}q^\gamma v^\delta\,. \end{aligned} \tag{6}$$

Here v^α is the four-velocity of the center of mass of the background medium, $\tilde{v}^\alpha=(g^{\alpha\beta}-q^\alpha q^\beta/q^2)v_\beta$, and Q is the Lorentz-invariant quantity defined by

$$Q=(\Omega^2-q^2)^{1/2},\quad \Omega=q\cdot v\,. \tag{7}$$

In general, the form factors are functions of Ω and Q. However, in the present case, further simplification is obtained since we are interested in the case $q^2=0$, i.e., $\Omega=Q$.

In Ref. 4, the expressions for the form factors $\mathcal{T}_{T,L,P}$ have been derived. From that, it is easily seen[8] that $\mathcal{T}_{L,P}$ vanish when $q^2=0$. Therefore, the only relevant form factor in the present case is

$$\mathcal{T}_T=-\frac{eg^2}{2M_W^2}\int\frac{d^3p}{(2\pi)^3E}[f_-(p)+f_+(p)]\,, \tag{8}$$

where

$$p^\mu\equiv(E,\mathbf{p})\ \text{with}\ E=(\mathbf{p}^2+m_e^2)^{1/2} \tag{9}$$

and

$$f_\pm(p)=\frac{1}{\exp[\beta(p\cdot v\mp\mu)]+1}\,. \tag{10}$$

Using $g^2/8M_W^2=G_F/\sqrt{2}$, we thus obtain

$$\mathcal{T}_T^{(\mathrm{NR})}\simeq-\sqrt{2}eG_Fn_e/m_e \tag{11}$$

in the case of a nonrelativistic (NR) background of electrons, and

$$\mathcal{T}_T^{(\mathrm{ER})}\simeq-\frac{eG_FT^2}{3\sqrt{2}} \tag{12}$$

in the extreme relativistic (ER) case, when the temperature $T\gg m_e$ so that both electrons and positrons are present. Using the polarization sum

$$\sum_{\lambda=\text{transverse modes}} e_\alpha^*(q\lambda)e_\beta(q\lambda)=-R_{\alpha\beta}\,, \tag{13}$$

we then obtain

$$|\mathcal{M}'|^2=m^2|U_{ev}^*U_{ev'}|^2|\mathcal{T}_T|^2\left(\frac{(k+k')\cdot v}{\Omega}-\frac{m^2}{2\Omega^2}\right). \tag{14}$$

The differential decay rate in any frame is given by[9]

$$d\Gamma'=\frac{1}{2k_0}(2\pi)^4\delta^4(k-k'-q)|\mathcal{M}'|^2\times\frac{d^3k'}{(2\pi)^32k_0'}\frac{d^3q}{(2\pi)^32q_0}\,, \tag{15}$$

where we present the background-induced rate only. We integrate it in the rest frame of the medium where $v^\alpha=(1,\mathbf{0})$. In this frame, the magnitude of the three-velocity of the decaying neutrino is denoted by $\mathcal{V}$. Then we obtain

$$\Gamma'=\frac{m}{16\pi}|U_{ev}^*U_{ev'}|^2|\mathcal{T}_T|^2F(\mathcal{V})\,, \tag{16}$$

where the velocity dependence is given by

$$F(\mathcal{V})=(1-\mathcal{V}^2)^{1/2}\left[\frac{2}{\mathcal{V}}\ln\left(\frac{1+\mathcal{V}}{1-\mathcal{V}}\right)-3\right]. \tag{17}$$

Using Eq. (11), we then get that for a NR electron background,

$$\Gamma'^{(\mathrm{NR})}\simeq\tfrac{1}{2}\,\alpha G_F^2|U_{ev}^*U_{ev'}|^2F(\mathcal{V})\frac{mn_e^2}{m_e^2}\simeq(8\times10^{21}\ \mathrm{s})^{-1}|U_{ev}^*U_{ev'}|^2F(\mathcal{V})\frac{m}{1\ \mathrm{keV}}\left(\frac{n_e}{10^{24}\ \mathrm{cm}^{-3}}\right)^2, \tag{18}$$

whereas for an ER background, use of Eq. (12) gives

$$\Gamma'^{(\mathrm{ER})}\simeq\tfrac{1}{2}\,\alpha G_F^2|U_{ev}^*U_{ev'}|^2F(\mathcal{V})\frac{mT^4}{36}\simeq(5\times10^4\ \mathrm{s})^{-1}|U_{ev}^*U_{ev'}|^2F(\mathcal{V})\frac{m}{1\ \mathrm{MeV}}\left(\frac{T}{1\ \mathrm{MeV}}\right)^4. \tag{19}$$

To appreciate how different these rates are compared to the rate in the vacuum, we recall that in the vacuum[10]

$$\Gamma\simeq\frac{1}{2}\alpha G_F^2\left(\frac{3}{32\pi^2}\right)^2m^5\left|\sum_{l=e,\mu,\tau}\frac{m_l^2}{M_W^2}U_{lv}^*U_{lv'}\right|^2. \tag{20}$$

In the sum, the factor proportional to the charged-lepton mass squared is the GIM suppression factor. Assuming that

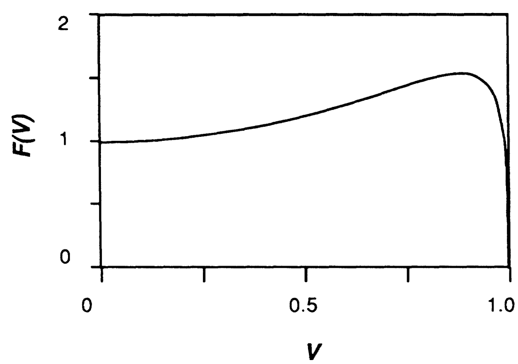

FIG. 1. The velocity dependence of the decay rate. The function $F(\mathcal{V})$ is defined in Eq. (17) of text.

the contribution of the τ dominates in the sum, we obtain

$$\frac{\Gamma'^{(\mathrm{NR})}}{\Gamma} \simeq 1.3\times 10^{19} r F(\mathcal{V}) \left(\frac{n_e}{10^{24}\,\mathrm{cm}^{-3}} \right)^2 \left(\frac{1\ \mathrm{eV}}{m} \right)^4 ,$$
$$\frac{\Gamma'^{(\mathrm{ER})}}{\Gamma} \simeq 1.5\times 10^{9} r F(\mathcal{V}) \left(\frac{T}{m} \right)^4 , \tag{21}$$

where

$$r \equiv |U^*_{e\nu}U_{e\nu'}|^2 / |U^*_{\tau\nu}U_{\tau\nu'}|^2 . \tag{22}$$

From Fig. 1, we see that $F(\mathcal{V})$ is a very flat function of $\mathcal{V}$, taking values between 1 and 1.55, for almost the entire range of $\mathcal{V}$. Thus, if r is not very small, then Eq. (21) shows that Γ'/Γ could be very large for interesting values of the background temperature and density.[9]

This enormous increase in the rate can be quite important when one considers the fate of a neutrino emitted in the stellar core. In fact, since the stellar material contains photons in equilibrium with the electrons, the stimulated emission factor[9] must be taken into account. This will increase the rate of the process further.

The issue is particularly interesting for a heavy neutrino decaying at the cosmological era when the temperature is comparable to its mass. The factor T/m in Eq. (21) is then of order unity, and the matter-induced decay rates are very large. For the radiative decay modes to be an acceptable mechanism for the cosmological depletion of neutrinos, one needs the lifetime to be smaller than about 10^6 s (Ref. 11) so that the photons get enough time to be thermalized. The photonic decay rates calculated in the vacuum are too slow for this purpose. However, from Eq. (19), one sees that unless the mixing is too small, neutrinos with masses larger than 1 MeV can always radiatively decay with the aid of the background electrons. Of course in this case the cosmological background contains thermalized photons and neutrinos. So a complete analysis of the effect should take into account the stimulated emission of photons, the Pauli blocking of neutrinos, and the inverse process as well.[9] However, our calculations indicate that the rate can be large.

The calculation presented in this paper provides an illustration of how a medium can affect the properties of elementary particles, particularly neutrinos. Specifically, the GIM suppression is not operative in a medium that consists of ordinary matter. Although the importance of these effects in concrete situations needs further study, our calculations provide the starting point for such studies and the effect that we have found should be kept in mind.

Both J.C.D. and J.F.N. did most of the present work during their stay at the Universidad de Valencia. They thank the theory group there. P.B.P. did most of his work while he was at the University of Massachusetts. The work was supported by the Dirección General de Investigación Científica y Técnica of the Spanish Ministry of Science and Education, and the National Science Foundation and the Department of Energy of the United States.

[1]L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).

[2]S. P. Mikheyev and A. Y. Smirnov, Nuovo Cimento **9C**, 17 (1986).

[3]J. F. Nieves and P. B. Pal, Phys. Rev. D **40**, 1693 (1989).

[4]J. C. D'Olivo, J. F. Nieves, and P. B. Pal, Phys. Rev. D **40**, 3679 (1989).

[5]A. de Rújula and S. L. Glashow, Phys. Rev. Lett. **45**, 942 (1980); F. W. Stecker, Phys. Rev. Lett. **45**, 1460 (1980); R. Kimble, S. Bowyer, and P. Jakobsen, Phys. Rev. Lett. **46**, 80 (1981).

[6]M. Fukugita, Phys. Rev. Lett. **61**, 1046 (1988).

[7]T. Matsumoto *et al.*, Astrophys. J. **329**, 329 (1988).

[8]In Ref. 4, it was shown that $\mathcal{T}_{T,L,P}$ can be expressed in terms of the Lorentz-invariant quantities A, B, and C, the expression of which are given. Of these, $C=0$ when $q^2=0$. Of the other two, B always appears in the combination $B/\tilde{v}^2 = -q^2B/\mathbf{q}^2$, which vanishes since B is finite as $q^2\to 0$. So, only A is nonzero, and its contribution to $\mathcal{T}_T$ is given in Eq. (8).

[9]If the background contains photons, one should put a factor of $1+(e^{\beta q\cdot v}-1)^{-1}$ on the right-hand side of Eq. (15) to take care of the stimulated emission of photons. Since this factor is always larger than unity, this will further increase the rate. Similarly, if the background contains neutrinos, one should put the Pauli-blocking factor on the right-hand side. In the presence of both neutrinos and photons, the inverse process should also be taken into account. For a preliminary estimate, we neglect these effects.

[10]P. B. Pal and L. Wolfenstein, Phys. Rev. D **25**, 766 (1982), and references therein.

[11]D. A. Dicus, E. W. Kolb, and V. L. Teplitz, Astrophys. J. **221**, 327 (1978).