Single Mechanism for Generating Large-Scale Structure and Providing Dark Missing Matter

William H. Press and Barbara S. Ryden
Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138

David N. Spergel
Princeton University Observatory, Princeton, New Jersey 08544
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We consider a boson field with a phase transition driven by a cosmologically small quartic self-coupling. While its short-wavelength components are stabilized, its very-long-wavelength components start a "slow-rolling" phase transition as soon as they come within their horizon. The Universe becomes filled with a critical density of "soft-boson" particles with a Compton wavelength of tens of kiloparsecs. Large-scale structure with present size tens of megaparsecs is inescapably formed by the wave-packet dynamics of such soft bosons. Baryons are gravitationally coupled to this structure. The Heisenberg uncertainty principle prevents soft bosons from falling into clusters of galaxies.

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We consider a complex scalar field with quadratic and quartic self-couplings. Its Lagrangian has the form

$$\mathcal{L}=-\frac{1}{4\pi}\left[\frac{1}{2}\phi_{,\alpha*}\phi^{,\alpha}+V_0\left(1-\frac{|\phi|^2}{\phi_0^2}\right)^2\right]. \tag{1}$$

We will show that for $\phi_0(\hbar c^3)^{1/2}\sim 10^{17}$ GeV (comparable to the grand unification scale), if V_0 has the tiny value $V_0(\hbar c)^3\sim 10^{-4}$ eV^4, then a number of interesting cosmological effects occur. In particular, a late-time phase transition ($z\sim 10^2$–10^3) can then accomplish all of the following: (1) Provide the "missing" matter needed to give the Universe a critical density $\Omega=1$. (2) Explain how the missing matter can be nonrelativistic (as seems required for galaxy formation to proceed[1]), yet not fall into galaxies or even clusters of galaxies.[2] (3) Explain the origin of the "large-scale structure" that is so strikingly apparent[3] on scales ~ 30 Mpc; and do so in a manner that is potentially compatible with stringent limits on the quadrupole cosmic microwave anisotropy.[4] (4) Suggest why the large-scale structure seems characterized by "holes" rather than by "lumps."[3] This requires that we live in a special epoch, only a few Hubble times after a phase transition, but the observations of large-scale structure may simply demand this "fine tuning."

The parameters V_0 and ϕ_0 define a length scale,

$$\lambda_L\sim\phi_0 c/\sqrt{V_0}, \tag{2}$$

which we will want to be on the order of 30 kpc, for the following reason: The present comoving size L of the scale that had size λ_L when it first entered its horizon (at redshift z_L, say) is given by

$$z_L=\frac{L}{\lambda_L}=\left(\frac{3000\text{ Mpc}}{\lambda_L}\right)^{2/3}\left(\frac{z_m}{z_L}\right)^{1/3}, \tag{3}$$

where a Hubble constant of 100 km/s Mpc is assumed, and where z_m is the redshift of matter dominance. Below we will see that $z_m/z_L\sim 0.1$, and so $\lambda_L\sim 30$ kpc is chosen to correspond to a present scale of large-scale structure $L\sim 30$ Mpc. The idea of invoking a late-time phase transition with a built-in large scale is not new;[5] however, previous investigators have focused on topological defects such as domain walls,[6] leading to contradictions with observation.[7] By contrast, we focus on the dynamical degrees of freedom of the ϕ field.

In an expanding universe, the scalar field's evolution is governed by

$$\frac{\partial^2\phi}{\partial t^2}+\frac{3}{a}\frac{\partial a}{\partial t}\frac{\partial\phi}{\partial t}-\frac{c^2}{a^2}\nabla^2\phi=-\frac{\partial V}{\partial\phi}. \tag{4}$$

Here the spatial coordinates are comoving in a Friedman cosmology, with $a(t)$ the expansion factor. If the ϕ field had coupled to any component of the primordial plasma, then this thermal coupling would confine ϕ to the top of the hill at some early epoch. After this component drops out of equilibrium, $\langle\phi\rangle$ will remain near 0 until z_L, since the ∇^2 term dominates on all wavelengths smaller than λ_L.

When $z<z_L$, the ϕ field begins rolling slowly down the hill. At the beginning of the slow-rolling phase, the energy density in vacuum energy is only a small fraction of the closure density,

$$\Omega_\phi(z_L)=\frac{V_0}{c^2/6\pi G\lambda_L^2}=6\pi\eta^2, \tag{5}$$

where $\phi_0\equiv\eta M_{\rm Pl}$.

The slow-rolling phase, similar to those considered in the context of inflation,[8] lasts many expansion times. It is worth reiterating that only the very-long-wavelength components ϕ_L contribute to the rolling dynamics, the thermal bulk of the ϕ field has no awareness of the phenomenon, except that its mean value is shifted from zero, coherently on a large length scale.

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During the early stages of this phase, $\phi \ll \phi_0$, $V(\phi) \approx V_0(1-2\phi^2/\phi_0^2)$, and Eq. (4) is linear in ϕ. Treating H as slowly varying yields

$$\phi(t)=\begin{cases}\phi(t_L)\exp[(t-t_L)/t_L], & t\ll\lambda_L/c\equiv t_L\,,\\ \phi(t_L)\exp[(t-t_L)/t_c], & t\gg\lambda_L/c\equiv t_L\,,\end{cases} \tag{6}$$

where $t_c=\lambda_L^2H^2/c^2$. The implication is that there is effectively no growth of ϕ_L until the scale λ_L comes within its horizon, and that it subsequently exponentiates with a time scale t_L.

Because of thermal fluctuation, we expect that at the beginning of the slow-rolling phase, $\phi(t_L)\sim kT=(2.7\ \mathrm{K})\tau(1+z)$, where the present photon temperature is 2.7τ K. Thus, the field rolls for ξ e-folding times, where

$$\xi=-\ln[\phi_L/\phi_0]\approx 70-\ln\left[\left(\frac{\tau}{0.1}\right)\left(\frac{\eta}{10^{-2}}\right)^{-1}\left(\frac{z}{10^3}\right)\right]. \tag{7}$$

During this slow-rolling phase, the vacuum-energy contribution grows and at the end of the slow-rolling phase reaches

$$\Omega_\phi\simeq\Omega_\phi(z_L)\xi^2\approx 6\pi\eta^2\xi^2\,. \tag{8}$$

If η is sufficiently small, the Universe remains photon, or baryon, dominated during the rolling. Then, the relationship between expansion factor and time is $a\propto t^{1/2}$ (photon) or $\propto t^{2/3}$ (baryon). Therefore, in the time that it takes ϕ_L to reach its true minimum, the Universe expands by about $\xi^{1/2}\approx 10$ or $\xi^{2/3}\approx 20$. For the parameters previously mentioned, rolling starts at about $z\approx 10^3$ and is completed by $z\approx 10^2$. It is necessary that the Universe *not* become dominated by the potential V_0 during this time; otherwise it will go over to exponential inflation. For both the photon- and baryon-dominated cases, the required limit on η is $\eta\lesssim\xi^{-1}\approx 10^{-2}$. In fact, $\eta\approx 10^{-2}$, corresponding to the already mentioned scale of 10^{17} GeV, makes the ϕ_L field come to dominate the cosmological density just at the *end* of its rolling phase. This is our preferred value for η, although somewhat smaller values might also be tolerable.

After the rolling phase, the nature of the ϕ_L field is that it represents "particles" with Compton wavelength $\sim\lambda_L$ (corresponding to an extraordinarily small mass) and mean energy density $\sim V_0$. We call these particles "soft bosons" since their large scale is reminiscent of so-called "soft domain walls."[6] The soft bosons were semirelativistic when they began the slow-rolling phase, but c^2k^2/a^2 has decreased with increasing a during that phase, and so they are nonrelativistic, with velocities $\sim c/\xi^{1/2}$ or $c/\xi^{2/3}$ at the time they form. For the parameters mentioned, the soft bosons dominate the mass of the Universe (i.e., provide the missing mass to make $\Omega=1$) from the time they form.

In the photon-dominated case, during the factor of 10 in z of slow rolling, the comoving horizon scale increased by a factor of 10. Thus, after one additional Hubble time, the velocities of $0.1c$, which are coherent on the comoving scale L, make order unity density fluctuations in the ϕ matter on this scale. In the baryon-dominated case, the velocity decreases somewhat faster than the horizon increases, and the magnitude of the density fluctuations is $\xi^{-1/3}\sim 0.2$. In either case, the origin of the large-scale structure is an immediate consequence of the fact that a propagating field with random phases cannot avoid energy fluctuations on its wavelength scale, even if its initial conditions are exactly uniform in energy density. Since the large-scale structure forms well within the horizon, and at relatively small redshifts ($z\lesssim 100$), it can be made consistent with experimental limits on the quadrupole microwave anisotropy.

The reason that soft bosons cannot fall into galaxies or clusters of galaxies is amusing: Their Compton wavelength, $\lambda_L\sim 30$ kpc, is too large. The minimum size of a wave packet with velocity v is the de Broglie wavelength $\lambda_L c/v$. Since the depth of a galaxy or cluster potential is at most 1000 km/s $\approx c/300$, the minimum confinement scale of a ϕ wave packet is ~ 10 Mpc, larger than the core radius of any clusters. One can thus regard the soft bosons as being excluded from galaxies by the Heisenberg uncertainty principle. However, it is somewhat misleading to think of the effect as quantum mechanical: The ϕ field is essentially classical, and Eq. (4) does not contain $\hbar$ if its potential is expressed in terms of Compton wavelength rather than mass. In that case, it is the "classical" uncertainty principle of Fourier analysis (relating a wave packet's size and velocity) which operates.

We have not made any use of the U(1) symmetry of the vacuum manifold implicit in making ϕ a complex field. In fact, the phase transition in ϕ produces very thick, light cosmic strings. These strings contain a very small fraction of the energy density of the Universe and do not affect its evolution. We could just as well have assumed other symmetries for ϕ, in which case other (irrelevant) structures would be produced: monopoles,[9] strings,[9] domain walls,[6] or textures.[10] Except for domain walls, which are dangerous, these defects are not cosmologically significant or observable in our model. The fact that we are making use only of the "radial" dynamics of ϕ suggests that ϕ could also have a gauged, rather than global, symmetry; we have not yet calculated this, however.

While we do not present an explicit particle-physics model, the physical scales in the problem, the grand unification scale and the Mikheyev-Smirnov-Wolfenstein neutrino mass scale,[11] suggest that the symmetry breaking may be associated with lepton-number violation and the origin of the neutrino mass,[5] perhaps in a modified version of the Majoron model.[12] Since the Lagrangian in Eq. (1) is renormalizable, the large range in scales represents not a fine-tuning problem, but rather another piece of the hierarchy problem.

Turn now to the state of baryonic matter in the

Universe. Nucleosynthesis data convincingly argue that the present baryon density Ω_B (relative to the critical density) lies between 0.01 and 0.03; the lower limit is based on the D and ^{3}He abundances, while the upper limit follows from ^{7}Li/H and ^{7}Li/D ratios.[13] (Here, as before, we assume a Hubble constant of 100 km/s Mpc.) At the lower end of the allowed range, the Universe remains radiation dominated during rolldown in our model. At the upper end of the range the Universe is baryon dominated during most of the rolldown. While this is tolerable, the model has less flexibility (smaller large-scale perturbation amplitudes and more fine tuning of η). Our model thus weakly favors small Ω_B. However, one is also able to have a radiation-dominated rolldown if L (size of structure that we are explaining) is decreased by a factor of 3, to 10 Mpc, so that z_L increases to $\sim 10^4$ [Eq. (3), eliminating λ_L].

A broad-brush version of galaxy formation in the present model is the following: Baryonic matter is gravitationally coupled to the order-unity fluctuations that develop in the ϕ field. In about a Hubble time, the baryons flow out of underdense regions and into overdense regions. While the soft bosons are collisionless, the baryonic matter is not: It shocks and forms thin sheets. Galaxy formation then proceeds according to the Zel'dovich pancake picture.[14] Since the soft bosons do not cluster significantly in the caustics, the growth of very dense structures is somewhat suppressed, thus avoiding one of the weaknesses of the hot dark-matter model.

Preliminary numerical results suggest that the model may make somewhat more detailed predictions, in particular, explaining the "bubbly" nature of the large-scale structure and also making contact with the similarity-solution blast-wave models of galaxy formation.[15] It shows the development of "smooth" structure on the comoving scale L. In addition, small "lagging" regions, where counteracting gradient forces temporarily delay the rolldown, evolve into coherent *outgoing-wave* disturbances of the ϕ field. These leave behind lacunae (voids) in the energy density, which do not become filled until after several Hubble expansion times (if at all). The one-dimensional calculations performed to date probably exaggerate the prevalence of the lagging regions. Work in progress will extend the analysis to more realistic cases. Calculations thus far do not include self-gravity. Were it included, the lacunae could be the seeds of gravitationally powered blast waves.[15] In its previous incarnations, the blast-wave theory has had the problem of moving a large density of dark matter by gravitational coupling to a small baryonic seed void. In the present context, the seed void is *in* the massive dark component, and the baryonic matter is strongly swept out of the growing hole gravitationally.

If the present large-scale structure is the result of several generations of hole mergings,[16] then the size L can be reduced—a desirable feature, as already explained.

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