Soft-Pion Theorems and a Light-Cone Quark Model

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It is shown that in a light-cone representation of nucleons soft-pion theorems which are violated in instant-form quark models can be recovered with suitable choices of parameters of the model. Instantaneous terms must be included for the soft-pion limit. The proton form factors are fit up to $q^2=3$ $(\text{GeV}/c)^2$ in this model.

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One of the important goals of medium-energy physics is to understand how quark-gluon QCD is realized in hadron and nuclear physics. Studies of nuclear form factors are promising for this task. Explicit pion-exchange currents as well as quark degrees of freedom must be introduced for consistency with electromagnetic form factors of few-nucleon systems at momentum transfers up to 1–2 GeV/c, since it is not yet possible to deal quantitatively with long-range two-body currents in terms of quarks and gluons only.

Different choices of hadronic models in treating meson-exchange currents at medium- and high-momentum transfer (larger than about 1 GeV/c) could change the theoretical predictions by orders of magnitude. One method for removing this uncertainty is the use of hybrid models, with mesons treated as local fields and baryons described in chiral bag models. In such models form factors are obtained as part of the description of the baryons without additional form-factor parameters. Several years ago it was pointed out[1] that quark models of pion-exchange currents have a very different isospin structure than hadronic models. As a result, hybrid-quark-model predictions for the ^{3}H vs ^{3}He structure functions are very different than hadronic models with pion-exchange currents. However, these hybrid quark models violate certain soft-pion theorems,[2] and it is the discrepancy with model-independent soft-pion theorems which produces the main difference in the three-body structure functions at medium-momentum transfers. This problem of violation of soft-pion theorems seems to be a difficult one for quark models,[2] including the so-called chiral quark models. The objective of the present work is to constrain quark models of nucleons by satisfying soft-pion relations within a few percent, as demanded by soft-pion theorems.

In addition to chiral symmetry, soft-pion theorems require Lorentz covariance. In the present paper we show that one can recover soft-pion theorems in a Lorentz-covariant light-cone hybrid quark model to within the required accuracy with a particular set of parameters. A major problem in standard (instant-form) quark models is the separation of center-of-mass from internal motion and boost properties. Scherer, Drechsel, and Tiator suggested that center-of-mass corrections might be the source of discrepancy.[2] Calculations of corrections to hadron properties have been carried out by several groups.[3] We believe it is the ability of light-cone representations to separate center-of-mass momentum and to boost states of composite systems properly which has enabled us to develop a nucleon model which is in agreement with experiments on the spin-independent nucleon form factors up to 3 GeV^2 and also agrees with soft-pion theorems.[4]

A most important point to note here is that the soft-pion theorems are model independent in the sense that if a theory satisfies Lorentz covariance and partial conservation of axial-vector current (PCAC), soft-pion theorems must be satisfied regardless of the details of the interior structure of the states. Although our Lagrangian and wave functions satisfy Lorentz covariance, the wave functions are not derived from the chirally invariant Lagrangian. Therefore the theory as a whole does not necessarily satisfy PCAC. It is for this reason that we use the model-independent soft-pion theorems to develop the model which is to be used in nuclear calculations. Soft-pion theorems also have often been used to constrain hadronic models, such as the choice of pseudovector or pseudoscalar coupling.[5] These theorems give relationships between certain amplitudes as an expression in the ratio of the pion to nucleon masses.

The basic ingredients of pion-exchange currents are pion photoproduction amplitudes. In Chew-Goldberger-Low-Nambu notation[6] the four independent $N(\gamma,\pi)N$ amplitudes have an isospin structure given by three amplitudes, $A^{(+)}$, $A^{(-)}$, and $A^{(0)}$. The Kroll-Ruderman[7] low-energy expansion in the parameter $\mu=m_\pi/M$ gives the result that $A^{(-)}$ is of order unity, while $A^{(+)}$ and $A^{(0)}$ are first order in μ. From PCAC one obtains additional information on the $N(\gamma,\pi)N$ amplitudes:

$$A^{(-)}=1+O(\mu^2),\quad A^{(+)}=-\mu/2+O(\mu^2),\quad A^{(0)}=-\mu/2+O(\mu^2)\,. \tag{1}$$

Since the π^0 photoproduction amplitudes are given by $A(\gamma p\to\pi^0 p)=A^{(+)}+A^{(0)}$ and $A(\gamma n\to\pi^0 n)=A^{(+)}-A^{(0)}$, while the amplitudes for photoproduction of charged pions involve $A^{(-)}$ $[A(\gamma p\to\pi^+ n)=\sqrt{2}(A^{(0)}+A^{(-)})$ and $A(\gamma n\to\pi^- p)=\sqrt{2}(A^{(0)}-A^{(-)})]$, the experimental measurements of neutral-pion production test the PCAC constraint of Eq. (1) for $A^{(+)}$ and $A^{(0)}$, while charged-pion production constrains $A^{(-)}$. The theory is consistent[8] with experiment to $O(\mu^2)$. Thus it has been shown in the soft-pion limit that

$$A^{(-)}=1,\quad A^{(+)}=A^{(0)}. \tag{2}$$

In the low-energy limit it is the spin-isospin operator $\sigma_3\tau_3$ for the fermions which is used in the representation of the baryon states. For a standard SU(6) proton wave function one obtains

$$\langle N(3q)|\sum_i \sigma_3^i\tau_3^i|N(3q)\rangle=\tfrac{5}{3}\langle N|\sigma_3\tau_3|N\rangle, \tag{3}$$

and

$$\langle N(3q)|\sum_i \sigma_3^i|N(3q)\rangle=\langle N|\sigma_3|N\rangle. \tag{4}$$

In turn, these give the low-energy limit in standard quark models

$$A^{(+)}=\tfrac{9}{5}A^{(0)} \tag{5}$$

rather than the model-independent PCAC result of Eq. (2), which requires equality of $\langle N(3q)|\sum_i\sigma_3^i\tau_3^i|N(3q)\rangle$ and $\langle N(3q)|\sum_i\sigma_3^i|N(3q)\rangle$ at the quark level. This serious violation of soft-pion theorems for pion photoproduction amplitudes is also found[2] in the chiral quark models.[9] Therefore, these quark models do not provide a reliable basis for developing meson-exchange currents.

As pointed out by Dethier *et al.*,[3] in nonrelativistic models the mean radius scales as $[(A-1)/A]^{1/2}$, and since the magnetic moment scales in the same way in many quark models, one may expect that nonrelativistic center-of-mass corrections to Eq. (3) would give approximately

$$\langle N(3q)|\sum_i\sigma_3^i\tau_3^i|N(3q)\rangle^{\text{c.m.}}=\sqrt{\tfrac{2}{3}}\left(\tfrac{5}{3}\right)\langle N|\sigma_3\tau_3|N\rangle. \tag{6}$$

This gives almost half of the needed correction and suggests that the violation of Lorentz covariance in quark models might be the source of the problem.

We use the form introduced by Chernyak and Zhitnitsky[10] for the nucleon wave function, which is analogous to the three invariant currents of Ioffe.[11] Our wave function for a proton with positive helicity in its rest frame is

$$|p\uparrow\rangle=[a_0\phi_0^\uparrow(1,2,3)+a_1\phi_1^\uparrow(1,2,3)+a_2\phi_2^\uparrow(1,2,3)]\times[q_{\text{LC}}^\dagger(1)q_{\text{LC}}^\dagger(2)q_{\text{LC}}^\dagger(3)]|0\rangle, \tag{7}$$

where the ϕ_i are the three independent light-cone spinors given in Ref. 10. In the present work we assume $a_2=0$ since this is adequate for the purpose of testing soft-pion theorems. It is convenient to work in the following Pauli spinor representation:

$$|p\uparrow\rangle=\frac{N}{(x_1x_2x_3)^{1/2}}\sum_i\Phi(x_i,\mathbf{k}_{\perp i})\sum_{I=0}^{1}\Big(A_I(1,2,3)\{[q_\uparrow^\dagger(1)q_\uparrow^\dagger(2)]^I q_\downarrow^\dagger(3)\}^{1/2,1/2}+B_I(1,2,3)\{[q_\downarrow^\dagger(1)q_\uparrow^\dagger(2)]^I q_\uparrow^\dagger(3)\}^{1/2,1/2}+C_I(1,2,3)\{[q_\uparrow^\dagger(1)q_\downarrow^\dagger(2)]^I q_\uparrow^\dagger(3)\}^{1/2,1/2}\Big)|0\rangle, \tag{8}$$

where I is the isospin of the first pair of quarks. The coefficients are

$$\begin{aligned}A_0(1,2,3)&=ma_0(k_{2R}-k_{1R})k_{3L},\\ B_0(1,2,3)&=a_0(k_{3+}+m)(k_{1+}k_{2+}+k_{1L}k_{2R}+m^2)\\ &=-C_0^\dagger(1,2,3),\end{aligned} \tag{9}$$

$$\begin{aligned}A_1(1,2,3)&=-ma_1(k_{2R}+k_{1R})k_{3L},\\ B_1(1,2,3)&=a_1(k_{3+}-m)(k_{1+}k_{2+}+k_{1L}k_{2R}-m^2)\\ &=C_1^\dagger(1,2,3).\end{aligned}$$

In Eq. (9) the light-cone quark momenta are $k_+=k_0+k_3$ and $k_{R,L}=k_1\pm ik_2$, and m is the quark mass. The spatial part of the wave function is taken from the Brodsky-Huang-Lepage prescription,[12]

$$\Phi(x_i,\mathbf{k}_{\perp i})=\exp\left[-\frac{\mathbf{k}_{\perp i}^2+m^2}{6\alpha^2x_i}\right], \tag{10}$$

with α a parameter which will be determined later in this Letter by fitting the proton form factors up to $q^2=3$ $(\text{GeV}/c)^2$. This wave function was used recently in a study of proton form factors by Dziembowski.[13] For the photopion production process we use a light-cone pseudoscalar interaction

$$L_{\pi q}=g_{\pi q}\bar{q}_{\text{LC}}\Gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi}_\pi q_{\text{LC}}, \tag{11}$$

with the pion-quark operators given by

$$\begin{aligned}\Gamma_5&=\sum_i U^\dagger(k_i)\gamma_5^iU(k_i),\\ \tau_3\Gamma_5&=\sum_i\tau_3^iU^\dagger(k_i)\gamma_5^iU(k_i).\end{aligned} \tag{12}$$

Here the pion-quark coupling $g_{\pi q}$ is determined by fitting the pion-nucleon coupling constant in our model, and the Melosh rotation $U(k)$ relates light-cone spinors to Dirac spinors.[12]

The calculation of the amplitudes $A^{(-)}$, $A^{(+)}$, and $A^{(0)}$ of Eq. (1) involve the calculation of the matrix element $\langle\pi N'|j_\mu|\gamma N\rangle$, which is of the form

$$M_\mu=\langle N'|j_\pi Dj_\mu|N\rangle+\text{c.c.}, \tag{13}$$

where j_π and j_μ are, respectively, the pionic and electromagnetic currents, D is the propagator, and $|N\rangle$ is the nucleon state, such as that given by Eq. (7) for a positive-helicity proton.

There are several interesting observations which result from our study of $A^{(-)}$ at the Kroll-Ruderman limit in Eq. (2): (1) The entire Kroll-Ruderman term comes from the instantaneous term in the light-cone perturbation theory;[12,14] (2) the Kroll-Ruderman term vanishes for the j^+ current in our s-state quark model; and (3) one must use the $j_\perp$ current in the calculation. It should not be surprising that one must use the transverse current for real photons; however, in most light-cone calculations the j_+ current has been used. The instantaneous matrix element needed for the calculation of $A^{(-)}$ is of the form

$$M_\perp \propto \langle n\downarrow | \gamma_5 \gamma^+ \gamma_\perp | p\uparrow\rangle + \text{c.c.} \qquad (14)$$

To obtain the soft-pion relationship of Eq. (2) the proton matrices $\langle p|\Gamma_5|p\rangle$ and $\langle p|\tau_3\Gamma_5|p\rangle$ should be equal to each other up to the order of μ^2. After carrying out the integrals over $\mathbf{k}_{\perp i}$, we obtain

$$\langle p|\Gamma_5|p\rangle = 4N^2(3\pi\alpha)^3\langle u\uparrow|\Gamma_5|u\uparrow\rangle \int \prod_i dx_i \exp\left[-\frac{m^2}{3\alpha^2}\sum_i \frac{1}{x_i}\right](L_1+0.5L_3)\,, \qquad (15)$$

and

$$\langle p|\tau_3\Gamma_5|p\rangle = 4N^2(3\pi\alpha)^3\langle u\uparrow|\Gamma_5|u\uparrow\rangle \int \prod_i dx_i \exp\left[-\frac{m^2}{3\alpha^2}\sum_i \frac{1}{x_i}\right](-L_1+1.5L_3)\,. \qquad (16)$$

Here L_1 and L_3 are functions of x,

$$L_1 = 9\alpha^4 x_1 x_2 [a_0(x_3 M - m) + a_1 m]^2 + 9\alpha a_1^2 (x_1 M + 2m)^2 x_2 x_3 + [a_0(x_3 M - m)(x_1 x_2 M^2 - m^2) - a_1(x_1 M + m)(x_2 x_3 M^2 + m^2)]^2\,, \qquad (17)$$

$$L_3 = 18\alpha^4 x_1 x_3 [a_0 m - a_1(x_2 M - m)]^2 + 4a_1^2(x_1 M + m)^2 (x_2 x_3 M^2 + m^2)^2\,, \qquad (18)$$

with m the quark mass.

To determine the parameter α in the quark wave function as given by Eq. (10), we calculate proton elastic form factors $F_1(q^2)$ and $F_2(q^2)$. They may be extracted from

$$\langle N(p')|\gamma_0 j^+(0)|N(p)\rangle = \bar{u}_{\text{LF}}(p')\left[F_1(q^2)\gamma^+ + \frac{\kappa}{2M}F_2(q^2)\sigma_\mu^+ q^\mu\right]u_{\text{LF}}(p)\,, \qquad (19)$$

where $\gamma^+ = \gamma_0 + \gamma_3$, $\sigma^+ = \sigma_0 + \sigma_3$, and $\kappa = 1.79$ nuclear magnetons. In Drell-Yen coordinates we can write

$$\langle n(p')|\gamma_0 j^+(0)|N(p)\rangle = p^+ \sum_n \int \prod_i dx_i\, d\mathbf{k}_{\perp i}\, \Psi_\lambda^{P\dagger}(x'_m, \mathbf{k}'_{\perp m}, \lambda'_m)\Psi_\lambda^P(x_m, \mathbf{k}_{\perp m}, \lambda_m)\frac{u_n^\dagger(k'_n)}{(k_n'^+)^{1/2}} e_n \gamma_0 \gamma^+ \frac{u_n(k_n)}{(k_n^+)^{1/2}}\,, \qquad (20)$$

with $p^\mu = (p^+, M^2/p^2, \mathbf{0}_\perp)$ and $q^\mu = (0, 2pq/p^+, \mathbf{q}_\perp)$. The quark momenta are $\mathbf{k}'_{\perp n} = \mathbf{k}_{\perp n} + (1-x)\mathbf{q}_\perp$ and $\mathbf{k}'_{\perp m} = \mathbf{k}_{\perp m} - x_m \mathbf{q}_\perp$, with n and m denoting the struck and spectator quarks, respectively.

As shown in Fig. 1, for the best parameters, $\alpha = 320$ MeV, $m = 40$ MeV, and $a_0 = a_1$, our light-cone quark model provides a reasonable description of the data up to $q^2 = 3$ (GeV/c)^2. For $q^2 > 3$ (GeV/c)^2 we believe that hard gluon exchange must be included. The results of $A^{(-)}$, $A^{(+)}$, and $A^{(0)}$ are shown in Table I. For $\alpha = 320$ MeV and $m = 40$ MeV, we obtain $A^{(-)} = 0.982$, $A^{(+)} = -0.490\mu$, and $A^{(0)} = -0.492\mu$. Thus we reproduce soft-pion predictions in our light-cone quark model up to the expected accuracy. We also note that the results are not very sensitive to the quark mass.

In Table I we give our results of E_{0+} for the $\gamma p \to \pi^0 p$ process, which is expressed by

$$E_{0+} = \frac{ef}{4\pi m_\pi}[1 - \mu + O(\mu^2)](A^{(+)} + A^{(0)})\,, \qquad (21)$$

with $e^2/4\pi = 1/137$ and $f^2/4\pi = 0.079$. A recent experi-

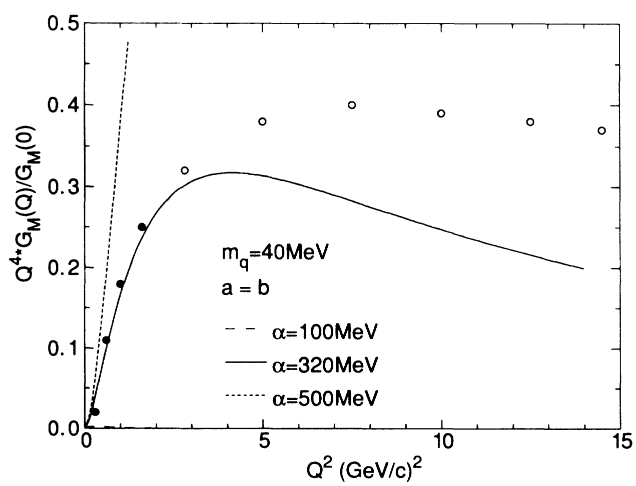

FIG. 1. Proton magnetic dipole form factor up to $q^2 = 15$ (GeV/c)^2. The solid dots are taken from Ref. 15 and the open ones come from Ref. 16.

TABLE I. Test of soft-pion predictions for various quark masses.

m_q (MeV)	$A^{(-)}$	$A^{(+)}$	$A^{(0)}$	$E_{0+}(\gamma p \to \pi^0 p)$
40	0.982	-0.490μ	-0.492μ	$-2.967\times10^{-3}m_\pi^{-1}$
100	1.270	-0.467μ	-0.423μ	$-2.689\times10^{-3}m_\pi^{-1}$
320	1.134	-0.405μ	-0.462μ	$-2.619\times10^{-3}m_\pi^{-1}$

ment[17] on threshold π^0 photoproduction with proton targets gives the result $E_{0+} = -0.35\times10^{-3}m_\pi^{-1}$, which is in disagreement with soft-pion predictions and our results given in Table I. This indicates that there might be other sources of chiral-symmetry breaking.

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