Glueball Contribution to $K_L \to 2\gamma$ and Its Possible Detection in D_s Decay

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Considering the $\iota(1460)$ to be the physical glueball G and to mix with η', we then find the mixing angle to be restricted to $0.15 > \sin\theta_2 > -0.43$ from the data of $\pi^0,\eta,\eta' \to 2\gamma$ and the upper bound on $\iota(1460) \to 2\gamma$. The transition matrix element for $K_L \to G^0$ (bare glueball) is also computed (and found to be comparable to that for $K_L \to \pi^0$). These permit the glueball contribution to $K_L \to 2\gamma$ and $\Delta m_{K_L\text{-}K_S}$ to be significant. Finally, for large mixing angles, the decay of D_s^+ into a glueball is sizable, e.g., $B(D_s^+ \to \iota(1460)\pi^+)$ can be as large as 1%, and searches for glueballs in these decays are highly desirable.

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It is well known that the short-distance contribution to $K_L \to 2\gamma$ is small in the standard model.[1,2] To explain the experimentally observed branching ratio of $K_L \to 2\gamma$, the long-distance contribution must be taken into account. The meson-pole model provides a good framework[1-5] for calculating long-distance effects. It has been pointed out, however, that the contributions from π, η, and η' by themselves give, in the nonet-symmetry limit, a branching ratio which is a few times larger than the observed value.[3] Furthermore, the $K_L \to 2\gamma$ amplitude is very sensitive to the nonet-symmetry-breaking parameter ρ, to be defined later on.[3-5] Letting ρ be a free parameter and fitting the experimental data, one obtains $\rho \approx 0.3$ or 0.8,[5] to be compared with $\rho = 1$ in the exact-symmetry limit, which shows that nonet symmetry is broken. We emphasize that in the discussions of Refs. 1-5 only contributions from π, η, and η' poles were considered. To obtain a better understanding of the situation, it is necessary to consider other possible pole contributions. One such contribution is a pseudoscalar glueball.[6] As a matter of fact, if the glueball exists, it can mix with η_1 and η_8, as we will show later on. The existence of glueballs is a clear prediction of QCD, but experimental detection of such particles is still very unclear. There are indications that the $\iota(1460)$ is a possible candidate to be a glueball; its 2γ decay rate was determined to be less than ~ 2 keV.[7] If $\iota(1460)$ is indeed a glueball and its 2γ decay rate is close to the upper bound just mentioned, then it mixes strongly with the other pseudoscalars and its effect on $K_L \to 2\gamma$ can be significant. In this Letter, we consider the effects of the glueball on the $K_L \to 2\gamma$ amplitude, its pole contribution to the K_L-K_S mass difference, and the detection of glueballs in D_s^+ decays.

The plan of the paper is as follows. First, we determine the amplitude $A(K_L \to G^0)$ by calculating the decay $K_L \to gg$ and then, using results from QCD sum rules to convert $\tilde{G}^a_{\mu\nu}G^a_{\mu\nu}$ into the decays of pseudoscalars to 2γ and the upper bound on the decay $\iota(1460) \to 2\gamma$, we can restrict the mixing angle. At this stage we have all the ingredients needed to proceed and compute glueball effects in the decays $K_L \to 2\gamma$, $D_s^+ \to G\pi^+$ and the mass difference $\Delta m_{K_L\text{-}K_S}$.

The lowest-order contribution to $K_L \to 2g$ (gluons) is shown in Fig. 1. The evaluation of these diagrams is straightforward and can be carried through in a way similar to the decay $K_L \to 2\gamma$.[1,2] One only needs to change the coupling of the quark-photon vertex $e\bar{q}\gamma_\mu q A^\mu$ to the quark-gluon vertex $g_s\bar{q}\gamma_\mu\lambda^a/2qG^a_\mu$, with $\lambda^a/2$ the $SU(3)_c$ generator. Taking the matrix element between the K^0 and the vacuum, we obtain

$$A(K_L \to 2g) = \frac{G_F\alpha_s}{2\pi} f_k \tilde{G}^a_{\mu\nu}G^a_{\mu\nu} G\,\mathrm{Re}(V_{id}V^*_{is})[A^l_i + A^R_i]\,, \tag{1}$$

where V_{ij} is the Kobayashi-Maskawa matrix element, $\tilde{G}^a_{\mu\nu}G^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^a_{\alpha\beta}G^a_{\mu\nu}$, and

$$C = \frac{\langle 0|\bar{s}(\lambda^a/2)\gamma_\mu(\lambda^a/2)d|K^0\rangle}{\langle 0|\bar{s}\gamma_\mu d|0\rangle} = \frac{1}{6}\,. \tag{2}$$

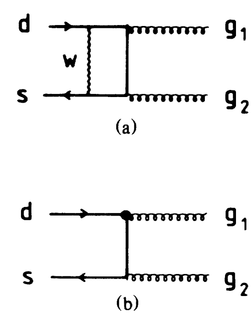

FIG. 1. The lowest-order contribution of K_L-glueball transition amplitude. (a) The irreducible contribution and (b) the reducible contribution from the penguin diagram.

A_i^I is the integral function from Fig. 1(a) and is given by

$$A_i^I = \frac{4x_i}{x_K}\int_0^1 \frac{dy}{y}\ln\left[1 - y(1-y)\frac{x_K}{x_i}\right], \quad (3)$$

$$x_i = \left(\frac{m_i}{m_W}\right)^2, \quad x_K = \left(\frac{m_K}{m_W}\right)^2.$$

A_i^R is from Fig. 1(b),

$$A_i^R = \xi\left[\frac{1-5x_i-2x_i^2}{(1-x_i)^3} - \frac{6x_i^2\ln x_i}{(1-x_i)^4}\right]. \quad (4)$$

ξ is the charge radius of the kaon and is estimated to be $\xi = 0.8\cos\theta_c$.[2] To convert $\tilde{G}^a_{\mu\nu}G^a_{\mu\nu}$ into a glueball, we parametrize

$$\langle 0 | \alpha_s \tilde{G}^a_{\mu\nu}G^a_{\mu\nu} | G^0\rangle = f_G m_G^2. \quad (5)$$

Using the QCD sum-rule results from Ref. 8, f_G is equal to ≈ 0.242 GeV for $m_G \approx 1.4$ GeV.

Using the value $\langle \pi^0 | H_W | K_L\rangle = 1.4\times 10^{-7} m_K^2$ (Ref. 4) determined from the experimental value of $K_S \to 2\pi$ through the use of PCAC (partial conservation of axial-vector current), we obtain

$$\beta = \frac{\langle K_L | H_W | G^0\rangle}{\langle K_L | H_W | \pi^0\rangle} \approx 0.34. \quad (6)$$

This result shows that the glueball-K_L transition is significant in comparison with the π^0-K_L transition.

In general, π^0, η, η', and G^0 will mix. However, it has been shown that the mixing of π^0 with others is very small and can be neglected.[9] There have also been estimates[10] on G^0, η_8, and η_1 mixings. Because of uncertainties in the estimates, we will treat the mixing parameters as free and constrain them by experiments. To begin with, it has been argued[6] that since $m_\eta \approx m_{\eta_8} = 4m_K^2/3 + m_\pi^2/3 \approx 580$ MeV, η is probably strongly dominated by η_8. η' contains most of the remaining small η_8 component and other terms with η_1 and G^0. The heavier physical glueball state G should be primarily a superposition of G^0 and η_1. In the case that the small η_8 component in G can be neglected, the mixing of η_1, η_8, and G^0 is described by two angles θ_1 and θ_2 as follows:

$$\eta = c_1\eta_8 - s_1c_2\eta_1 + s_1s_2G^0,$$
$$\eta' = s_1\eta_8 + c_1c_2\eta_1 - c_1s_2G^0, \quad (7)$$
$$G = s_2\eta_1 + c_2G^0,$$

where $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$. Constraints on the θ_i's can be obtained from the experimental bound $\Gamma(\iota(1460) \to 2\gamma) \le 2.2$ keV [in our discussion we will treat $\iota(1460)$ as the physical glueball state G], and data on $\Gamma(\pi^0 \to 2\gamma) = 7.48 \pm 0.34$ keV, $\Gamma(\eta \to 2\gamma) = 0.56 \pm 0.04$ keV, and $\Gamma(\eta' \to 2\gamma) = 4.47 \pm 0.29$ keV.[11] The 2γ decay amplitudes of π^0, η_8, and η_1 are given by

$$A(P_i \to 2\gamma) = \frac{\alpha}{2\pi F_i} C_i \epsilon_{\mu\nu\alpha\beta}\epsilon_1^{\mu *} q_1^\nu \epsilon_2^{\alpha *} q_2^\beta, \quad (8)$$

with $P_i = \{\pi^0, \eta_8, \eta_1\}$, $C_i = \{1, 1/\sqrt{3}, 2(\frac{2}{3})^{1/2}\}$, and $F_i = \{F_\pi, F_8, F_1\}$.

In order to compute the $G \to 2\gamma$ decay we must also estimate the amplitude $G^0 \to 2\gamma$. This was done recently in the chiral Lagrangian, where the leading term in the $1/N$ expansion gave[12]

$$R = \frac{A(G^0 \to 2\gamma)}{A(\pi^0 \to 2\gamma)} = 0.23. \quad (9)$$

One may, further, worry about a short-distance contribution, which, however, was estimated from the Euler-Heisenberg diagram to be small,[13] i.e.,

$$A(G^0 \to 2\gamma)_{\rm short}/A(\pi^0 \to 2\gamma) \sim 0.03.$$

It follows now that

$$A(G \to 2\gamma) = A(\pi^0 \to 2\gamma)\left[s_2\frac{F_\pi}{F_1}2(\tfrac{2}{3})^{1/2} + c_2R\right]. \quad (10)$$

With the above formulas we can use the data to constrain the mixing angles. The bound on the decay $\iota(1460) \to 2\gamma$ restricts the mixing angle θ_2 to be small: $|\theta_2| < 0.43$. With this small θ_2, we find that the analyses for π^0, η, and η' to 2γ are not altered very much and θ_1 is determined to be $\theta_1 \approx -20°$ and $F_\pi/F_1 \approx 1$.[14] Finally, the above values and $\Gamma(G \to 2\gamma) \le 2.2$ keV, yield the restriction $-0.43 < s_2 < 0.15$.

We are now ready to calculate the glueball effects on $K_L \to 2\gamma$ and the K_L-K_S mass difference. The pole contributions to $A(K_L \to 2\gamma)$ and the $\Delta m_{K_L\text{-}K_S} = 2\,\mathrm{Re}\,M_{12}$ are given by

$$A(K_L \to 2\gamma) = \sum_{P_i}\langle K_L | H_W | P_i\rangle \frac{1}{m_K^2 - m_{P_i}^2} A(P_i \to 2\gamma) \quad (11)$$

and

$$2m_K\,\mathrm{Re}\,M_{12} = \sum_{P_i}\frac{|\langle P_i | H_W | K^0\rangle|^2}{m_K^2 - m_{P_i}^2}, \quad (12)$$

where the summation on P_i is over all possible poles, that is, π^0, η, η', and G.

The different transition matrix elements appearing in the decay amplitudes are related to each other by the nonet symmetry. Of course the symmetry is broken in the real world. We parametrize the breaking in terms of two parameters δ and ρ as

$$\frac{\langle \eta_8 | H_W | K^0\rangle}{\langle \pi^0 | H_W | K^0\rangle} = \frac{1+\delta}{\sqrt{3}},$$
$$\frac{\langle \eta_1 | H_W | K^0\rangle}{\langle \pi^0 | H_W | K^0\rangle} = -2(\tfrac{2}{3})^{1/2}\rho; \quad (13)$$

for $\langle G^0 | H_W | K_L\rangle$, we use Eq. (6), and for the $\langle P_i | H_{\rm em} | 2\gamma\rangle$, the experimentally determined values and the signs determined by theory. It has been noticed that the $A(K_L \to 2\gamma)$ is very sensitive to the breaking parameters δ and ρ.[4] We will use $\delta \approx 0.17$ calculated in Ref. 4 and then use the decay $K_L \to 2\gamma$ to constrain ρ as a function of s_2. In the pole model the amplitudes in terms of θ_i, δ, and ρ are given by

$$A(K_L \to 2\gamma) = \frac{A(\pi^0 \to 2\gamma)\langle \pi^0 | H_W | K_L \rangle}{m_K^2 - m_\pi^2} \left[1 + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_\eta^2} \frac{A(\eta \to 2\gamma)}{A(\pi^0 \to 2\gamma)} \left(c_1 \frac{1+\delta}{\sqrt{3}} + s_1 c_2 (\tfrac{8}{3})^{1/2} \rho + s_1 s_2 \beta \right) + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_{\eta'}^2} \frac{A(\eta' \to 2\gamma)}{A(\pi^0 \to 2\gamma)} \left(s_1 \frac{1+\delta}{\sqrt{3}} - c_1 c_2 (\tfrac{8}{3})^{1/2} \rho - c_1 s_2 \beta \right) + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_G^2} \frac{A(G \to 2\gamma)}{A(\pi^0 \to 2\gamma)} \left(-s_2 (\tfrac{8}{3})^{1/2} \rho + c_2 \beta \right) \right] \tag{14}$$

and

$$2m_K \operatorname{Re} M_{12} = \frac{|\langle \pi^0 | H_W | K^0 \rangle|^2}{m_K^2 - m_\pi^2} \left[1 + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_\eta^2} \left(c_1 \frac{1+\delta}{\sqrt{3}} + s_1 c_2 (\tfrac{8}{3})^{1/2} \rho + s_1 s_2 \beta \right)^2 + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_{\eta'}^2} \left(s_1 \frac{1+\delta}{\sqrt{3}} - c_1 c_2 (\tfrac{8}{3})^{1/2} \rho - c_1 s_2 \beta \right)^2 + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_G^2} \left(-s_2 (\tfrac{8}{3})^{1/2} \rho + c_2 \beta \right)^2 \right]. \tag{15}$$

With all the relations collected so far, we can use Eq. (14) together with the experimental value $\Gamma(K_L \to 2\gamma) = 6.22 \times 10^{-12}$ eV (Ref. 11) and the constraint for s_2 in Eq. (10) to obtain values for ρ and $2m_K \operatorname{Re} M_{12}$ as a function of s_2. For a given s_2, there are two solutions for ρ, ρ_+ and ρ_-, which correspond to $A(K_L \to 2\gamma)/A(K_L \to \pi \to 2\gamma)$ being positive and negative, respectively. Without the glueball contribution ($\beta = 0$ and $s_2 = 0$), and for $\delta = 0.17$, we find

$\rho_+ = 0.82$ and $(2m_K \operatorname{Re} M_{12})_+ = 0.29 \times 10^{-15}\,\text{GeV}^2$

or

$\rho_- = 0.29$ and $(2m_K \operatorname{Re} M_{12})_- = -0.32 \times 10^{-15}\,\text{GeV}^2$.

A similar analysis for $\delta = 0$ gives

$\rho_+ = 0.69$ and $(2m_K \operatorname{Re} M_{12})_+ = 0.94 \times 10^{-15}\,\text{GeV}^2$

or

$\rho_- = 0.16$ and $(2m_K \operatorname{Re} M_{12})_- = 0.14 \times 10^{-15}\,\text{GeV}^2$.

When the glueball contributions are included, we obtain Table I. It is seen that the variation of ρ_+ can be 20% when s_2 varies from -0.43 to 0.15 and $\Delta m_{K_L\text{-}K_S}$ is very sensitive to ρ_+ and s_2. The solution for ρ_- indicates that ρ can be reduced by a factor of 2 or more from its symmetric value and $\Delta m_{K_L\text{-}K_S}$ is also sensitive to ρ_- and s_2.

The selection of a value for ρ is unclear at this time, with all values in Table I being possible. Arguments for restoring ρ to its symmetry value are rather weak, because there is no convincing evidence that a large value for ρ will help[5] in explaining the $\Delta I = \frac{1}{2}$ rule in $K \to 2\pi$ decays. On the other hand, ρ_- is a sensitive function of s_2 and when $0.30 \le \rho \le 0.40$, which corresponds to the large mixing angle, the glueball contribution to $\Delta m_{K_L\text{-}K_S}$ is significant.

Finally, when the glueball has a large pseudoscalar component, then its detection in Cabibbo-favored decays of D_s^+ is possible. A simple estimate gives

$$\Gamma(D_s^+ \to G\pi^+)/\Gamma(D_s^+ \to \eta'\pi^+) \approx |s_2|^2 \times \text{(phase-space factor)}.$$

TABLE I. Fitting the decay $K_L \to 2\gamma$ determines ρ as a function of s_2. The dependence of ρ and $2m_K \operatorname{Re} M_{12}$ on s_2 is shown.

	ρ_+		$2m_K \operatorname{Re} M_{12}$ (10^{-15} GeV^2)		ρ_-		$2m_K \operatorname{Re} M_{12}$ (10^{-15} GeV^2)	
s_2	$\delta=0.17$	$\delta=0.0$	$\delta=0.17$	$\delta=0.0$	$\delta=0.17$	$\delta=0.0$	$\delta=0.17$	$\delta=0.0$
0.15	0.80	0.67	0.27	0.92	0.26	0.14	−0.33	0.14
0.0	0.82	0.69	0.24	0.90	0.29	0.16	−0.35	0.12
−0.15	0.86	0.73	0.19	0.85	0.32	0.19	−0.38	0.09
−0.30	0.91	0.78	0.11	0.79	0.36	0.23	−0.43	0.06
−0.45	1.00	0.86	−0.03	0.68	0.41	0.27	−0.50	0.01

With the experimental data on $\Gamma(D_s^+ \to \eta'\pi^+)/\Gamma(D_s^+ \to \phi\pi^+) \approx 5 \pm 3$ (Mark II)[15] and $\approx 5.7 \pm 1.5$ (NA14)[15] and on $B(D_s^+ \to \phi\pi^+) \sim 3.5\%$, the branching ratio $B(D_s^+ \to G\pi^+)$ can be as large as 1%. This value can be reached with the sensitivity of the present experiments.

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