Nucleon and Nuclear Anapole Moments

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The leading T-conserving, P-nonconserving (PNC) electromagnetic coupling to the nucleon or nucleus is known as the anapole moment. We evaluate the pion-cloud contribution to the nucleon anapole moment, and the enhancements in nuclei associated with meson-exchange currents and with the mixing of the nuclear ground state with opposite-parity excited states. We find that the anapole moment becomes the dominant PNC spin-dependent coupling in heavy nuclei.

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For spin-$\frac{1}{2}$ fermions two parity-nonconserving (PNC) static couplings to the electromagnetic field can arise, the electric dipole moment (EDM), which violates both parity (P) and time-reversal (T) invariance, and the anapole moment, which violates P but conserves T. Although first discussed by Zel'dovich thirty years ago,[1] the anapole moment has received less attention than the EDM. Naively, one expects the experimental effects of the anapole moment [generated by weak radiative corrections like those of Fig. 1(a)] to be swamped by tree-level neutral-current processes, the former being suppressed by roughly a factor of α over the latter. However, nuclear or atomic many-body effects may enhance the size of the anapole moment to the point where it competes effectively with Z^0 exchange.[2,3] Thus, the anapole moment could be important in experimental tests of the standard model that use charged particles (e.g., electrons) as probes. Moreover, nucleon and nuclear anapole moments are of fundamental interest, providing a new test of the PNC meson-nucleon couplings that govern the weak NN interaction.[4,5]

In this Letter we estimate the anapole moments of the free nucleon and selected nuclei (^{19}F and ^{133}Cs). We show that the form of the nuclear anapole operator depends on fully enforcing the constraints of current conservation, which we implement through an extended Siegert's theorem.[6-8] We also show that meson-exchange currents can generate anapole moments in heavy nuclei that greatly exceed the one-body or valence-nucleon value. We evaluate these one- and two-body terms and the additional anapole contributions from PNC wave-function admixing (a second important source of enhancement[2]) by using shell-model density matrices and by exploiting an elegant algorithm for inverting linear operators. The resulting PNC V(electron)-A(nucleus) interaction generated by the nuclear anapole moment, a weak radiative correction, is shown to be as large as the corresponding tree-level Z^0 interaction in many nuclei.

For an on-shell spin-$\frac{1}{2}$ fermion (and therefore a virtual photon), current conservation and Lorentz invariance require PNC corrections to matrix elements of the electromagnetic current to have the form[3]

$$\langle p'|\, j_\mu^{\text{em}}(0)\,|p\rangle_{\text{PNC}} = \frac{a(q^2)}{m_N^2}\bar{u}(p')(\not{q}q_\mu - q^2\gamma_\mu)\gamma_5 u(p)\,, \tag{1}$$

where $q_\mu = p' - p$ is the momentum transfer to the fermion. The form factor $a(q^2)$ evaluated at $q^2 = 0$ defines the fermion anapole moment. For interactions with on-shell external particles, only the $\gamma_\mu\gamma_5$ term will contribute to PNC amplitudes. The explicit q^2 factor in this term cancels the q^{-2} from the photon propagator, so that a contact interaction results. Thus, the anapole moment produces the same type of coordinate-space contact interaction as low-q^2 neutral-current processes.

The electron[9] (and constituent quark[10]) anapole mo-

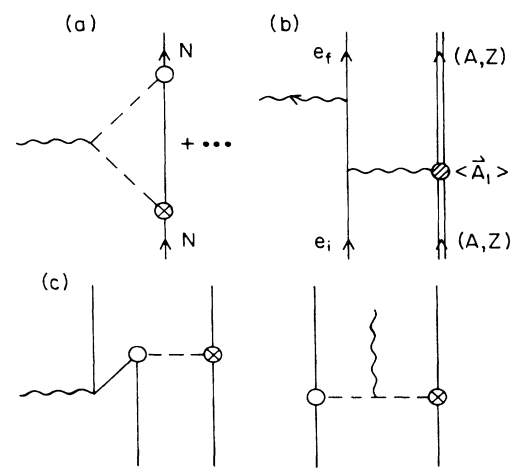

FIG. 1. Examples of (a) one-body and (c) exchange-current contributions to the nuclear anapole moment, and (b) an induced PNC atomic decay.

ment calculated in the standard model is gauge dependent; in physical processes like e-e scattering radiative corrections (e.g., two-boson exchange) must be combined with $a(q^2)$ to produce a gauge-independent result.[10] This complication does not arise in the present work, where we restrict our attention to the effects generated by an on-shell PNC πNN vertex. The pion one-loop corrections [Fig. 1(a)] to the γNN vertex then provide a gauge-independent estimate of the meson-cloud contribution to the nucleon anapole moment. The results for pseudovector and pseudoscalar strong couplings agree up to an ambiguity associated with the linear divergence of the pseudovector loop integral,

$$a(0)_{\pi\,\text{cloud}} = e\left(\frac{f_\pi g_{\pi NN}}{8\sqrt{2}\pi^2}\right)(\alpha_s+\alpha_v\tau_s)\equiv a_s(0)+a_v(0)\tau_3\,, \tag{2}$$

where $g_{\pi NN}$ is the usual strong coupling and f_π is the weak PNC coupling (but defined as minus that of Ref. 4). The terms $\alpha_{s,v}$ contain logarithms $\ln(m_\pi/m_N)$; their numerical values are $\alpha_s \simeq 1.6$ and $\alpha_v \simeq 0.4$. Similar logarithms suppress the contributions from heavier mesons. While, as noted above, other terms contribute to nucleon anapole amplitudes, we will use Eq. (2) to estimate the scale of $a(0)$. The PNC electron-proton potential generated by $a(0)$ is smaller than the isovector tree-level V(electron)-A(proton) neutral-current interaction by a factor of $3.8\alpha(f_\pi/f_\pi^{\text{DDH}})$, where $f_\pi^{\text{DDH}}=-4.5\times10^{-7}$ is the best-value coupling of Ref. 4. Despite the distinctive isoscalar contribution, the nucleon anapole moment may be impossible to isolate experimentally. [Although neglected here, the ρ-meson contribution to the nucleon anapole moment has also been estimated by one of us (M.J.M.) using vector-meson dominance.]

A more tractable task may be the observation of anapole moments in nuclei, where many-body effects enhance the anapole coupling. Two distinct effects are of interest: the "polarization" contribution due to the mixing of the nuclear ground state with nearby excited states with the same angular momentum but opposite parity,[2] and two-body (and, in principle, higher-order) currents arising from the interaction of the photon with $N\bar{N}$ pairs and virtual mesons. Long-range pion-exchange contributions should dominate these nuclear amplitudes.

Time-reversal symmetry restricts the electromagnetic static moments of a nucleus to even multipoles of the electromagnetic charge operator ($C0,C2,\dots$) and odd multipoles of the magnetic and electric current operators ($M1,M3,\dots;E1,E3,\dots$). (Note that the T-odd electric dipole moment is a $C1$ coupling.) If we consider only those PNC photon couplings arising from the weak interaction in first order, parity further restricts the nonzero matrix elements to the odd electric projections of the axial-vector (anapole) current $\langle \text{g.s.}|\hat{E}_J^A|\text{g.s.}\rangle$ and to the odd electric projections of the ordinary vector current that can arise because of wave-function polarization:

$$\sum_n \frac{\langle \text{g.s.}^+|\hat{E}_J^V|n^-\rangle\langle n^-|\hat{H}_{\text{PNC}}|\text{g.s.}^+\rangle}{E_{\text{g.s.}}-E_n} + \frac{\langle \text{g.s.}^+|\hat{H}_{\text{PNC}}|n^-\rangle\langle n^-|\hat{E}_J^V|\text{g.s.}^+\rangle}{E_{\text{g.s.}}-E_n}\,. \tag{3}$$

Here $|\text{g.s.}^+\rangle$ is the ground state, $|n^-\rangle$ denotes an excited state having the same spin but opposite parity, and H_{PNC} is the parity-nonconserving NN interaction.[5]

A proper treatment of current conservation is crucial in evaluating matrix elements of $\hat{E}_J^A$ and $\hat{E}_J^V$. These operators can be written as $\hat{E}_J(q)=\hat{S}_J(q)+\hat{R}_J(q)$, where all components of the electromagnetic current operator that are constrained by current conservation have been isolated in $\hat{S}_J$ and expressed as a commutator of the charge operator with the nuclear Hamiltonian. The static matrix elements of $\hat{S}_J(q)$ then vanish ($q_0=0$). To lowest order in q^2, $\langle \text{g.s.}|\hat{E}_J|\text{g.s.}\rangle = \langle \text{g.s.}|\times\hat{S}_J|\text{g.s.}\rangle=0$, which we recognize as Siegert's theorem.[6] The extended Siegert's theorem[7,8] determines $\hat{R}_J$,

$$\hat{R}_J \underset{q^2\to0}{\longrightarrow} \frac{q}{J+2}\int d\mathbf{r}\,\frac{(qr)^J}{(2J+1)!!}\mathbf{Y}_{JJ1}(\Omega_r)\cdot[\mathbf{r}\times\mathbf{j}_{\text{em}}(\mathbf{r})]\,,$$

yielding in the long-wavelength limit

$$\langle \text{g.s.}\,\|E1\|\,\text{g.s.}\rangle \underset{\mathbf{q}^2\to0}{=} -\frac{i\mathbf{q}^2}{9(6\pi)^{1/2}}\int d\mathbf{r}\,r^2\langle \text{g.s.}\,\|\,\mathbf{j}_{\text{em}}(\mathbf{r})+(2\pi)^{1/2}[Y_2(\Omega_r)\otimes j_{\text{em}}(\mathbf{r})]_1\,\|\,\text{g.s.}\rangle\,, \tag{4}$$

where $\otimes$ denotes a spherical tensor product and $\|\ \|$ denotes a reduced matrix element. Thus current conservation requires the ground-state $E1$ moment to have the same leading q^2 behavior as the transverse part of the single-nucleon current of Eq. (1). Like its free-nucleon counterpart, the nuclear anapole moment generates a contact interaction between the nucleus and an on-shell external particle. The explicit form of Eq. (4) depends on properly removing those $O(q^2)$ terms in the $E1$ matrix element that vanish because of current conservation.

The one-body contribution to Eq. (4) is derived by reducing the axial single-nucleon current operator [Eq. (1)] nonrelativistically and transforming into coordinate space,

$$\langle \text{g.s.}\,\|E1\|\,\text{g.s.}\rangle^{\text{1-body}} \underset{\mathbf{q}^2\to0}{=} \frac{i}{(6\pi)^{1/2}}\frac{\mathbf{q}^2}{m_N^2}\langle \text{g.s.}\,\|\,\sum_{i=1}^{A}[a_s(0)+a_v(0)\tau_3(i)]\sigma(i)\,\|\,\text{g.s.}\rangle\,. \tag{5}$$

The anapole operator within the matrix element, which we will denote $\mathbf{A}_{\text{nuc}}(1)$, is the direct analog of $a(0)\gamma\gamma_5$ appearing in Eq. (1). In a naive nuclear model with a single nucleon outside a spin-paired core, the one-body contribution to the nuclear anapole moment is just the anapole moment of the valence nucleon.

The nuclear current operator appearing in Eq. (4) also contains important two-body currents. The $N\bar{N}$ and pionic diagrams [Fig. 1(c)] yield

$$\mathbf{A}_{\rm nuc}(2) = -\frac{m_N e f_\pi g_{\pi NN}}{108\pi\sqrt{2}}\frac{1}{2}\sum_{\substack{ij\\ i\neq j}}[\tau(i)\cdot\tau(j)-(\tfrac{3}{2})^{1/2}[\tau(i)\otimes\tau(j)]_{20}]$$
$$\times\Bigg[r_i^2\sigma(i)+r_j^2\sigma(j)+(2\pi)^{1/2}\{r_i^2[Y_2(\hat{\mathbf{r}}_i)\otimes\sigma(i)]_1+r_j^2[Y_2(\hat{\mathbf{r}}_j)\otimes\sigma(j)]_1\}$$
$$-\tfrac{1}{2}[\sigma(i)\cdot\nabla(i)-\sigma(j)\cdot\nabla(j)]$$
$$\times\left((r_i^2+r_j^2)\mathbf{r}+(2\pi)^{1/2}\{r_i^2[Y_2(\hat{\mathbf{r}}_i)\otimes\mathbf{r}]_1+r_j^2[Y_2(\hat{\mathbf{r}}_j)\otimes\mathbf{r}]_1\}+\frac{3}{2}\frac{r}{m}\mathbf{r}\right)\Bigg]\frac{e^{-rm_\pi}}{r}, \quad (6)$$

where $\mathbf{r}=\mathbf{r}_i-\mathbf{r}_j$. Although the full operator is used in our calculations, much insight is gained by reducing this operator to an approximate one-body form,

$$\langle\alpha|\mathbf{A}_{\rm nuc}^{\rm eff}(1)|\beta\rangle=\sum_{\delta<F}\langle\alpha\delta|\mathbf{A}_{\rm nuc}(2)|\beta\delta-\delta\beta\rangle,$$

where the sum is taken over the nuclear core. The Fermi-gas model, with a spin-symmetric but isospin-asymmetric core, yields for the $N\bar{N}$ contribution

$$\mathbf{A}_{N\bar{N}}^{\rm eff}(1)=2.74a_s(0)\frac{m_N}{m_\pi^2}\sum_{i=1}^{A}\rho(\mathbf{r}_i)r(i)^2\{\sigma(i)+(2\pi)^{1/2}[Y_2(\hat{\mathbf{r}}_i)\otimes\sigma(i)]_1\}$$
$$\times\left[\frac{Z}{A}\omega_Z^\pi[1-\tfrac{2}{3}\tau_3(i)]+\frac{N}{A}\omega_N^\pi[1+\tfrac{2}{3}\tau_3(i)]\right], \quad (7)$$

with $\rho(\mathbf{r}_i)$ the nuclear density operator and ω_Z^π (ω_N^π) a proton (neutron) Fermi-gas response function that depends on $k(i)/k_F$, the nucleon momentum as a fraction of the Fermi momentum.[11] The ω's vary only gently, ranging from 0.33 to 0.19 as $k(i)/k_F$ increases from 0 to 1. Thus we can approximate $\omega\sim0.25$.

Using a nuclear density of 0.195/fm^3, we conclude that nuclei will exhibit enhanced isoscalar anapole moments $\sim0.9A^{2/3}a_s(0)$ due to the $N\bar{N}$ exchange current. The inclusion of short-range correlations reduces this estimate by 25%.[5] Thus we expect a net isoscalar $N\bar{N}$ anapole moment for ^{133}Cs (an example discussed below) 17 times the single-nucleon value. The isovector anapole moment is smaller by a factor of $2(Z-N)/3A$ due to a cancellation between contributions from core neutrons and protons.

Finally, we turn to the third piece of the anapole moment, the polarization contribution of Eq. (3). This contribution is similar to the exchange currents in that the net effect of interactions with core nucleons is a PNC polarization of orbits of valence nucleons. It differs in that the energy denominators governing the mixing are determined by the spectrum of excited nuclear states. Thus, if one selects a nucleus where an excited state of the same spin but opposite parity appears very near the ground state, a substantial enhancement can result. The form of the vector-current anapole operator for Eq. (3) is again provided by the extended Siegert's theorem,[8]

$$\mathbf{A}^v(1)=\frac{-m_N e}{6\sqrt{2}}\sum_{i=1}^{A}\left[\frac{1}{\sqrt{2}}\mathbf{r}(i)\tau_3(i)+[\mathbf{r}(i)\otimes\mathbf{l}(i)]_1[1+\tau_3(i)]+\tfrac{3}{2}[\mathbf{r}(i)\otimes\sigma(i)]_1[\mu_s+\mu_v\tau_3(i)]\right],$$

where the isoscalar and isovector magnetic moments are $\mu_s=0.88$ and $\mu_v=4.706$. For the $H_{\rm PNC}$ we take the pion-exchange piece of the Hamiltonian of Ref. 5. Except in cases where a ground-state doublet is important, it is argued in Ref. 2 that the polarization contribution also scales like $A^{2/3}$.

Calculations were performed for two nuclei, ^{19}F and ^{133}Cs. The former is an example of a nucleus where a ground-state parity doublet (the $\frac{1}{2}^+$-$\frac{1}{2}^-$ splitting is 110 keV) could lead to an enhanced polarization contribution to the anapole moment. The latter was the subject of a recent atomic experiment on anapole moments.[12] We describe the ^{19}F ground state in the shell model, diagonalizing the Brown-Wildenthal interaction[13] in a basis consisting of all $2s1d$-shell configurations. Equations (5) and (6) were evaluated from the shell-model one- and two-body density matrices using a harmonic-oscillator basis ($b=1.78$ fm). The polarization contribution [Eq. (3)] requires us to sum over the complete set of $1\hbar\omega$ states that connect to the ground state through $\mathbf{A}^v$. We solve this difficult numerical problem by exploiting a variation of the Lanczos algorithm to evaluate the effect of $(E_{\rm g.s.}-H)^{-1}$ on the vector $H_{\rm PNC}|{\rm g.s.}\rangle$, where H is the shell-model Hamiltonian acting in the full $1\hbar\omega$ space. This procedure involves a rapidly converging expansion in terms of the Lanczos vectors, with the expansion coefficients being products of continued fractions

TABLE I. Shell-model estimates of the one-body, polarization, and exchange-current contributions to the anapole matrix element $\langle \text{g.s.} \| A_1 \| \text{g.s.} \rangle$ in units of ef_π. The last column gives the ratio of the anapole interaction with an on-shell electron to that generated by Z^0 exchange, assuming $f_\pi = f_\pi^{\rm DDH}$ and $\sin^2\theta_W = 0.23$.

Nucleus	One body	Polarization	$N\bar{N}$	Pionic	Total	V^{AN}/V^{Z^0}
^{19}F	0.55	20.03	1.79	−0.62	21.8	1.07
^{133}Cs	−0.58	−41.97	−9.90	0.76	−51.7	2.72

formed from the entries of the tridiagonal Lanczos matrix.[11,14] A complete description of this work will appear elsewhere.[11]

The ^{133}Cs ground-state wave function was determined by diagonalizing the Baldridge-Vary interaction[15] in the $1g_{7/2}$-$2d_{5/2}$-$1h_{11/2}$-$3s_{1/2}$-$2d_{3/2}$ shell-model space. The valence protons were restricted to the first two of these orbits, and the neutron holes to the last three. The one-body and exchange-current contributions were evaluated as in ^{19}F, using $b = 2.27$ fm. As no ground-state parity doublet exists in ^{133}Cs and the $E1$ strength is concentrated in the giant-resonance region, we replaced E_n in Eq. (3) by an average value, $E_n - E_{\rm g.s.} = 15.2$ MeV. By invoking closure, the resulting "core-polarization" expression can be evaluated from the ground-state one- and two-body density matrices. (No three-body terms arise because matrix elements of $\langle A^v \rangle$ vanish within the model space.)

The results of these calculations are summarized in Table I. In the case of ^{19}F, mixing with the lowest $\frac{1}{2}^-$ state was treated separately: The mixing matrix element can be determined from the measured ^{19}Ne $\frac{1}{2}^+ \to \frac{1}{2}^-$ β-decay rate, as described in Ref. 5. We also employed the correlation function of Ref. 5 to modify all ^{133}Cs and ^{19}F two-nucleon densities. The total anapole moment is dominated by the core polarization and exchange-current terms, with these contributing constructively in the ratio of about 4 to 1 in ^{133}Cs. The contribution of the ^{19}F 110-keV doublet to the polarization sum is significant but not extraordinary, accounting for 53% of the total.

In the last column of Table I we compare the strength of the interaction between the nuclear anapole moment and an on-shell electron with that arising from V(electron)-A(nucleus) Z^0 exchange. For $f_\pi = f_\pi^{\rm DDH}$ the former exceeds the latter by about a factor of 3 for ^{133}Cs, a ratio smaller than that found in Ref. 2. The comparison would be more favorable for nuclei with very degenerate doublets (e.g., ^{229}Pa) and, of course, for $T=0$ nuclei. In a later publication[11] we will present calculations of PNC atomic decay rates [Fig. 1(b)] and electron scattering cross sections arising from the nuclear anapole moment.

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