Accelerating Observer and the Hagedorn Temperature

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We investigate thermal effects occurring when an inertial string vacuum is described in an accelerating frame. It is shown that there is a critical acceleration corresponding to a temperature equal to T_H/π, where T_H is the Hagedorn temperature for a bath of free strings.

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It is well known that an observer at constant acceleration a in a Minkowski vacuum will feel the existence of a heat bath with a temperature $T=a/2\pi$.[1,2] In this paper we investigate the nature of these thermal effects if such an observer accelerates in a vacuum of free strings. In particular, we study the occurrence of a critical temperature akin to the so-called Hagedorn temperature $T_H=O(1/\sqrt{\alpha'})$ above which the string partition function is known to diverge.[3-5]

We start by defining local thermodynamic variables in the accelerating frame associated with the string vacuum. We find that they diverge above a critical acceleration corresponding to a temperature $T_c=T_H/\pi$, rather than T_H, as one might expect. We show how this peculiar factor of $1/\pi$ can be understood in a geometrical way, and interpret it in terms of winding strings. This allows us to speculate about thermal effects of strings in de Sitter space.

Consider an observer A uniformly accelerating in the (x^0,x^1) plane along a world line given by $(x_A^0=a^{-1}\sinh a\tau,\ x_A^1=a^{-1}\cosh a\tau)$. The Rindler coordinate system (τ,ξ,x^i),

$$\begin{aligned} x^0&=(a^{-1}+\xi)\sinh(a\tau)\equiv\rho\sinh(a\tau)\,,\\ x^1&=(a^{-1}+\xi)\cosh(a\tau)\equiv\rho\cosh(a\tau)\,,\\ x^i&=x^i\quad(i=2,\ldots,d)\,,\end{aligned}\tag{1}$$

is the coordinate system associated with the Fermi-Walker nonrotating tetrad carried by A.[6] Actually, the trajectory ρ,x^i constant corresponds to an observer accelerating in the x_1 direction with acceleration ρ^{-1}. The most important property is the occurrence of a horizon for these observers. It was shown by Fulling[7] and Unruh[2] that because of this, second quantization is inequivalent for inertial and accelerating observers. Specifically, the distinction between positive and negative frequencies is frame dependent. Consequently, the two observers will define creation and annihilation operators, and thus their vacua, differently.

If one tries to study the thermodynamic properties of this thermal bath, one encounters a problem. Normally local thermodynamic variables can be defined using homogeneity of space. For instance, the free energy density for a bath of free scalar particles can be calculated by doing a path integral over one-loop diagrams, and dividing by the volume in the end. For the accelerating observer such a procedure makes no sense, as the coordinate system describes observers at different accelerations and hence different temperatures. In fact, the temperature felt by the observer at constant ρ is $T=(2\pi\rho)^{-1}$. This highlights the necessity of formulating the thermodynamics of accelerating systems in local terms.

To this end, let us, for the moment, restrict our attention to scalar particles. We observe that physical processes measured by A in the Minkowski vacuum are controlled by the Feynman propagator

$$G_F(x,y)=\int\frac{d^dp}{(2\pi)^d}\frac{e^{ip\cdot(x-y)}}{p^2+m^2+i\epsilon}\,.\tag{2}$$

Expectation values of relevant physical quantities, e.g., the stress-energy tensor, can be directly determined by performing certain operations on the propagator.[8]

A careful study of the Feynman propagator in a Rindler frame was carried out by Troost and Van Dam.[9] They examined the expression one gets if one substitutes Rindler coordinates (1) for x,y in (2). It was shown that the resulting expression, in the case of massless particles in four dimensions, and when the end points are taken on the world line of A, is exactly a flat-space thermal propagator. Specifically, between two points parametrized by τ and τ' (with ξ and the transverse coordinates zero), one finds

$$\begin{aligned} G_F(\tau,\tau')&=-\frac{1}{4\pi^2}\sum_{k=-\infty}^{\infty}\frac{1}{(\tau-\tau'+i\beta k)^2-i\epsilon}\\ &=\frac{1}{(2\pi)^4}\int dE\,d^3p\,e^{iE(\tau-\tau')}D_\beta(E,p)\,,\end{aligned}\tag{3}$$

where $\beta=2\pi/a$ and

$$D_\beta(E,p)=\frac{i}{E^2-p^2+i\epsilon}+\frac{2\pi\delta(E^2-p^2)}{e^{\beta|E|}-1}$$

is the thermal propagator in four dimensions.

Furthermore, they showed that each of the terms in the sum over k in (3) can be given an attractive interpretation by writing the propagator, using the Schwinger proper-time method, as a sum over paths. It turns out that the kth term in (3) is exactly equal to analytic continuation of the sum over paths in Euclidean space between the end points with the winding number around the origin in the (x_0,x_1) plane restricted to k (see Fig. 1). Clearly the total propagator is equal to the sum over contributions of all possible winding numbers: $G_F(x,y) = \sum_{k=-\infty}^{\infty} G_F^{(k)}(x,y)$. The zero winding term corresponds to the zero-temperature Feynman propagator.

The same decomposition can be done for the case of massive particles[10] and in dimensions different from four. The resulting propagator again has a thermal character, which is basically due to the fact that in Euclidean space Rindler coordinates are just polar coordinates: The time variable is the angle variable, and is periodic. The zero winding contribution is not equal to the flat-space Feynman propagator in the general case, but describes propagation of Rindler quanta in the Rindler vacuum. Specifically, one can show that the k winding contribution yields for the Euclidean propagator[11]

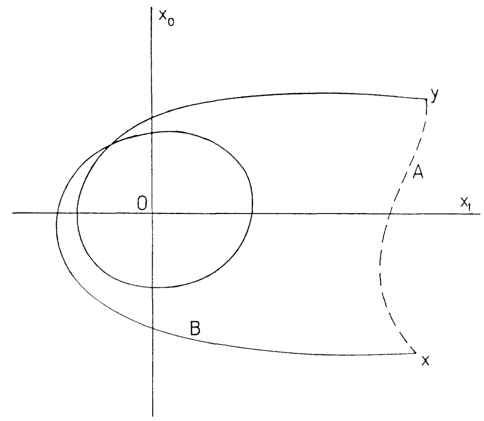

FIG. 1. Two paths in the Euclidean (x_0, x_1) plane contributing to the total propagator. Path A has winding number 0, path B has winding number -2.

$$G_E^{(k)}(\tau,\rho,x_\perp;\tau',\rho',x'_\perp) = \frac{1}{\pi^2}\int \frac{d^{d-2}p_\perp}{(2\pi)^{d-2}} e^{-ip_\perp \Delta x_\perp} \int_0^\infty d\nu K_{i\nu}(\kappa\rho) K_{i\nu}(\kappa\rho') \sinh(\pi\nu) \exp\{-\nu[(a\tau - a\tau' + 2\pi k)^2]^{1/2}\}. \quad (4)$$

Here $\kappa = (p_\perp^2 + m^2)^{1/2}$, $x_\perp$ denotes the transverse coordinates, the quantity $\rho = a^{-1} + \xi$ becomes, in the Euclidean (x^0,x^1) plane, the radial coordinate, and $K_{i\nu}$ is the modified Bessel function.

It is illuminating to make a comparison with the flat-space thermal propagator. Here the Euclidean time direction is compactified to a circle with circumference $\beta = 1/T$. The total propagator also splits into contributions with definite winding, this time around the circle. However, the interpretation of the winding terms is the same. In fact, it can be shown that in both cases the $\pm k$ winding contributions correspond exactly to the k particle contributions to the thermal trace in the second quantized expression for the propagator.

Having obtained the propagator in the above form, it is straightforward to extract expectation values of the stress-energy tensor. As an example, we have the trace of the stress-energy tensor, which is given by[8]

$$g^{\mu\nu}(x)\langle T_{\mu\nu}(x)\rangle = \tfrac{1}{2} m^2 G_F(x,x). \quad (5)$$

Here the expectation value is to be taken in the Minkowski vacuum. As it stands, (5) is ill defined, and requires regularization. As we want to describe the physics of the accelerating observer, it is natural to normal order $T_{\mu\nu}$ relative to the Rindler vacuum. This automatically sets the energy density of the Rindler vacuum to zero, which is clearly what we would want. It follows form the discussion above that this normal ordering corresponds to dropping from the Feynman propagator the zero winding (vacuum) contribution. In other words, we replace in (5) $G_F \to G_F - G_F^{(0)}$.

Let us return to the case of strings. We want to give a meaning to the stress-energy tensor for a vacuum of free strings. The most straightforward thing to do is to work in the light-cone gauge. The string vacuum just becomes the direct product of the vacua of all components of the spectrum of the string. As they are noninteracting, the value of a local quantity measured by an accelerating observer just becomes the sum of the values for all components.

To find the critical acceleration (and thus, the critical temperature) above which the canonical variables diverge, we recall that such a divergence is caused by summing over the exponential asymptotic degeneracy of the string spectrum. Consequently, the critical behavior is controlled by the high mass end of the spectrum. Evaluating (4) for large mass $[m \gg a = O(1/\sqrt{\alpha'})]$, when the end points of the propagator are on the world line of the observer, one finds

$$G_E^{(k)}(\Delta\tau,\rho) \approx \frac{\Gamma[(d-2)/2]}{2\pi}\frac{1}{m}\left(\frac{m}{\rho}\right)^{(d-1)/2} e^{-2m\rho}\frac{1}{(a\Delta\tau + 2k\pi)^2 - \pi^2} \quad (k \neq 0). \quad (6)$$

Now it is an easy matter to obtain the critical temperature. From (5) we see that we need to take the end points to coincide in (6). Recall the asymptotic level density for the closed bosonic string is

$$d(N) \propto m^a \exp(\beta_0 m), \quad (7)$$

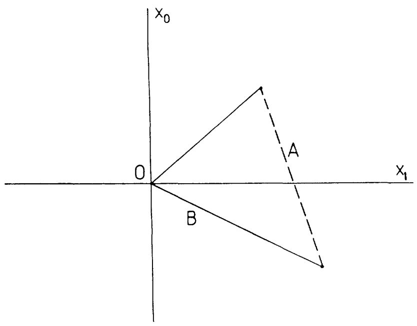

FIG. 2. Paths with the dominating contributions for winding number zero (path A) and winding number nonzero (path B) in the large-mass limit.

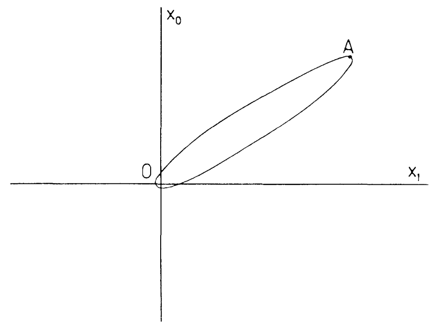

FIG. 3. "Winding string" giving rise to divergences in local thermodynamic variables for the accelerated observer. The distance between O and A is ρ.

where $\beta_0 = 4\pi\sqrt{\alpha'}$ is the inverse Hagedorn temperature defined through the singular behavior of thermodynamic functions for inertial observers. Summing (6) with degeneracy (7) yields a critical temperature defined through acceleration

$$T_c = \frac{1}{2\pi\rho_c} = \frac{1}{4\pi^2\sqrt{\alpha'}} = \frac{\beta_0^{-1}}{\pi}. \tag{8}$$

Note that the exponential dependence of (6) will carry over to the components of the stress-energy tensor: These merely pick up powers of the mass and the distance.

It is not hard to understand the exponential dependence in (6) which is responsible for the peculiar value of the critical temperature. In flat space, the Euclidean propagator for a particle with mass m between two points at a distance r ($mr \gg 1$) behaves like $G_E(r) \sim m^\alpha r^\beta \exp(-mr)$ (α and β are constants irrelevant for the present purposes). We could interpret r as the length of the shortest path between the end points. For the constrained propagator the shortest path is the direct distance for zero winding number; in the case of nonzero winding number, however, it is equal to the sum of the radii of the end points. In the latter case the direct path is "not allowed" (see Fig. 2).

This picture allows for an illuminating comparison with the inertial string bath at temperature $1/\beta$. As remarked above, in that case the thermal contributions to the propagator wind around the compactified imaginary-time direction S^1_β.[12] There the Hagedorn divergence will occur when the (negative) exponent in the winding ± 1 contributions to the propagator are canceled by the exponential level degeneracy. The minimal distance is the circumference β of the circle, rather than twice the radius, as in the case of the accelerated frame. This is the origin of the factor π in (8).

This brings us to an interesting difference between the two cases. In (6) the exponential does not depend on the winding number k. Consequently the critical acceleration is the same for all winding numbers, and thus the divergence gets a contribution from all k, rather than just from the $k = \pm 1$ terms.

It has been shown that the Hagedorn divergence can be interpreted as the temperature where a string that winds once around the imaginary-time direction becomes massless, and gives a diverging contribution to the partition function.[13-16] Applying this interpretation to the accelerated frame, we are led to view the critical acceleration as the point where strings that wind around the origin and pass by the observer (see Fig. 3) become massless.

This discussion makes it clear that the occurrence of this peculiar critical temperature is a "stringy," nonlocal effect. Because of the geometrical origin, the prefactor will be universal, and extend to other string models.

It is interesting to speculate about generalizations of the results obtained for the accelerating observer to other models. For instance, for de Sitter space a temperature effect also arises because the Euclidean time variable is periodic.[17] More precisely, Euclidean–de Sitter space

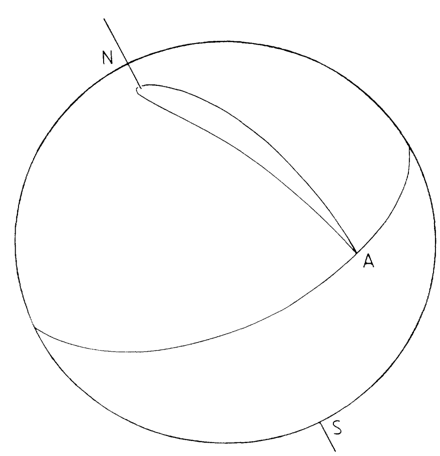

FIG. 4. "Winding string" in (Euclidean) de Sitter space. The transverse directions are not shown. For an observer A, located at the equator, time translation is rotation along the N-S axis.

can be represented as a d sphere, and (Euclidean) time translation is rotation along great circles. As we are working in curved space here, string quantization may be hard to carry out explicitly, but we will assume here that it can be done consistently. If we assume furthermore that the high-mass degeneracy will possess an exponential behavior $[d(N) \sim \exp(\beta_{\text{de Sitter}}\sqrt{N})]$, a critical temperature will occur as well. In analogy with the Rindler space, we expect the critical temperature to be controlled by the winding strings indicated in Fig. 4. From this we may infer that the geometric factor relating the temperature associated with the periodicity of the metric (the Gibbons-Hawking temperature) with the string critical temperature is $\frac{1}{2}$ rather than $1/\pi$. If string theory and the Hagedorn transition play a role in the very early Universe, this result may be important in studying its evolution.

One may derive similar effects in any situation in which the time coordinate is periodic in imaginary time. We mention, for instance, a stationary observer in a Schwarzschild background metric. After completion of this paper, we became aware of Ref. 18 in which the occurrence of a maximum acceleration is also discussed in a somewhat different context.

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