Experimental Observation of Crisis-Induced Intermittency and Its Critical Exponent

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Critical behavior associated with intermittent temporal bursting accompanying the sudden widening of a chaotic attractor was observed and investigated experimentally in a gravitationally buckled, parametrically driven, magnetoelastic ribbon. As the driving frequency, f, was decreased through the critical value, f_c, we observed that the mean time between bursts scaled as $|f_c - f|^{-\gamma}$.

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Chaotic attractors can undergo sudden changes as a system parameter is varied.[1] Such "crises" have been experimentally observed in a number of systems.[2-8] In particular, the attractor can collide with an unstable periodic orbit which *lies in the interior* of its basin of attraction. When this occurs, the attractor typically experiences an explosion in size. This type of event has been termed an interior crises (as opposed to a boundary crises in which the attractor is destroyed). Accompanying an interior crisis is a type of intermittent behavior in which the chaotic motion for parameter values f near the critical crisis value f_c lies predominantly in the smaller phase-space region which the attractor occupied before the crisis but which has occasional bursts of activity into the expanded region. The mean time τ between these intermittent bursts has been predicted to have a characteristic power-law dependence,[9] $\tau \propto |f_c - f|^{-\gamma}$, for f near f_c. The theory further says that the scaling of τ with $|f_c - f|$ arises from the scaling structure of the attractor just before the crisis.[9] While such power-law scaling of the times should be common, their realization in physical experiments has previously been documented in only two papers. Carroll, Pecora, and Rachford[8] observed chaotic transients in a spin-wave system, with lifetimes given by $\tau \propto |f_c - f|^{-\gamma}$. Rollins and Hunt[5] observed crisis-induced intermittency in a diode resonator circuit, which is well modeled by a smooth *one-dimensional* map, in which case γ is generically restricted to be $\frac{1}{2}$. We have observed crisis-induced intermittency in a nonlinear infinite-dimensional magnetically driven mechanical system.

The main contributions of this paper are the following: (1) observation of crisis-induced intermittency; (2) observation of its characteristic power-law dependence; (3) the first observation of the critical exponent γ in a case where the dynamics is not described by a one-dimensional map (for us, $\gamma > \frac{1}{2}$); (4) the first experimental determination of an unstable periodic orbit mediating a crisis; (5) experimental determination of the tails of the distributions of intervals between bursts as consistent with Poisson statistics; and (6) measurement of γ', the scaling exponent for the attractor, at one value of the frequency before the crisis, consistent with the measured value of γ.

The experimental system was a gravitationally buckled, amorphous magnetoelastic ribbon[10] driven parametrically by a time-varying magnetic field (Fig. 1). The ribbon used in the experiment is from a new class of amorphous materials that have been found to exhibit very large reversible changes of Young's modulus, $E(H)$, with the application of a small magnetic field (cf. inset in Fig. 1). The critical height, h_c, at which a vertical column will buckle depends on its stiffness. $E(H)$ can be altered by changing the value of the vertical field H, thereby setting the value of h_c alternately greater than and less than the actual height of the ribbon.[10]

For the experiment reported here we have used a transversely annealed, $Fe_{81}B_{13.5}Si_{3.5}C_2$, amorphous magnetoelastic ribbon, 25 μm thick, 3 mm wide, and 100 mm long clamped at the base to yield a 65-mm free vertical height. A three-axis set of Helmholtz coils driven by a bipolar operational power supply and a frequency syn-

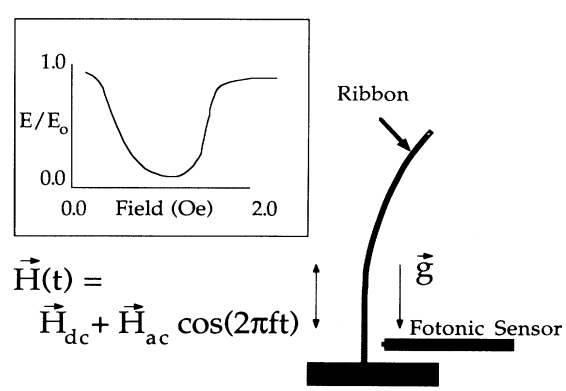

FIG. 1. Experimental configuration. Inset: Ratio of Young's modulus E of the ribbon to zero-field modulus E_0, against applied field (after Ref. 11).

thesizer (stable to better than 10^{-5} Hz) were used to produce the vertical magnetic field $H = H_{dc} + H_{ac} \cos(2\pi f t)$, with $H_{dc} = 1.20$ Oe and $H_{ac} = 1.08$ Oe (after compensation for the Earth's field as measured by a Hall probe). An MTI 1000 Fotonic Sensor, positioned 6 mm from the clamped base of the ribbon, was used to measure the horizontal displacement. The data collected were time series of voltages from the Fotonic Sensor, monotonically related to the displacement of the ribbon from the vertical. Time series of 100000 points, each over a period of approximately 39 min, were obtained for 60 different values of the driving frequency f. The ribbon was clamped so that it asymmetrically favored buckling on one side.

We have observed that a crisis occurs in our system as the driving frequency f is decreased through a critical value f_c. For $f \gtrsim f_c$, the motion of the ribbon is chaotic, oscillating around an initial buckled state and never passing the vertical position. For f slightly below f_c, the phase portrait for the chaotic motion typically remains for a long time in a region corresponding to the initial buckled state in what we term the *core attractor*. After a period of time, the trajectory bursts into a larger outer region and then returns to the core attractor, where it remains until the next burst. The bursts were observed to consist of oscillating around the initial buckled position, passing through an approximately vertical position (corresponding to 4.1 V in the signal), buckling on the opposite side, and then returning to the side of the initial buckling.

While a proper model of the ribbon motion must be a partial differential equation and thereby an infinite-dimensional system, our analysis of the experimental results is consistent with the existence of a three-dimensional model.

Phase portraits were constructed from the voltage as a function of time, $V(t)$, by a delay-coordinate construction into $\mathbf{R}^d$. In this method,[11] the voltage time-series values $V_i = V(i\Delta)$, where Δ is the sampling time, are arranged into data-state vectors whose d components at time $i\Delta$ are $(V_i, V_{i+n}, V_{i+2n}, \ldots, V_{i+(d-1)n})$. The tip of this vector traces a phase portrait in $\mathbf{R}^d$. The delay selected was the first minimum of the autocorrelation function of the voltage time series $\{V_i\}$, which was determined to be fifteen data sampling times (yielding a delay of 0.35 sec).

We measured the correlation dimension for the phase portrait for $f = 0.836$ Hz (just before the crisis—see below) by the maximum-likelihood method of Takens,[12] and again by a method incorporating treatment of undercounting effects near the attractor boundary.[13] Both gave values of 2.3 ± 0.1. In addition, a new local singular-value-analysis technique was applied to the same time series to determine the dimension of the manifold in which the dynamics lies.[14] The number of unit vectors needed to span the tangent spaces was estimated at various points of the phase portraits. The result was 3; i.e., the number of degrees of freedom required to describe the dynamics is 3. Finally, we found that embedding in $\mathbf{R}^5$ was sufficient to remove all apparent attractor self-intersections normally associated with projection effects.

Using a data-state vector $(X_1, \ldots, X_5)$, where $X_j(t) = V(t + 15j\Delta)$, we constructed Poincaré sections. The hypersurface of section, S, was defined by $S = \{(X_1, X_3, X_4, X_5) \mid X_1 = X_2 \text{ and } dX_1/dt > dX_2/dt\}$. Figure 2 exhibits projections of the four-dimensional sec-

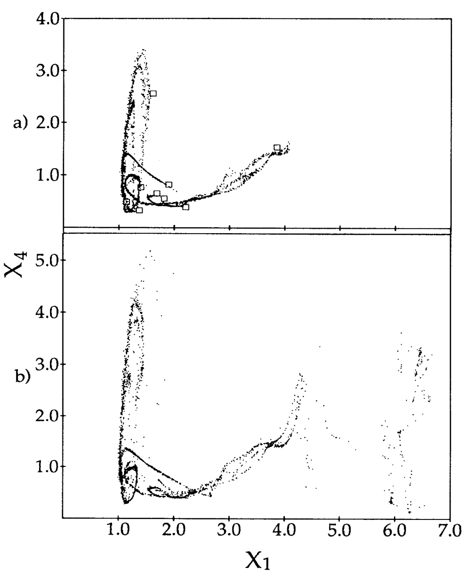

FIG. 2. Projections onto the X_1-X_4 plane of Poincaré sections using the hypersurface of section defined in text for (a) $f = 0.836$ Hz $> f_c$ and (b) $f = 0.830$ Hz $< f_c$. The open squares in (a) indicate where the period-9 orbit will appear at $f = f_c$.

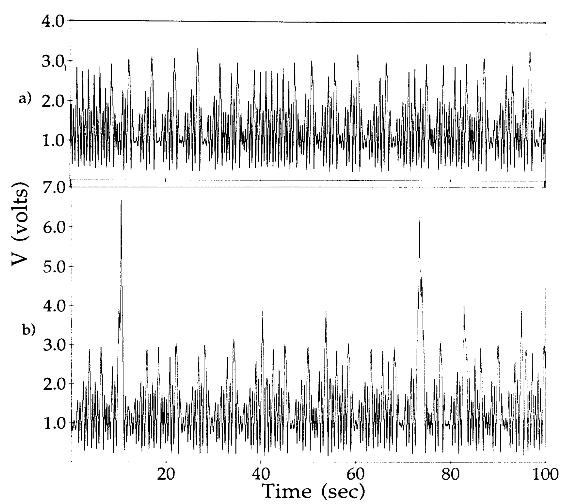

FIG. 3. Time series data for (a) $f = 0.836$ Hz $> f_c$ and (b) $f = 0.830$ Hz $< f_c$.

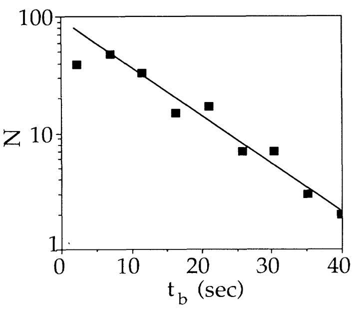

FIG. 4. Distribution of bursting times at $f=0.820$ Hz $<f_c$.

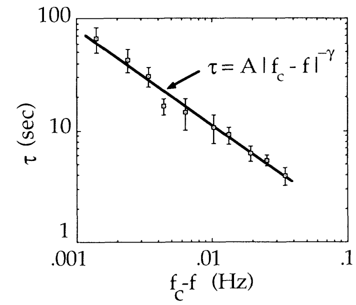

FIG. 5. Mean time between bursts τ vs $|f_c-f|$. Fit gives $\gamma=0.86\pm0.03$.

tions onto the X_1-X_4 plane before and after the crisis. Portions of the time-series data yielding the attractors in Fig. 2 are shown in Fig. 3.

Bursting was not observed at $f=0.835$ Hz but was observed at $f=0.834$ Hz, with only two bursts 30 min apart; hence the crisis occurred above $f=0.834$ Hz. At each value of $f<f_c$, the values of t_b, the time between bursts, exhibited distributions with Poisson tails (Fig. 4), for $t_b \gtrsim 10$ sec. The mean times between bursts τ, averaged over each time series of 39 min, for times between bursts longer than a short-time cutoff $t_0=11.5$ sec, were determined at several frequencies close to f_c. We cut off the times used in this determination of τ so that only the bursting times in the Poisson tails are included. The precise values of f_c and t_0 were chosen to minimize the scatter in the graph of $\log(\tau)$ vs $\log(f_c-f)$, shown in Fig. 5. In Fig. 5 the mean time clearly follows the power law $\tau \propto |f_c-f|^{-\gamma}$, as predicted in Ref. 9, with the best fit given by using a value of $f_c=0.8344\pm0.0003$ Hz, yielding $\gamma=0.86\pm0.03$.

In our Poincaré sections we have identified the existence of an unstable period-9 orbit [indicated in Fig. 2(a)], which appears on the attractor very prominently after the crises. In the time series we see that a burst is typically preceded by motion near this orbit. This is consistent with an important component of the theory,[9] that the intermittency crisis is mediated by the collision of the attractor with the stable manifold of an unstable periodic orbit which is not on the attractor for $f>f_c$.

The scaling of the mean time between bursts with $|f_c-f|$ is thought to arise from the fractal structure of the attractor in the region where it collides, as f is increased, with the stable manifold of the crisis-mediating unstable orbit (cf. Fig. 16 of Ref. 9). The assumption is that the core attractor shifts its position relative to this stable manifold nearly linearly with $|f-f_c|$, while the basic structure of the attractor changes little.[9] Thus, as f falls, more of the core attractor extends beyond the stable manifold of the crisis-mediating unstable orbit, increasing the probability of a burst and lowering the mean time between bursts.[14] This means that from the distribution of points near the outer edge of the attractor for $f \gtrsim f_c$ one can estimate how τ should scale with $|f_c-f|$. Accordingly, we drew a pair of parallel lines on a portion of the attractor (for $f=0.836$ Hz $>f_c$), separated by a distance ϵ, near a position where the core attractor is observed to pull off to the expanded attractor. One of the lines is tangent to the outer edge of the attractor near where it is later observed to burst out; the other intersects a portion of the attractor (see Fig. 6). We measured the number of data points, $N(\epsilon)$, between these lines for several values of ϵ.[15] Figure 7 shows a plot of $\log_2[N(\epsilon)]$ vs $\log_2(\epsilon)$ with slope $\gamma'=0.84\pm0.06$. This is consistent with the interpretation that $\gamma=\gamma'$ and with our value $\gamma \approx 0.86$.

We conclude that our experimental results have pro-

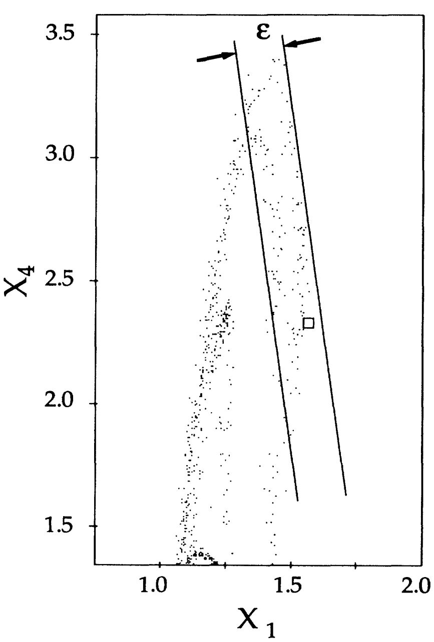

FIG. 6. Portion of the attractor in the $X_1=X_2$ surface of section, for $f=0.836$ Hz, slightly above f_c, seen in projection onto the X_1-X_4 plane. $N(\epsilon)$ is constructed by counting the points between the tangent line and lines parallel to it separated by ϵ. The ϵ value shown is the maximum used in our analysis. The open square marks one point on the period-9 orbit.

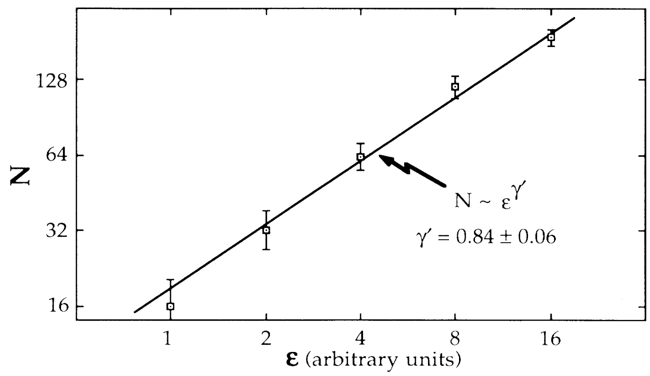

FIG. 7. $N(\epsilon)$ vs ϵ. The error bars are $\sqrt{N}$ and the bound on γ' is statistical.

vided evidence of crisis-induced intermittency in an infinite-dimensional mechanical system, with a scaling law for τ and dynamics consistent with the predictions of the theory.

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[15]This assumes that upon reentering the core the trajectory does not land near the stable manifold of the crisis-mediating unstable orbit, in which case it would be captured and quickly reejected. We have avoided contamination from these effects on τ by excluding the short ($t_b < t_0$) interburst times.