Experimental Determination of Total H^0-H^0 Scattering Cross Section at 2 K from Intrabeam Collisions

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The total H^0-H^0 scattering cross section at 2 K is inferred from intrabeam scattering to be about 100 Å^2. This value is somewhat higher than computations published during the past two decades. These results provide the first experimental clue regarding the magnitude of H^0-H^0 scattering at low temperatures.

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A number of calculations for the total scattering cross section in H^0-H^0 collisions at temperatures covering the range 0–5 K have been performed.[1-4] The predicted values of these cross sections, which are of interest to astrophysics[5] (transport phenomena), Bose-Einstein condensation,[6] hydrogen masers, polarized fusion,[6] and spin-physics research[7] (sets limits on polarized-hydrogen-gas density), differ by well over an order of magnitude. Pioneering theoretical work by Dalgarno,[5] for H^0-H^0 scattering at higher temperatures, showed that cross sections in diffusion have anomalously high values, and that they increase with decreasing impact energy (from 19×10^{-16} cm^2 at 100 eV to 41×10^{-16} cm^2 at 0.1 eV). Later calculations[2] yielded at 2 K, e.g., values of somewhat below 10^{-14} cm^2. Calculations by Friend and Etters[3] reveal, at 0 K, much lower values of 6.5×10^{-16} and 4.9×10^{-16} cm^2 for polarized and unpolarized H^0 gas, respectively (compared to the values of Allison and Smith[2] of 8.7×10^{-15} and 6.8×10^{-15} cm^2). Lhuillier[4] computed partial-wave phase shifts, from which the scattering cross section computed at 2.3 K is estimated to be about 3×10^{-15} cm^2.

In this Letter, the first experimental determination of the total H^0-H^0 scattering cross section at 2 K is presented. The data consist of observations of the dependence of H^0 beam focusing, by a strong magnetic field gradient, on the unfocused beam density. Focusing decreases as beam density increases to a point where collisionality prevents any focusing. It is shown that intrabeam H^0-H^0 scattering, in a beam with 2-K thermal spread, is the dominant collisional effect. The cross section is first inferred from equating the mean free path for intrabeam scattering to the size of the magnet at the focusing cutoff point, and then it is estimated from focusing reduction with increase in beam density for lower-density values. Both of these results strongly suggest that the total H^0-H^0 scattering cross section at 2 K is about 10^{-14} cm^2, a value close to that of Allison and Smith,[2] although their interaction potentials did not include adiabatic corrections (which were not available until later).

As part of a program to develop a ground-state mA source of $\vec{H}^-$ ions with polarized protons ($\vec{H}^-$), a cold, high-intensity, atomic-hydrogen beam source was built.[8] Figure 1 shows the experimental apparatus which is an improved version of that source. Atomic hydrogen is produced in a conventional room-temperature rf dissociator. The H^0 atoms then flow through a transition section in contact with liquid nitrogen, and into a copper accommodator channel which could be cooled to as low as 3.2 K. The accommodator is followed by a skimmer at 2.5 K, and ten charcoal-coated cryopanels at the same temperature, having a combined area of about 4500 cm^2. This tremendous pumping (of about 40000 l/sec for H_2 at these temperatures) ensures that scattering by any gas other than H^0 is insignificant. Immediately following this stack of cryopanels, there is a small cryostat, filled with liquid helium, which houses a superconducting solenoid whose entrance is 15.5 cm downstream from the accommodator exit. The solenoid consists of three coils in series with an i.d. of 9.4 cm and a length of 10 cm. The current in the outer two coils flows counter to the current in the middle coil resulting in a large magnetic field with strong gradients, which focus[9] H^0 atoms like conventional sextupoles.

With this atomic beam stage at an accommodator temperature of 6 K, without focusing, a peak H^0 density of 6.1×10^{11} cm^{-3} was measured via a residual gas analyzer (RGA) located 90.5 cm from the accommodator exit. This peak density is an improvement by a factor of 34 over the maximum density measured with the atomic beam in Ref. 8. When the accommodator channel was reduced to the configuration of Ref. 8, with a copper insert, the peak H^0 output was reduced exactly to that obtained previously. Furthermore, by using two different inserts in the flared section of the accommodator (and adjusting the "normal" controls of gas and rf power), the H^0 density measured at the RGA could be varied from as low as 2×10^9 cm^{-3} to as high as 6.1×10^{11} cm^{-3}. Time-of-flight measurements[8] of the velocity distribution showed that the beam had a most probable velocity of 680 m/sec and a FWHM of 196 m/sec corresponding to a *beam temperature of* 2.3 K. The perpendicular thermal spread is also 2.3 K as the experimentally determined sextupole acceptance angle indicated. (Previously a chopper and a sextupole magnet could be easily installed alternately.) No provisions exist for time-of-flight measurements, since the H^0 beam ve-

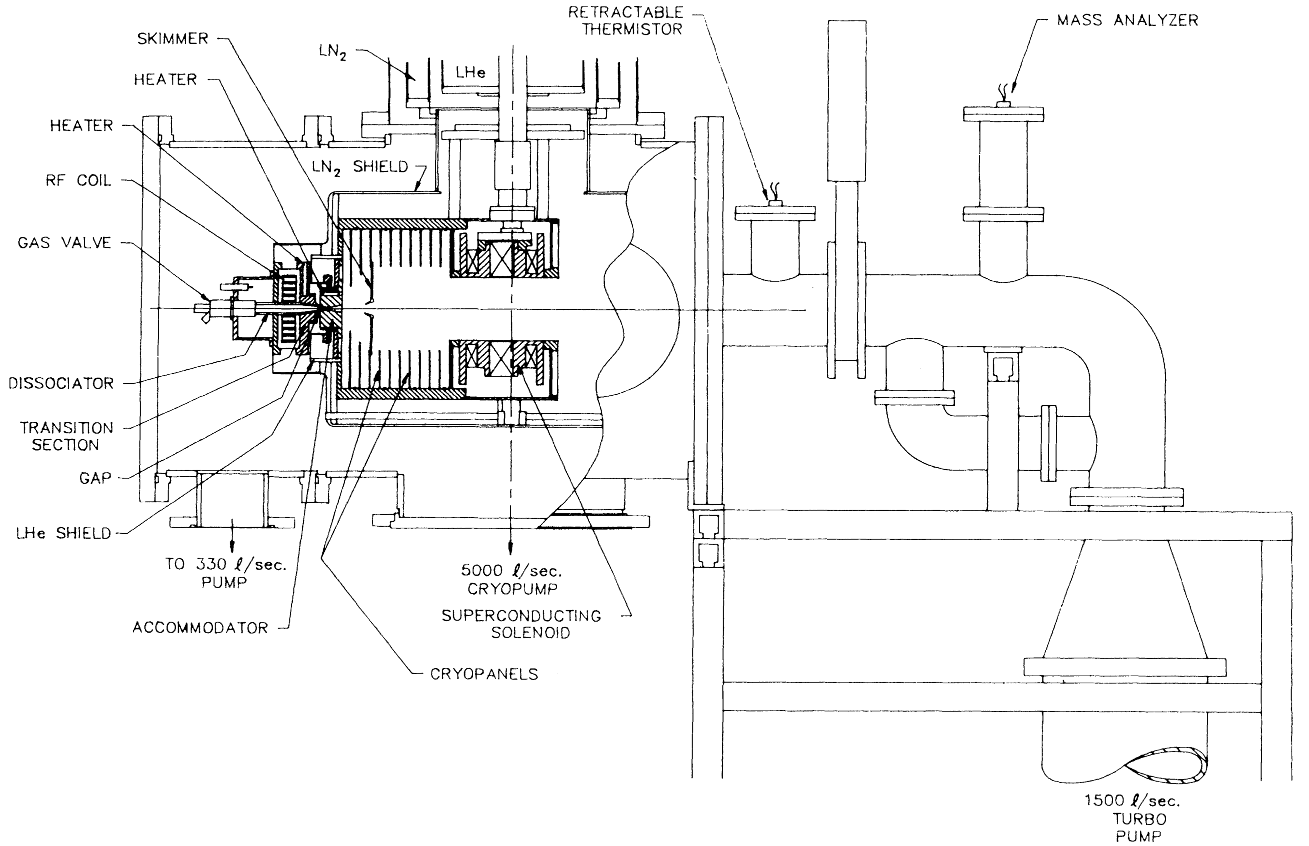

FIG. 1. Schematic of the experimental arrangement.

locity is expected to be the same as in Ref. 8 for the same accommodator temperature. The accommodator is designed to result in frictionally choked flow, which ensures that the outlet Mach number is independent of density; therefore subsequent to supersonic expansion, the final beam velocity distribution depends only on outlet Mach number and accommodator temperature.[10] Even though this design is based on the excellent agreement[8] between theory and experiment, an additional test was done to prove that the H^0 beam velocity in this setup is the same as that of the previous configuration, where time-of-flight measurements were done. In this check, the thermistor H^0 detector signal, which measures beam flux,[11] was compared to the H^0 density measurements and showed a linear relation. This indicates that the H^0 beam velocity is unchanged for all densities including those at which the H^0 beam velocity was measured. Furthermore, if the most probable velocity is indeed 680 m/sec, the atomic beam forward flux of 3.2×10^{20} H^0/sr sec matches the atomic beam flux of Belov *et al.*[12] A higher velocity would imply an even higher flux, which is very unlikely due to the similarity of this dissociator and its mode of operation to that of Belov *et al.*[12]

While the peak output from the atomic beam reached all projected parameters, the focusing of H^0 atoms exhibited an unexpected limitation. Figure 2 shows the "focusing factor" (the ratio of the H^0 density at the RGA with the solenoid peak magnetic field $B=4.38$ T to the H^0 density with $B=0$) versus the H^0 density at the RGA with $B=0$. Focusing is observed at lower densi-

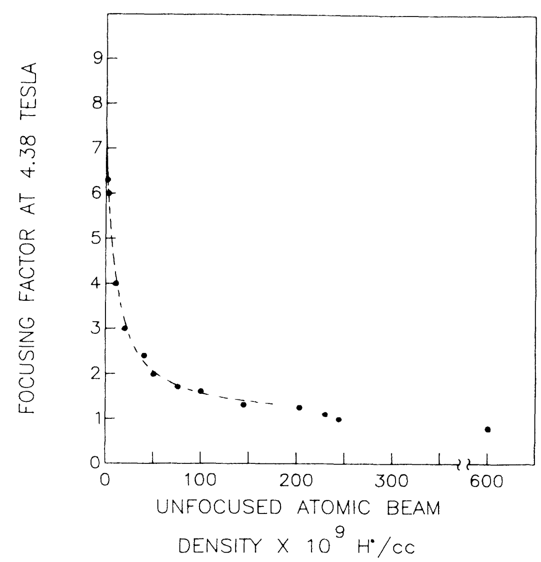

FIG. 2. Variation of the focusing factor (ratio of H^0 density at $B=4.38$ T to density at $B=0$) vs H^0 density at $B=0$. Dashed curve is a plot of Eq. (6) with the cross-sectional value of Eq. (7).

ties, but it falls off as density is increased. Eventually, a complete "cutoff" in focusing, i.e., the focusing factor becomes 1, is reached at an unfocused density of 244 $\times 10^9$ H^0/cm^3 at the RGA. Taking into account the $1/R^2$ falloff in density as one moves away from the accommodator, cutoff occurs at an H^0 density of 8.32 $\times 10^{12}$ cm^{-3} at the solenoid entrance. Any analysis and measurements presented here refer to the beam on axis; therefore, beam density is governed by $1/R^2$ falloff (any angular contributions[13] have a $\cos^n\theta$ dependence and $\cos^n 0 = 1$).

As it follows from the cryopanel description, intrabeam H^0-H^0 scattering is the only significant collisional process to prevent separation of atoms of opposite electron spin, and hence focusing. A cutoff in focusing results when the mean free path λ for H^0 scattering is reduced to the length of the solenoid, i.e., $\lambda = 10$ cm. From the simple relation

$$\lambda = 1/n\sigma, \tag{1}$$

the total scattering cross section σ can now be estimated. Since the density $n = 8.34\times 10^{12}$ cm^{-2} at cutoff, the estimated cross section is

$$\sigma = 1.2\times 10^{-14}\,\mathrm{cm}^2. \tag{2}$$

Computing σ from the Fig. 2 data can alternatively be done from the dependence of the focusing factor F_F on the unfocused density n_u. Downstream from the skimmer n_u varies as

$$n_u = n_0 (R_0/R)^2, \tag{3}$$

where n_0 and R_0 are the density and radius (distance from the accommodator exit) of a "freezing surface," a point beyond which the velocity distribution freezes, i.e., no further cooling occurs and intrabeam scattering (in absence of focusing) can be neglected. The focused beam has a rather complex functional dependence on R; its density n_f can be expressed in the following relation:

$$n_f = C_1 n_0 \left(\frac{R_0}{R}\right)^2 G(R) + C_2 \frac{n_0}{1 + n_0 \sigma M^{-1}(R - R_0)}. \tag{4}$$

The first term on the right-hand side of this expression describes a beam which has undergone supersonic expansion, following which, the beam is subjected to spin selection and to focusing which enhances the density by a factor $G(R)$. Since all measurements in Fig. 2 were done at a fixed magnetic field and at a fixed position, G (for gain) is a constant. However, because of focusing, collisions beyond the freezing surface can no longer be ignored. The second term on the right-hand side of Eq. (4) accounts for intrabeam collisions. It has its origin in a differential equation describing like-particle losses due to scattering $dn/dt = -n^2\sigma v_{\mathrm{th}}$, where v_{th} is thermal velocity. A transformation of this relation from time to R coordinates ($R = v_0 t$) results in an equation $dn/dR = -n^2\sigma/M$ describing collisional losses from a volume element as it moves downstream (v_0 is the most probable velocity, $M = v_0/v_{\mathrm{th}}$ is the Mach number). Solution of this equation subject to the constraint $n = n_0$ at $R = R_0$ yields the second term in Eq. (4). Since both elements in Eq. (4) have their origin in equations governed by linear operators, the principle of superposition can be used to combine contributions of both density falloff and collisions, with C_1 and C_2 as constants that account for the relative importance of each effect. Next, from Eq. (3), $n_0 = n_u (R/R_0)^2$ is substituted into Eq. (4), which is then divided through by n_u. Also, constants C_1, C_2, G, R, and R_0 are lumped, where appropriate, into K and C to obtain

$$\frac{n_f}{n_u} = F_F = K + \frac{c}{1 + n_u (R/R_0)^2 \sigma M^{-1}(R - R_0)}. \tag{5}$$

Equation (5) shows the functional dependence of the focusing factor on n_u with all other quantities being constant. From the abundant experience[14] with atomic beams undergoing supersonic expansion, modern sources have been designed with a skimmer just beyond the freezing-surface position R_0; in this device $R_0 = 5$ cm. Constants K and C can be evaluated from extrapolation of the Fig. 2 data: As $n_u \to \infty$, $F_F \to 1$, i.e., no focusing; hence $K = 1$. Close to the other extreme ($n \to 0$), the lowest-density data point is $F_F = 6.3$ at $n_u = 2\times 10^9$ cm^{-3}. Substituting these values and $M = 3.4$ into Eq. (5) yields $C = 5.3(1 + 1.65\times 10^{13}\sigma)$. Thus, Eq. (5) becomes (in cgs units)

$$F_F = 1 + \frac{5.3(1 + 1.65\times 10^{13}\sigma)}{1 + n_u 8.24\times 10^3 \sigma}. \tag{6}$$

A least-squares fit of the Fig. 2 data by Eq. (6) using the MIGRAD code (performed by D. Weygand, Applied Math, BNL) yields

$$\sigma = 1.25\times 10^{-14}\,\mathrm{cm}^2. \tag{7}$$

Total H^0-H^0 scattering cross-sectional values obtained from either focusing cutoff [Eq. (2)] or focusing attenuation [Eq. (7)] due to intrabeam scattering are in excellent quantitative agreement. RGA calibration[8] has a possible error of $\pm 20\%$, and from Eq. (1) it is obvious that σ has the same error. This is the largest error in the Fig. 2 data which were very reproducible; e.g., $F_F = 3$ was recorded for $n_u = 2.0\times 10^{10}$ and 1.98×10^{10} cm^{-3} (the overlap value of two different inserts in the accommodator flared section). The RGA, which is made by Riber, has a wide dynamic range and a very (μsec) fast response. Saturation effects start to occur at a density which is a factor of 100 higher than the highest measured. The background pressure in this system, as measured by an ionization gauge mounted on a tube connected to the vacuum chamber, is 10^{-8} Torr. Pressure in the volume enclosed by the superconducting solenoid at 4.2 K and the cryopanels at an even lower temperature is most likely orders of magnitude lower. Additionally, all measurements were done with the leading edge of the beam pulse (at a duty factor of 0.01 with 2 sec between pulses). Scattering by H_2 can be discounted for the following reasons: (1) Very little H_2 is produced since modern dissociators are close to 100% efficient. Walraven and Silvera[15] measured a degree of dissociation as high as 94%. Correlation of input gas pulses with beam outputs in Refs. 8 and 12 corroborate those results. (2) The highest possible residual pressure that the leading edge of the beam encounters is about 8×10^{-7} Torr inside the magnet (vapor pressure of H_2 at 4.2 K). At this pressure the H_2 density is 3 orders of magnitude lower than that of the beam atoms. (3) The upper limit on the H^0-H_2 scattering cross section can be taken to be 5.8×10^{-15} cm^2. This value was measured by Harrison.[16] Later studies[17] proved it to be too large. Thus, the combination of the H_2 density and the H^0-H_2 scattering cross section strongly suggests that H_2 effects can be neglected. (4) Finally, depolarization effects, e.g., spin exchange, are unimportant due to their lower cross sections (orders of magnitude).

At 2 K, Allison and Smith[2] predict $\sigma=8.7\times10^{-15}$ cm^2 for $\vec{H}^0$ gas. Since polarization is never 100%, the value of σ to which Eqs. (2) and (7) should be compared is slightly lower (but definitely higher than 6.8×10^{-15} cm^2 for unpolarized H^0). Lhuillier[4] performed a partial-wave analysis for $\vec{H}^0$-$\vec{H}^0$ collisions in the temperature range 0.04–10 K. From the collision phase-shift results (Fig. 1 in Ref. 4, with $k^*=0.806$ at 2.3 K) the scattering cross section $[\sigma=(4\pi/k^2)\sum_l(2l+1)\sin^2\delta_l]$ can be calculated to be $\sigma\simeq3\times10^{-15}$ cm^2 at 2.3 K. The values of Eqs. (2) and (7) are most likely lower because of contributions which can be only qualitatively[18] addressed. In Eq. (2), λ was arbitrarily chosen to be the physical length of the solenoid (10 cm). However, by equating λ in Eq. (1) to a path length during which atoms are subjected to significant focusing effects, e.g., an e-folding distance of total B, adds 1.857 cm to each side of the solenoid, and yields in Eq. (2) $\sigma=8.75\times10^{-15}$ cm^2. Similarly, since the solenoid focusing has a radial dependence, and because Eqs. (3)–(6) refer to the beam on axis, there are small underestimates of both the effective density and intrabeam scattering length, based on which qualitative arguments can be invoked to adjust the value of σ from Eq. (7) to slightly below 10^{-14} cm^2.

Chronologically, the initial sequence of events was determined by the purpose of this endeavor to maximize H^0 output while minimizing scattering to optimize yield of short H^0 pulses (intrabeam scattering was considered to be insignificant[7] throughout the design process). First, H^0 output was optimized, after which the magnet was energized. Consequently, the first data point to be recorded was the highest value of n_u in Fig. 2. Since no focusing was observed, a long series of experiments was conducted at maximum H^0 output to ascertain that every component functioned properly and that every parameter reached its expected value. In the absence of any other explanation for the lack of focusing, no choice was left but to consider intrabeam scattering. The next series of experiments is described in this paper. Their data support σ values close to 100 Å^2, which are larger than predicted values used in astrophysics, as well as in the fields of hydrogen masers, polarized fusion, and spin-physics research. These experimental results suggest that more theoretical work is needed.

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