Inflation Can Save Cosmic Strings

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A new scenario of cosmic strings is presented which is free from the gravitational-radiation constraints imposed on their line density by the primordial nucleosynthesis and the timing data of a millisecond pulsar. In this scenario the phase transition is induced nonthermally during the inflation so that it is not necessary to assume a Friedmann-Robertson-Walker universe in thermal equilibrium at the outset.

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The cosmic-string scenario is one of the major proposed formation mechanisms of the large-scale structures in the Universe.[1-3] One of the remarkable features of this scenario is that it has essentially only one parameter, namely, the line density of the string μ. It should satisfy $G\mu = \mu/M_{\rm Pl}^2 \approx 10^{-6}$ in order to meet various observational requirements, where G and $M_{\rm Pl} = 1.2\times 10^{19}$ GeV are the gravitational constant and the Planck mass, respectively.[1-5] This value, however, causes some problems. We first point out three problems in the conventional scenario, and then present a new scenario which resolves them by abandoning the thermal phase transition and utilizing inflation. That is, we shall show that inflation is not only compatible with the cosmic-string scenario by may also be a *necessary ingredient* of it.

The first problem we would like to point out concerns the implicit assumption of thermal equilibrium in previous literatures.[1-3,6] Consider the following Abelian Higgs model which allows a string solution:

$$\mathcal{L} = \tfrac{1}{2}(D_\mu\chi)^\dagger(D^\mu\chi) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - V[\chi]\,,$$
$$V[\chi] = (\lambda/4)(|\chi|^2 - v^2)^2\,,$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu - ieA_\mu$ with e the gauge coupling constant. Since the line density is $\mu \approx v^2$, v should be $\sim 10^{16}$ GeV for $G\mu \approx 10^{-6}$. This means that the critical temperature of the *thermal* phase transition is $T_c \sim [\lambda/(\lambda+e^2)]^{1/2} v \sim ev \sim 10^{15-16}$ GeV. In the conventional scenario, it has been taken for granted that the scalar field χ was already in thermal equilibrium well before $T = T_c$ so that string formation is described by the Kibble mechanism.[6] However, in order to attain thermal equilibrium from an arbitrary initial condition, it is necessary that the particle interaction rate with χ, Γ, exceed the cosmic expansion rate H.[7] That is,

$$\Gamma = \langle n\sigma c\rangle \approx \frac{N}{\pi^2} T^3 \frac{\alpha^2}{T^2} > H = \left(\frac{8\pi^3 g_* T^4}{90 M_{\rm Pl}^2}\right)^{1/2}.$$

Hence T should satisfy

$$T < T_{\rm eq} \equiv 3\times 10^{14} \left(\frac{\alpha}{0.02}\right)^2 \left(\frac{N}{10}\right) \left(\frac{g_*}{200}\right)^{-1/2} \text{GeV}\,,$$

where α, N, and g_* are (gauge) coupling strength, number of relativistic modes interacting with χ, and total relativistic degrees of freedom, respectively. Though all these parameters are model dependent, it is not always true that $T_{\rm eq}$ is as large as T_c.

A more serious problem caused by the high critical temperature is the well-known incompatibility[8] of the string scenario with inflation, which is now widely accepted as the sole known mechanism to explain the observed homogeneity, isotropy, and flatness of the Universe.[9] Thus in order for the conventional scenario to work, among various possible initial states of the classical Universe, we must single out a very particular one, namely, the Friedmann-Robertson-Walker universe in thermal equilibrium with its energy density equal to the critical density with high accuracy. In other words, one should give up the scenario based on the thermal phase transition to study cosmic strings in more realistic cosmologies.

For the purpose of overcoming the second problem, some people have proposed mechanisms of nonthermal phase transitions such as the "inflation"-induced phase transition[10,11] or the curvature-induced phase transition.[12] The latter mechanism is especially plausible since it is a natural consequence of reanalyzing the phase transition in the curve spacetime.

Finally, in addition to the above two difficulties, to which a natural resolution has been proposed, there is yet another difficulty, which may be a problem for the conventional cosmic-string scenario depending on the result of numerical simulations, and no resolution has been proposed yet, that is, constraints on the line density imposed by the gravitational radiation background from string loops.

We can calculate the spectrum in terms of the number density of loops and the radiation spectrum of a single loop. The former is give by the scaling solution of the loop distribution,[2,3,13-15] which gives the number density of loops with radius $R \sim R + dR$ at time t as

$$n(R,t)dR = \frac{\nu}{R^{5/2} t^{3/2}} dR\,,$$

in the radiation-dominated phase, where ν is a numerical coefficient determined by numerical simulations of string networks. As for the latter, it has been shown that a loop with radius R emits gravitational radiation whose typical angular frequency and lifetime are R^{-1} and $\tau = R/\gamma G\mu$, respectively, where $\gamma \approx 5$.[16,17]

We may obtain some constraints on $G\mu$ with the spectrum thus calculated from the primordial nucleosynthesis and the timing data of a millisecond pulsar. From the upper bound on the cosmic expansion rate at the primordial-nucleosynthesis era ($t \sim 1$ sec) we obtain[18]

$$G\mu < 2\times 10^{-5}\left(\frac{\gamma}{5}\right)\left(\frac{\nu}{0.01}\right)^{-2}\left(\frac{\beta}{10}\right)^{-2}, \tag{1}$$

where β is the ratio of the length to the average radius of a loop. From the timing data of the millisecond pulsar PSR 1937+21, we obtain the following constraint[17,19] depending on the duration of the observation D,

$$G\mu < 4\times 10^{-6}\left(\frac{\gamma}{5}\right)\left(\frac{\nu}{0.01}\right)^{-2}\left(\frac{\beta}{10}\right)^{-2}\left(\frac{2\pi}{D/(1\,\mathrm{yr})}\right)^{8}. \tag{2}$$

The observation began in November 1982[20] and the last factor in the inequality (2) is now becoming as large as unity.

Meanwhile, a recent development in the numerical simulation of string networks[15] has shown that the quantitative feature of the scaling solution is considerably different from the naive one in which the typical curvature scale of a long string is identified with the horizon scale.[2] That is, there appear as many as some tens of long strings in the horizon volume and their correlation length is so small that the average radius of loop formed at time t is given by $R = \epsilon t$ with $\epsilon \sim 10^{-3}$. Consequently the parameter ν is 10 times larger than the previously reported value[5] 0.01, or $\nu = 0.1$. Thus, taking this result literally, the conventional scenario with $G\mu \approx 10^{-6}$ seems to be ruled out. However, we should allow for some numerical ambiguities in γ, ν, β, and ϵ, taking into account the result of other groups' numerical simulations.[14] Hence it is too early to make a definite conclusion about this problem. Nevertheless it is true that the constraint on the line density would become much more stringent if the timing data of the pulsar should remain quiet in the future too.

Therefore, in this paper we regard the gravitational-radiation constraints on the line density as a problem and seek a way out below. First recall that the constraint (2) is derived from gravitational waves with frequency $\sim D^{-1} \sim 10^{-1}\ \mathrm{yr}^{-1}$ today.[20] There are various sizes of loops which may contribute to waves of this frequency. Some of them are small ones which originally emitted high-frequency gravitational waves. Their frequency was red shifted to the observed frequency in the course of cosmic expansion. But their energy density also suffered a large red shift at the same time. Hence the contribution from very small loops is minor. On the other hand, larger loops also contribute to this frequency, for they become smaller and emit arbitrarily high-frequency waves as they decay. Some of these waves may just red shift to the frequency $\sim D^{-1}$ today. The larger the original loop size is, the later they decay so that their contribution is not red shifted very much. However, larger loops are much smaller in number. Consequently their contribution is also minor.

Therefore the loops which dominantly contribute to the gravitational waves with frequency $\sim D^{-1}$ today are intermediate-sized loops whose radiation just before their decay time has red shifted to this frequency today. Their size and formation epoch are given, respectively, by

$$R \approx \frac{D^2}{(2\pi)^2\gamma G\mu t_{\mathrm{eq}}}\left(\frac{t_{\mathrm{eq}}}{t_0}\right)^{4/3} = (1.1\times 10^{11}\ \mathrm{cm})\left(\frac{D}{5\ \mathrm{yr}}\right)^2 \Omega^{-2/3},$$

$$t \approx (6\times 10^{2}\ \mathrm{sec})\left(\frac{5\times 10^{-3}}{\epsilon}\right)\left(\frac{D}{5\ \mathrm{yr}}\right)^2 \Omega^{-2/3},$$

where t_{eq}, t_0, and Ω are the equipartition time, present age, and density parameter of the Universe, respectively. Considering the effect of larger loops which may contribute to gravitational waves in the range of interest through the higher-order harmonics, we shall take the formation epoch of loops relevant to the pulsar constraint to be earlier than $t \sim 10^4$ sec.

This means that if there is yet no significant loop formation at this epoch, we are free from the constraints (1) and (2). Such suppression can be realized if the density of long strings is so small that they cannot intersect with each other frequently enough for their distribution to relax into the scaling solution.

Here we state that inflation can provide such a dilution mechanism. Indeed, if the phase transition takes place in a late inflationary stage, the subsequent exponential expansion may moderately dilute the string density as well as conformally stretch the long strings produced. After inflation, self-intersection of a long string starts when its curvature scale reenters the Hubble radius and intersection of two long strings practically starts when their density becomes as large as one per Hubble volume. Then loop production sets in to make the system relax into the scaling solution.

Hence what is necessary for a successful scenario is that the beginning of significant loop production is late enough to suppress the production of the small loops relevant to the gravitational-radiation constraints but early enough to produce the loops responsible for galaxy formation, which are produced just after the matter domination, according to the recent numerical simulation.[15] In the conventional scenario it is evident that there is no room for such an advantageous scenario. However, it is possible to obtain such a model if we make use of the mechanisms of nonthermal phase transitions which have been proposed to solve the inflation-string problem.[10-12]

Here let us consider the curvature-induced phase-transition scenario[12] as an example, though we could proceed with the following arguments in the inflaton-induced phase-transition scenarios[10,11] as well.

As discussed in our previous paper,[12] it is likely that the scalar field χ is nonminimally coupled with the scalar curvature $\mathcal{R}$ and that the phase transition takes place due to time variation of $\mathcal{R}$ rather than that of temperature. Thus the assumption of thermal equilibrium is unnecessary. Here we consider, for definiteness, the case where χ is conformally coupled with curvature in the chaotic-inflation scenario in which inflation is driven by a minimally coupled massive scalar field ϕ. That is, their potentials read

$$V_c[\chi]=(\lambda/4)(|\chi|^2-v^2)^2+\tfrac{1}{12}\mathcal{R}|\chi|^2,\quad U[\phi]=\tfrac{1}{2}m^2\phi^2,$$

respectively, where m is the inflaton's mass which should be smaller than $m\approx 10^{14}$ GeV to avoid the production of too much density fluctuation.[21]

In the inflationary stage $\phi(t)$, the scale factor $a(t)$, and $\mathcal{R}(t)$ are given by

$$\begin{aligned}\phi(t)&=\phi_0-\frac{mM_{\rm Pl}}{2(3\pi)^{1/2}}t,\\ a(t)&=a_0\exp\left\{\frac{2\pi}{M_{\rm Pl}^2}[\phi_0^2-\phi(t)^2]\right\},\\ \mathcal{R}(t)&=\frac{16\pi m^2}{M_{\rm Pl}^2}\phi(t)^2-2m^2\cong\frac{16\pi m^2}{M_{\rm Pl}^2}\phi(t)^2,\end{aligned}\tag{3}$$

respectively, where $\phi_0\approx M_{\rm Pl}^2/m$ is the typical value of ϕ at the Planck epoch.[12] Thus the scalar curvature decreases as ϕ evolves towards $\phi=0$. Therefore $\chi=0$ becomes unstable when $V_c''[\chi=0]<0$ or $\mathcal{R}/6<\lambda v^2$. However, while ϕ satisfies $H^2(\phi)>|V_c''[\chi=0]|$, quantum fluctuations intrinsic to the exponentially expanding spacetime dominate χ's evolution.[11,22] When ϕ becomes as small as to satisfy $H^2(\phi)=-V_c''[\chi=0]$, χ's evolution is governed by its potential so that its phase is practically fixed and we may predict where cosmic strings will appear after the phase transition. Thus the phase-fixing epoch of χ is given in terms of ϕ as

$$\phi=30\sqrt{\lambda}m_{14}^{-1}v_{16}M_{\rm Pl}\equiv\phi_{\rm st},\tag{4}$$

where $m_{14}=m/(10^{14}\text{ GeV})$ and $v_{16}=v/(10^{16}\text{ GeV})$.

One can obtain constraints on the e-folding numbers of inflation n after ϕ becomes as small as $\phi_{\rm st}$ from the two requirements stated above. To do this, however, we must know the average density of strings at the phase-fixing epoch. Let us now estimate the mean separation of strings at that time. First consider a point with arbitrary values of $\chi=\chi_R+i\chi_I$ and $\nabla\chi$. We can always make a gauge transformation so that χ has only the real part there. Then since a string is the loci of $\chi=0$, the following inequality must be satisfied in order that a string runs within a distance of r from this point:[11]

$$r\left|\frac{\nabla\chi_I}{|\nabla\chi_I|}\times\nabla\chi_R\right|>|\chi_R|.$$

Conversely, there is a string with the probability of $\frac{1}{2}$ if this inequality is satisfied. Thus the mean separation of "strings" d is given by $d^2\approx 3\langle\chi_R^2\rangle/\langle(\nabla\chi_I)^2\rangle$. Since χ's behavior is governed by quantum fluctuations during $H^2(\phi)>V_c''[\chi=0]>-H^2(\phi)$, when it has the power spectrum $\chi_{Rk}^2=\chi_{Ik}^2=H^2/2k^3$, $\langle\chi_R^2\rangle$ and $\langle(\nabla\chi_I)^2\rangle$ at the phase-fixing epoch are reasonably estimated as

$$\begin{aligned}\langle\chi_R^2\rangle&\approx\int_{He^{-l}}^{H}\frac{H^2}{2k^3}\frac{d^3k}{(2\pi)^3},\\ \langle(\nabla\chi_I)^2\rangle&\approx\int_{He^{-l}}^{H}\frac{H^2}{2k}\frac{d^3k}{(2\pi)^3},\end{aligned}\tag{5}$$

respectively, where l is the e-folding number of cosmic expansion while quantum fluctuations dominate χ's evolution, which is given by $l=\lambda v^2/m^2$. As will be revealed shortly, $l\approx 10^2$ in our model. We may estimate d from (5) as $d\approx 3\sqrt{2}/m$.

Knowing the mean separation of strings at the phase-fixing epoch, we may now derive the constraints on n. First, in order to avoid significant loop production before $t\sim 10^4$ sec, we demand the line density of long strings to be less than ~ 1 in the Hubble volume. Then n must satisfy

$$n\gtrsim 43+\tfrac{1}{3}\ln T_{R9}+\tfrac{1}{3}\ln m_{14},$$

where T_{R9} is the reheat temperature in units of 10^9 GeV. Meanwhile for successful galaxy formation, there should exist as many as ~ 10 long strings in the Hubble volume at the equipartition time, so that

$$n\lesssim 51+\tfrac{1}{3}\ln T_{R9}+\tfrac{1}{3}\ln m_{14}.$$

Using (3) and (4), the above inequalities are translated to a constraint on the model parameter λ as

$$7\times 10^{-3}m_{14}^2v_{16}^{-2}<\lambda<9\times 10^{-3}m_{14}^2v_{16}^{-2}.$$

Thus our model predicts the value of λ with an accuracy of $\approx 30\%$.

In conclusion, we have argued that the nonthermal phase transition during inflation surpasses the thermal phase transition as the formation scenario of cosmic strings, because it can simultaneously resolve the three problems of the conventional scenario, namely, the assumption of thermal equilibrium of the χ field before the phase transition, which is not necessarily justified; the incompatibility with inflation; and the gravitational-radiation constraints of loops on their line density which may be a problem for the conventional scenario according to the recent numerical simulation by Albrecht and Turok.[15] As a specific example, we have considered the curvature-induced phase-transition scenario and shown that it is both natural and successful. Finally we note that in the present scenario there may well exist two mechanisms of generating density fluctuations simultaneously, namely, relic quantum fluctuations due to inflation and those produced by cosmic strings. Hence it is worthwhile to investigate their combined effects on the formation of large-scale structure.

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