Gauge Field, Aharonov-Bohm Flux, and High-T_c Superconductivity

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In a spin-$\frac{1}{2}$ Heisenberg model with short-range antiferromagnetic order, a hole making a closed loop on one sublattice is subject to a slowly varying spin-quantization axis and picks up a phase equal to half the solid angle subtended by the spin orientation around the loop. The phase can be represented by an Aharonov-Bohm flux resulting in a U(1) gauge theory. For a finite hole density this model leads to superconductivity even in the presence of Coulomb repulsion. The gauge field also enhances low-energy particle-hole excitations, leading to a $T^{4/3}$ law for the normal-state resistivity.

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It is commonly accepted that the essential physics of the copper-oxide superconductors can be modeled by two-dimensional sheets of copper and oxygen orbitals. The insulating compounds consist of spin-$\frac{1}{2}$ local moments on the copper sites and doping introduces holes on the oxygen orbitals. Anderson[1] and Zhang and Rice[2] argue that a symmetrized orbital of the oxygen hole forms a singlet with the copper moment. The resulting complex can be considered a hole in a square lattice of copper orbitals, and described by the one-band t-J model where t describes nearest-neighbor hopping, subject to the constraint of no double occupation on any site, and J is the exchange. The inclusion of direct oxygen-oxygen overlap leads to a substantial further neighbor hopping between the singlet complexes due to direct overlap and to an expansion of the size of the singlet. Thus we propose that the one-band model should be extended to a t-t'-J model, where t_{ij} and t'_{jk} denote hopping on the opposite and the same lattice, respectively. (We divide the square lattice into A,B sublattices as labels only, without assuming Néel ordering.) t and t' are strongly renormalized and are very difficult to compute from bare values given by band calculations.

In a locally antiferromagnetic (AF) environment, the t and t' terms have very different physical consequences. A hole hopping from the A to B sublattice leaves behind a misoriented spin and therefore destroys the local AF order. By the same token, a hole can hop coherently only on the same sublattice, so that $t \gg J$ produces a $t' \approx J$.[3] In this paper we assume that the t term has done its job in disordering the spins and we focus on the effect of t' which we expect to be $\gtrsim J$ with contributions from both direct hopping and via t as mentioned earlier.

A number of workers[4-7] have pointed out that the spin fluctuation and its coupling to the holes should be described by a compact U(1) gauge theory. In particular, Weigmann[6] has studied the t'-J model with disordered spin (which he denotes as a quantum paramagnet). He pointed out that holes on opposite sublattices couple to a U(1) gauge field which represents spin fluctuations with opposite charges, resulting in superconducting pairing. Wen[7] recently developed a phenomenological description with similar results. The treatment so far is restricted to a few holes moving in a spin background. In this paper we first reconsider this limit and provide a description of the gauge field in terms of the instantaneous sublattice magnetization. This physical picture will prepare us for the second part of this paper, which treats the t'-J model with a finite concentration of holes.

We employed the Schwinger-boson–slave-fermion formalism to handle the constraint of no double occupation; i.e., in the t and t' terms the fermion operator is $c_{i\sigma}^\dagger = \hat{f}_i \hat{b}_{i\sigma}^\dagger$, where $\hat{b}_{i\sigma}$ is a boson operator which carries the spin index and $\hat{f}_i^\dagger$ is a spinless fermion operator which creates a hole on site i relative to half filling. We restrict the Hilbert space to the constrained subspace satisfying $\sum_\sigma \hat{b}_{i\sigma}^\dagger \hat{b}_{i\sigma} + \hat{f}_i^\dagger \hat{f}_i = 1$.

The quantum partition function can be written as a functional integral, $Z = \int db_{i\sigma} df_i d\lambda_i \exp(-\int_0^\beta L\, d\tau)$, over complex $b_{i\sigma}$, Grassmanian f_i, and time-independent Lagrange multiplier field λ_i, and $L = L_J + L_{t'} + L_\lambda$, with

$$L_J = \frac{1}{2}\sum_j \left(b_j^\dagger \frac{\partial}{\partial\tau} b_j - \frac{\partial b_j^\dagger}{\partial\tau} b_j \right) + J \sum_{\langle ij\rangle} b_i^\dagger \sigma b_i \cdot b_j^\dagger \sigma b_j , \quad (1)$$

$$L_{t'} = \frac{1}{2}\sum_j \left(f_j^* \frac{\partial}{\partial\tau} f_j - \frac{\partial f_j^*}{\partial\tau} f_j \right) + \sum_{ij} t'_{ij} f_i f_j^* b_i^\dagger b_j + \text{c.c.} , \quad (2)$$

and $L_\lambda = \sum_j i\lambda_j (b_j^\dagger b_j + f_j^* f_j - 1)$. We have used the complex spinor notation $b_j = (b_{j\uparrow}, b_{j\downarrow})$ so that $\hat{\mathbf{\Omega}}_j = b_j^\dagger \sigma b_j$ represents the instantaneous spin orientation (or quantization axis) on site j. Note that $b_j \to b_j e^{i\theta(j,\tau)}$ leaves $\hat{\mathbf{\Omega}}_j$ invariant. If holes are absent, $b_j^\dagger b_j = 1$, and we can parametrize $b_\uparrow = \cos(\theta/2)$, $b_\downarrow = \sin(\theta/2) e^{-i\phi}$, where θ, ϕ are the Euler angles of $\hat{\mathbf{\Omega}}$. For any pair of spinors we have the identity[8]

$$b_1^\dagger b_2 = e^{i\hat{\omega}_{12}/2} \left| \tfrac{1}{2}(1 + \hat{\mathbf{\Omega}}_1 \cdot \hat{\mathbf{\Omega}}_2) \right|^{1/2} , \quad (3)$$

where $\hat{\omega}_{12}$ is the solid angle subtended by the unit vectors $\hat{\mathbf{z}}$, $\hat{\mathbf{\Omega}}_1$, and $\hat{\mathbf{\Omega}}_2$. Using Eq. (3) we can readily see that the first term in Eq. (1) reproduces Haldane's Berry phase for a path-integral representation of spin $\frac{1}{2}$. Haldane[9] has shown in a large-S expansion that L_J reduces to the nonlinear σ model in the long-wavelength limit, i.e., $L_J = \int d\mathbf{r}\, c g^{-1} (\partial \hat{\mathbf{\Omega}} / \partial x_\mu)^2$, where $x_0 = c\tau$, $\hat{\mathbf{\Omega}}(r)$ is a unit vector corresponding to the local sublattice magnetization, c is the spin wave velocity, and g is an effective coupling. For $g > g_c$, the σ model has a disordered ground state[10,11] with a gap $2\Delta_s$ for triplet excitations and a correlation length $\xi_s = c/2\Delta_s$.

Now we are ready to interpret the Lagrangian in the presence of a few holes. We note that $L_{t'}$ corresponds to a tight-binding Hamiltonian on the A and B sublattices separately with a time-dependent hopping matrix element $t'_{ij} b_i^\dagger(\tau) b_j(\tau)$. Since i and j are on the same sublattice, b_i is slowly varying and corresponds to $\hat{\Omega}(r)$ and $-\hat{\Omega}(r)$ on the A and B sublattices, respectively. Using Eq. (3) we see that $b_i^\dagger(\tau) b_j(\tau) \approx e^{i\hat{\omega}_{ij}/2}$. It is natural to associate a lattice gauge field A_{ij} on the bond ij equal to $\hat{\omega}_{ij}/2$. The associated time-dependent magnetic flux through an elementary plaquette in sublattice A is then equal to the solid angle on the unit sphere subtended by the four $\hat{\Omega}(r,\tau)$ on the corners of the plaquette. *Thus we see that the fluctuations of the* $\hat{\Omega}$ *field gives rise to an effective electromagnetic field which acts on the holes.* It is also clear that the holes on sublattice B couple to the gauge field with a "charge" of opposite sign since the solid angle swept out by $-\hat{\Omega}$ is of opposite sign. Another way of understanding the sign difference is that the gauge field is associated with the *staggered* quantization axes.

We remark that while L obeys local gauge invariance $b \to b e^{i\theta}$, $f \to f e^{i\theta}$, $L_{t'}$ does not. Using the constraint, the first term of Eq. (1) becomes $-iA_0(f_j^\dagger f_j + b_j^\dagger b_j)$, where $A_0 = i b_j^\dagger \partial b_j/\partial\tau$. The $A_0 f_j^\dagger f_j$ term can be added to Eq. (2) to produce $\partial/\partial\tau \to \partial/\partial\tau - iA_0$ and a gauge-invariant Lagrangian in the limit of zero hole density. The time component A_0 can be related to the spin structure around a space-time plaquette in the same way as we did for a plaquette in space.

Now we can give a physical picture of the pairing mechanism discussed by Wiegmann. Consider the Feynman paths of two particles on opposite sublattices which begin and end close to each other. An Aharonov-Bohm phase factor $\exp(\pm i\oint \mathbf{A}\cdot d\mathbf{l})$ is associated with each path and the propagation amplitude will be suppressed by the factor $\exp(i\oint \mathbf{A}\cdot d\mathbf{l}) = \exp(i\phi)$, where ϕ is the total flux enclosed by the loop formed by the two paths. If the spins are disordered we expect strong suppression of the propagation amplitude, unless the two paths are always within ξ_s of each other. Thus the pair on opposite sublattices are bound and propagate essentially as a free boson. A finite density of these pairs will Bose condense. Note that the pairing mechanism relies on coherent propagation on a length scale $\gg \xi_s$ and is inherently associated with the disordered state. However, this state is analogous to the bipolaron limit, in that T_c is the Bose-condensation temperature of tightly bound pairs and even above T_c a finite energy is required to break up the pair into single-particle charged excitations. This does not agree with tunneling experiments which show a gap vanishing above T_c. Clearly this picture is valid only if the average distance between holes is larger than ξ_s, because the exchange between holes has been ignored. Experimentally it appears that in order to suppress the Néel ordering, sufficient density of holes must be introduced so that this criterion is violated.

In the remainder of this paper we treat the opposite limit $k_F\xi_s \gg 1$. Using the relation between spin solid angle and flux, the typical "magnetic" flux within an area of ξ_s^2 is unity, so that $k_F\xi_s \gg 1$ implies the magnetic flux per particle is small, so that a perturbative treatment is possible.

The t'-J model given by Eqs. (1) and (2) can be treated systematically in a $1/N$ expansion. We invert the quantization axis on the B sublattice by introducing $\bar{b}_\sigma = \epsilon_{\sigma\sigma'} b_{\sigma'}^*$ so that b_i and $\bar{b}_{i+1}$ form a slowly varying field. Following Arovas and Auerbach,[11] we extend the σ sum from 1 to N and introduce a mean field $D_{ij} = \sum_\sigma \langle \bar{b}_{\sigma i}^* b_{\sigma j}\rangle$, where i,j are nearest neighbors. For $N=2$ this corresponds to the formation of a singlet bond. For $2S/N < 0.19$ the mean-field theory corresponds to a short-range resonating-valence-bond (RVB) state with an energy gap Δ_s.[12] We shall assume that the effect of t and doping has stabilized this mean-field state. Fluctuations around this mean field were treated by Read and Sachdev,[12] who produced the effective Lagrangian

$$\tilde{L}_J = \int d^2r \{ |(\partial_\mu - iA_\mu) z_\sigma|^2 + (\Delta_s^2/c^2)|z_\sigma|^2 \}, \quad (4a)$$

where $z_\sigma(r_i) = \frac{1}{2}(b_{i\sigma} + \bar{b}_{i+1,\sigma})$ corresponds to the slowly varying sublattice magnetization and A_μ is related to the phase fluctuation of D_{ij}. By integrating out z from Eq. (4a) we obtain the Lagrangian which controls the fluctuation in the gauge field,

$$\tilde{L}'_J = \int d^2r\, c^2 F_{\mu\nu}^2/16\tilde{e}^2 \quad (4b)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\tilde{e}^2 = \Delta_s/8\pi$ (for $N=2$). This treatment can readily be extended to include Eq. (2). Suppose we start with second- and third-neighbor hopping t'_2 and t'_3. The mean-field D_{ij} implies $\langle b_i^\dagger b_j\rangle \neq 0$ so that hopping matrix elements $\tilde{t}'_2$ and $\tilde{t}'_3$ are generated, resulting in a band structure $4\tilde{t}'_2 \cos k_x \cos k_y + 2\tilde{t}'_3 \times (\cos 2k_x + \cos 2k_y)$. Depending on the sign and relative size of $\tilde{t}'_2$ and $\tilde{t}'_3$, the band minimum may be at (0,0) $[(\pi,0),(0,\pi)]$ or $(\pm\pi/2, \pm\pi/2)$. For simplicity we treat the case with a single minimum (otherwise additional band labels are needed) and expand about it. Including fluctuations about the mean field following Read and Sachdev produces short-range interaction terms plus the important coupling to the gauge field,

$$\tilde{L}_{t'} = \int d^2r \left\{ f_A^\dagger \left(\frac{\partial}{\partial x_0} - iA_0 \right) f_A + f_B^\dagger \left(\frac{\partial}{\partial x_0} + iA_0 \right) f_B + \frac{1}{2m} \left[f_A^\dagger \left(\frac{\partial}{\partial x_i} - iA_i \right)^2 f_A + f_B^\dagger \left(\frac{\partial}{\partial x_i} + iA_i \right)^2 f_B \right] \right\}. \quad (4c)$$

Equations (4a)-(4c) were first written down phenomenologically by Wen.[7]

We emphasize that Eq. (4) is valid only for $\omega, q < \Delta_s$. In the absence of holes Eq. (4b) predicts a mode with linear dispersion. It is a singlet excitation created by $F_{\mu\nu}$ which in spin language is a rather complex operator corresponding to the topological charge density[13] $\hat{\Omega}\cdot\partial_\mu\hat{\Omega}\times\partial_\nu\hat{\Omega}$, i.e., the solid angle subtended by three spins as discussed earlier. This mode does not enter $\langle\mathbf{S}(\mathbf{r},\tau)\mathbf{S}(0)\rangle$ but should contribute to the specific heat.[14] We shall now see that this mode is strongly modified in the presence of a finite areal density $n = n_A = n_B$ of holes.

As mentioned earlier if $k_F\xi_s \gg 1$, we can start with the Fermi sea and treat gauge fields perturbatively. We redefine A_μ with a factor of $\tilde{e}/c$ to conform with electromagnetic convention and adopt the transverse gauge $\nabla\cdot\mathbf{A}=0$. The longitudinal part A_0 decouples and gives rise to a Coulomb potential $4\pi\tilde{e}^2/q^2$. It is convenient to include the A^2 term in Eq. (4c) in defining the bare transverse field propagator $D^0_{\mu\nu}(\mathbf{x}) = \langle T[A_\mu(\mathbf{r},\tau)A_\nu(0)]\rangle$,

$$D^0_{\mu\nu}(\mathbf{q},\omega_n) = -\frac{4\pi c^2(\delta_{\mu\nu}+q_\mu q_\nu/|q|^2)}{\omega_n^2+c^2q^2+\omega_{sp}^2}, \tag{5}$$

where $\omega_{sp}^2 = 4\pi\tilde{e}^2 n/m \approx \Delta_s\epsilon_F/2\pi$. The coupling to the transverse field $\mathbf{A}$ is given by usual $\mathbf{A}\cdot\mathbf{J}$ and $(f_A^\dagger f_A - n_A)A^2$ terms. Unlike the electron gas where the coupling to transverse fields is small by v_F^2/c^2, here the spin-wave velocity $c\approx J$ is comparable or less than $v_F \approx n^{1/2}t'$; new physics emerges. Fortunately if $c < v_F$ the Migdal theorem is obeyed and a systematic diagrammatic analysis analogous to the phonon problem is possible.

The first step is to write down the screened propagator $D_{\mu\nu}$ which is obtained from $D^0_{\mu\nu}$ by replacing ω_{sp}^2 in Eq. (5) by $\omega_{sp}^2 - 2(\tilde{e}/mc)^2 2k_F^2\Pi_0(q,\omega)$, where Π_0 is the usual 2D free fermion polarization bubble. For $\omega \ll v_F q$ and $q < 2k_F$, the correction term is a constant which exactly cancels ω_{sp}^2 and we have

$$D_{\mu\nu}(q,\omega+i\eta) = 4\pi c^2(\delta_{\mu\nu}-q_\mu q_\nu/|q|^2) \times[\omega^2-c^2q^2+i\omega_{sp}^2\omega/v_F q]^{-1}. \tag{6}$$

For $\omega \gg v_F q$, Π_0 is negligible and $D_{\mu\nu}\approx D_\mu{}_\nu{}^0$. For a given q, it is clear that for both $\omega > v_F q$ and $\omega < cq$, $\mathrm{Re}D(q,\omega+i\eta)$ is negative for $\omega < \omega_{sp}$ and exchange of $D_{\mu\nu}$ will lead to pairing of the gauge-invariant pair propagation $\langle f_B^\dagger(r,\tau)f_A^\dagger(r,\tau)f_A(0)f_B(0)\rangle$. In the spirit of BCS, we construct a retarded interaction by replacing $D_{\mu\nu}$ by $D^0_{\mu\nu}$ $(\omega=q=0)$ for $\omega < \omega_c$ and zero otherwise. Clearly ω_c is at least Δ_s but its determination requires a Lagrangian valid for $\omega > \Delta_s$. Physical considerations lead us to set $\omega_c \approx \Delta_s$ because excitations with $\omega, cq > \Delta_s$ are indistinguishable from those of a Néel ordered state which we assume to be not superconducting. In analogy with phonon exchange, we obtain a dimensionless coupling constant $\lambda \approx (\tilde{e}/mc)^2k_F^2 m/\omega_{sp}^2 \approx 1$. [The restriction $q\xi_s < 1$ may reduce λ by $(k_F\xi_s)^{-1}$ upon averaging over Fermi surface.]

In addition to the exchange of the transverse gauge field, we also have to include the longitudinal field, which in the static limit is simply a screened Coulomb interaction $-4\pi\tilde{e}^2/(q^2+\tilde{\kappa}^2)$, where $\tilde{\kappa}^2 = 4\pi\tilde{e}^2 dn/d\mu$. Note that the dimensionless coupling constant is $\tilde{\mu} = -1$ in the $q\to 0$ limit, and exactly cancels $\mu=1$ of the ordinary screened Coulomb repulsion. Because of the different q and ω dependences, there will probably be some residual attraction left. An intrinsic difficulty of all previous attempts to raise T_c by exchanging high-frequency excitation ω_0 such as plasmons or excitons is that the renormalized Coulomb repulsion $\mu^* = \mu/[1+(\mu\ln(\epsilon_F/\omega_0)]$ grows with increasing ω_0. Our mechanism avoids this problem by canceling out the Coulomb repulsion, leaving $\lambda\approx 1$ so that the theory predicts a uniform energy gap Δ equal to a fraction of $\omega_c \approx \Delta_s$. For sufficiently large Δ_s, a substantial part of the Fermi sea may participate in the pairing, with a relatively short coherence length ξ given by $\xi k_F \approx \epsilon_F/\Delta_s$ or $\xi/\xi_s \approx v_F/c$ which can be of order 1.

The coupling to low-lying transverse gauge fields has a profound effect on the normal-state properties. Very recently, Reizer[15] pointed out that in ordinary metal, coupling to a transverse electromagnetic field leads to a scattering rate for electrons of order $(v_F/c)^2kT$. His argument can be directly applied to the present problem in the normal state. For simplicity let us consider a single fermion with energy ω above the Fermi energy. It decays by emitting Bose excitation with spectral density given by $\mathrm{Im}D(q,\nu)$, where $\nu < \omega$. In Eq. (6), $\mathrm{Im}D^{-1} \approx \nu/v_F q$ is the usual density of particle-hole excitations. The important point made by Reizer is that for transverse excitation, $\mathrm{Re}D^{-1}$ vanishes as q^2. Thus coupling to the transverse gauge field effectively enhances the matrix element for the excitation of long-wavelength particle-hole pair. A simple dimensional analysis yields the result that in 2D, the decay rate is $\tau^{-1} \approx \lambda\epsilon_F(\omega/\epsilon_1)^{2/3}$, where the energy scale is $\epsilon_1 = c^2k_F^2/\Delta_s$. We recall that the f Green's function is not gauge invariant and has no physical meaning. The physical quantity is the conductivity $\sigma(\omega)$ which is computed with an f bubble with self-energy and vertex correction, and the latter converts τ^{-1} to the transport time $\tau_{\rm tr}^{-1}$ which requires an additional factor of $1-\cos\theta$. This leads to $\tau_{\rm tr}^{-1} \approx \lambda\epsilon_F(\omega/\epsilon_1)^{4/3}$. At finite temperature $T > T_c$, we expect ω to be replaced by T so that the conductivity should be given by $ne^2\tau_{\rm tr}/m$, with $\tau_{\rm tr}^{-1} \approx \lambda\epsilon_F(T/\epsilon_1)^{4/3}$. A long-standing mystery about the normal-state resistivity of oxide superconductors is that $\tau_{\rm tr}^{-1}$ is approximately kT. We have identified a source of strong inelastic scattering and, while our result is limited to $\Delta_s > T > T_c$, it may well be consistent with the experimental data.

A second effect of the strong inelastic scattering is that by the Kramers-Kronig relation, the real part $\Sigma'(\omega)$ of the f self-energy also goes as $\omega^{2/3}$, so that the spectral weight $a = (1+\partial\Sigma'/\partial\omega)^{-1} \approx \omega^{1/3}$. The physical Green's function $\langle c_\sigma^\dagger c_\sigma\rangle$ is a convolution of the f and z

Green's functions in ω space, and the $\omega^{1/3}$ dependence should show up in the single-particle density of states observed by tunneling for ω above the energy gap. The convolution will smear the usual square-root singularity at the gap edge, and increase the gap for quasiparticle excitation to $\Delta+\Delta_s$. We should also include the attraction between $f^\dagger$ and z_σ mediated by the gauge field using Eq. (4). Our preliminary conclusion is that the joint density of states is constant so that a bound state is always formed below $\Delta+\Delta_s$ and the low-lying quasiparticles carry both spin $\frac{1}{2}$ and charge. Nevertheless, the tunneling density of states should show substantial deviation from BCS theory as well as the usual Fermi-liquid behavior far above the gap.

The superconducting state we find has s symmetry[16] and the energy gap is isotropic. Nevertheless, it differs from BCS theory in an important way; Anderson's theorem regarding nonmagnetic impurities is not obeyed. The pairing is between holes on A and B sublattices; the two experience different random potentials and the compensation leading to Anderson's theory does not occur. Experimentally it is known that the copper-oxide superconductivity is destroyed by nonmagnetic doping in the plane. This distinguishes the copper-oxide system from $BaPb_xBi_{1-x}O_3$ which is in the dirty limit.

The spin correlation $\langle \mathbf{S}(\mathbf{r},\tau)\mathbf{S}(0)\rangle$ is a convolution of z Green's functions and exhibits a gap of $2\Delta_s$ corresponding to triplet excitation.[11] The coupling to gauge field and to holes should modify this quantitatively but not qualitatively so that neutron scattering should be mainly sensitive to the spin gap $2\Delta_s$. Since the onset of superconductivity affects only Δ_s/ϵ_F of the holes, it should effect the neutron scattering only as a small correction.

Our model also raises an interesting possibility concerning the difference between single-layer and multilayer copper-oxide structures. Even if the electronic hopping between the layers is weak, the exchange between the spins on different layers may be sufficiently strong to correlate them if they are not frustrated as in La_2CuO_4. In this case our mechanism will lead to an additional pairing channel between holes on neighboring layers. The interlayer exchange may also reduce Δ_s, so that a quantitative treatment may be difficult. A second comment is that our pairing mechanism per se is not restricted to 2D. In 2D it may operate over a wider range in parameter space within the t-t'-J model because the 2D Heisenberg AF is easy to disorder. We speculate that the 3D copper-oxide systems are not superconducting because the hole concentration is so large that the ground states are Fermi liquids. It will be interesting to reduce the hole concentration in these systems by doping to see if an AF phase boundary can be reached and whether superconductivity can be found in a narrow region near this phase boundary with short-range AF order.

In summary, we believe the U(1) gauge theory[4-7] is the language to describe the original RVB picture of Anderson,[1] which envisions a liquid of holes in a background of singlets on opposite sublattices. Indeed, for short-range RVB states the sublattice label for holes is a topological property[17] and singlet pairing of holes on opposite sublattices is a natural consequence of this picture. Of course our theory is still incomplete in that the role of t is assumed and not fully elucidated. Our main worry is that the t term may produce a spiral state which induces a coherent admixture of the two sublattices and singlets on the same sublattice.[18] This latter effect is pair breaking for the superconductivity discussed here. Fortunately, the hopping matrix element is expected to be small, $\approx Jn$ and there should be significant parameter space in a t-t'-J model where the superconductivity survives. It will be interesting to study the t-t'-J model by the Monte Carlo method, because it may be far richer than the standard Hubbard model itself.

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