Full-Folding-Model Description of Elastic Scattering at Intermediate Energies

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Nucleon-nucleus optical potentials are calculated using full off-shell nucleon-nucleon free t matrices based on the Paris potential. Applications to proton scattering from ^{40}Ca at 200 and 300 MeV are presented. Significant differences are found between observables calculated using full-folding potentials and $t\rho$ approximations to them, demonstrating the importance of an accurate treatment of off-shell effects. The agreement between calculated and measured observables improves substantially using the full-folding model.

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One of the recent major thrusts in intermediate-energy nuclear physics has been directed towards understanding and interpreting the growing body of nucleon-nucleus (NA) scattering data within a microscopic framework. Particular attention has been devoted to the microscopic models of Kerman, McManus, and Thaler[1] (KMT) and Watson.[2] Assuming that the projectile-nucleus coupling is restricted to a sum of effective nucleon-nucleon (NN) interactions between the projectile and each target nucleon, these models lead to a structure for the optical potential involving a convolution of the target-state wave functions with the off-energy-shell NN t matrix. This form of the optical potential is known as the full-folding model. Potentials of similar structure have recently been applied to pion scattering.[3] Because of the complicated structure of the full-folding model several approximations have been introduced to simplify the calculation of the optical-model potential, resulting in a factorized $t\rho$ structure in momentum space. Extensive applications of these nonrelativistic $t\rho$ approximations to elastic scattering have met with varying degrees of success.[4,5] In fact, they encounter systematic difficulties in describing, even qualitatively, the experimental data at intermediate energies. The introduction of relativistic degrees of freedom through the relativistic $t\rho$ approximation[5,6] has shown some striking improvements; however, there remain deficiencies which need to be understood, especially below 500 MeV incident energy.

In order to draw more definitive conclusions about the intrinsic limitations of the nonrelativistic model, accurate calculations of the optical potential including all the relevant intrinsic properties of the NN effective interaction and nuclear ground state are required. At intermediate energies, where the nonlocal and energy-dependent free t matrix is expected to be a reasonable approximation to the effective internucleon force, the calculation of the full-folding optical potential is especially important in that it provides a natural base line from which further corrections may be introduced.

In this Letter we report results obtained from the calculation of the full-folding optical potential in momentum space for proton elastic scattering, including the intrinsic energy dependence of the NN effective interaction, its off-shell behavior, and knock-on exchange terms. Although medium corrections have been shown to be important at intermediate energies[7] when the NN force is *assumed* to be local, this type of correction is not explicitly considered here. Indeed, one of the long-term objectives is to reassess the role of medium corrections within a framework in which the off-shell properties of a realistic NN interaction are included. For example, medium corrections to the interaction usually associated with Fermi averaging[7] within the local-density approximation are naturally accounted for in the full-folding framework.

To first order in a multiple-scattering expansion,[1,2] the optical potential for elastic scattering is given by the antisymmetrized ($\mathcal{A}$) matrix elements[8]

$$U(E)=\sum_{i=1}^{A}\langle\Phi_0|\,T_i(E+E_0)\,|\Phi_0\rangle_{\mathcal{A}}\,, \tag{1}$$

where E is the beam energy, E_0 is the energy of the target ground state, $|\Phi_0\rangle$ is the correlated target wave function, and T_i is the many-body effective interaction[8] between the incident nucleon and the ith nucleon in the target. Neglecting medium corrections and assuming a free spectrum for the nucleons propagating in intermediate states, the T_i matrix is related in Ref. 9 to the two-body free t_i matrix which satisfies the Lippmann-Schwinger integral equation

$$t_i(\omega)=v_i+v_i\frac{1}{\omega-K_0-K_i}t_i(\omega)\,, \tag{2}$$

where v_i is the bare internucleon force and K_0 (K_i) is the incident (struck) nucleon kinetic energy. Total momentum conservation for the interacting pair allows the two-body t_i matrix to be expressed in terms of a one-body t matrix,

$$\langle\mathbf{k}'\mathbf{p}'|\,t_i(\omega)\,|\mathbf{kp}\rangle=\delta(\mathbf{k}'+\mathbf{p}'-\mathbf{k}-\mathbf{p})\langle\tfrac{1}{2}(\mathbf{k}'-\mathbf{p}')\,|\,t\left(\omega-\frac{(\mathbf{p}+\mathbf{k})^2}{2M}\right)|\,\tfrac{1}{2}(\mathbf{k}-\mathbf{p})\rangle\,, \tag{3}$$

with M the total mass of the pair. With these considerations and taking the target ground state to be a Slater determinant of single-particle wave functions $\{\varphi_\alpha\}$ one obtains the full-folding expression for the optical potential in momentum space

$$U(\mathbf{k}',\mathbf{k};E)=\sum_\alpha \int d\mathbf{p}\,\varphi_\alpha^\dagger(\mathbf{p}-\mathbf{k}')\langle\mathbf{k}'-\tfrac{1}{2}\mathbf{p}|\,t\left(E+\epsilon_\alpha-\frac{\mathbf{p}^2}{2M}\right)|\mathbf{k}-\tfrac{1}{2}\mathbf{p}\rangle_{\mathcal{A}}\varphi_\alpha(\mathbf{p}-\mathbf{k})\,, \quad (4)$$

where the sum runs over the occupied single-particle states α having single-particle energies ϵ_α.

We have calculated the optical potential using Eq. (4) for proton elastic scattering on ^{40}Ca. The single-particle energies ϵ_α were each replaced by an average value of -20 MeV. The off-shell t matrix was calculated from the Paris potential.[10] The single-particle wave functions φ_α were taken from a harmonic-oscillator model fit to the electron-scattering data;[11] the fit provides a reasonable description of the charge form factor for momentum transfers $q \lesssim 2$ fm^{-1}. The same set of wave functions $\{\varphi_\alpha\}$ was used throughout all calculations reported here. Proton and neutron densities were taken to be identical.

We have also calculated on-shell and off-shell φ approximations[12] to Eq. (4). The *on-shell tρ* potential is constructed by multiplying the nuclear density in momentum space by the on-shell free t matrix evaluated at an energy defined by the invariant mass in the Breit frame.[12] The *off-shell tρ* potential is calculated following Ref. 12. In this approximation to Eq. (4), $\mathbf{p}$ in the t-matrix elements is fixed by the optimum factorization prescription[12,13] resulting in an optical potential having the factorized form $\langle \boldsymbol{k}'|t|\boldsymbol{k}\rangle\rho(q)$, where the NN relative momenta $\boldsymbol{k}'$ and $\boldsymbol{k}$ are not constrained to be on the energy shell. In performing the latter $t\rho$ calculation the t matrix was evaluated at a fixed energy given as if the struck nucleon were at rest in the target system. A similar calculation has been recently performed by Elster and Tandy.[13]

When calculating the proton-nucleus scattering observables in momentum space, the Coulomb potential was treated following the method proposed by Vincent and Phatak[14] with the charge densities treated following the Eisenstein and Tabakin procedure.[15] We have found this procedure to be reliable in the range of momentum transfer studied here (0–3.7 fm^{-1}). Results were verified by transforming the optical potential to coordinate space and solving the corresponding integrodifferential equation.

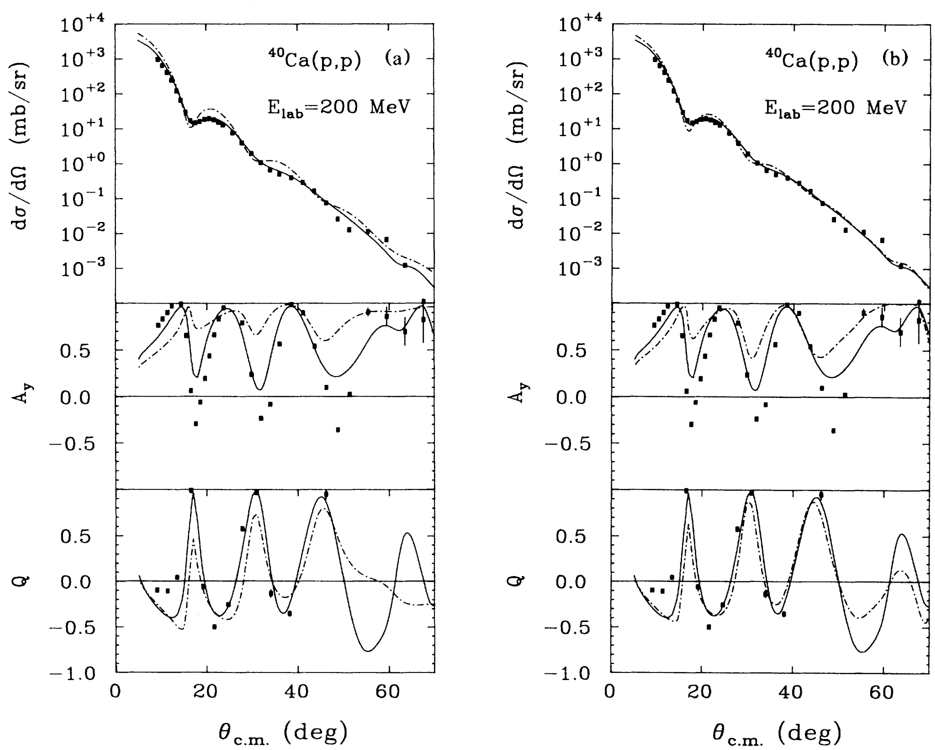

FIG. 1. (a) Full-folding (solid curve) and on-shell $t\rho$ (dash-dotted curve) calculated observables for $p+{}^{40}$Ca at 200 MeV. The data are taken from Refs. 16 and 17. (b) Full-folding (solid curve) and off-shell $t\rho$ (dash-dotted curve) calculated observables for $p+{}^{40}$Ca at 200 MeV. The data are taken from Refs. 16 and 17.

Calculations of optical potentials and scattering observables for the $^{40}Ca(p,p)$ reaction at 200 and 300 MeV were made following the outlined procedure. In Fig. 1(a) we compare results at 200 MeV using the full-folding optical potential with those obtained using the on-shell $t\rho$ approximation and with measured observables[16,17] for $\theta_{c.m.} \lesssim 70°$ ($q \lesssim 3.6$ fm^{-1}). For each observable the full-folding results differ significantly from those obtained with the on-shell $t\rho$ approximation. Moreover, the full-folding results are uniformly in much better agreement with the measured observables than are those based on the conventional $t\rho$ approximation. This is especially true for $d\sigma/d\Omega$ and A_y, where the $t\rho$ approximation invariably yields too much and too little structure, respectively. Even on-shell relativistic $t\rho$ models,[5,6] which provide varying degrees of improvement in A_y and Q, predict differential cross sections which have too much structure.

In Fig. 1(b) we compare results at 200 MeV using the full-folding model with those obtained from the off-shell $t\rho$ approximation. As in the comparison above, the two results differ significantly. The results calculated using the off-shell $t\rho$ approximation lie, as expected, between those of the on-shell $t\rho$ and the full-folding models with the full-folding results being in best agreement with the data.

In Fig. 2(a) results at 300 MeV using the full-folding model are compared with those generated by the on-shell $t\rho$ approximation and with measured observables[17] for $\theta_{c.m.} \lesssim 55°$ ($q \lesssim 3.7$ fm^{-1}). As at 200 MeV the full-folding and on-shell $t\rho$ results differ substantially; the full-folding results are in much better agreement with the data. Apart from the peak in A_y near 10°, the full-folding calculations provide a very reasonable description of the measured A_y for $\theta_{c.m.} \lesssim 40°$. As at 200 MeV, the problem with the overstructured $d\sigma/d\Omega$ characteristic of the on-shell $t\rho$ approximation is largely (but not entirely) resolved by the full-folding model.

In Fig. 2(b) we compare results at 300 MeV using the full-folding and off-shell $t\rho$ models with each other and with the data. At this higher energy the two types of calculations are in much closer agreement than at 200 MeV. Nevertheless, the full-folding results provide a superior description of the data in the region where the target wave functions are most reliable ($\theta_{c.m.} \lesssim 40°$).

Our calculations at the two energies considered suggest that the off-shell $t\rho$ and full-folding calculations are converging (albeit slowly) with each other with increasing energy. This may become an important practical point as the computational time required to perform the off-shell $t\rho$ calculation is nearly 2 orders of magnitude shorter than required to calculate the full-folding poten-

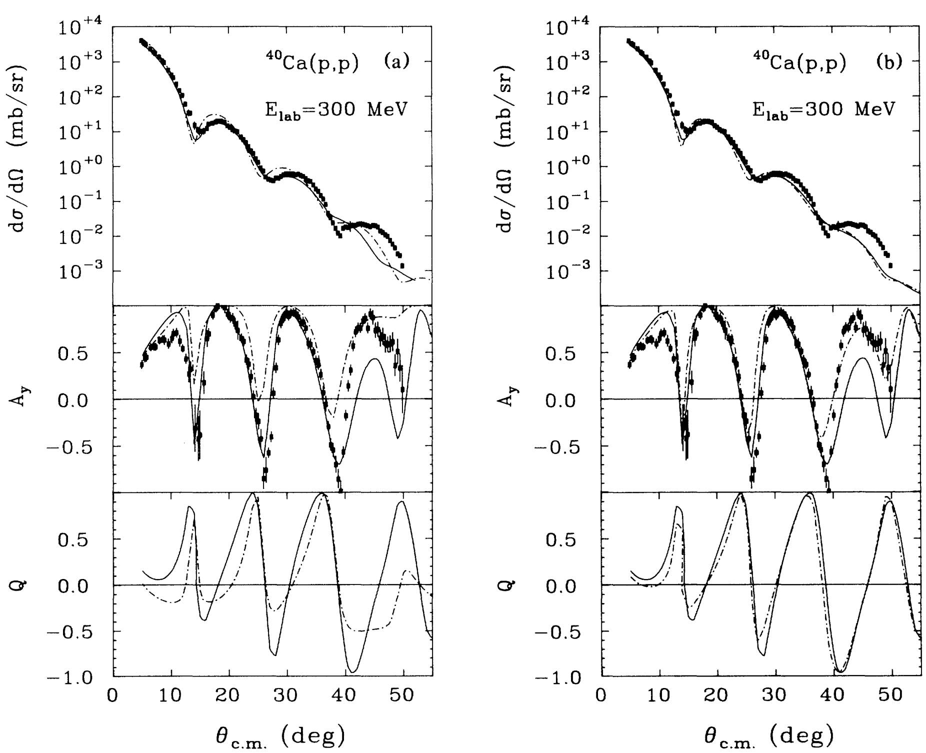

FIG. 2. (a) Full-folding (solid curve) and on-shell $t\rho$ (dash-dotted curve) calculated observables for $p+{}^{40}Ca$ at 300 MeV. The data are taken from Ref. 17. (b) Full-folding (solid curve) and off-shell $t\rho$ (dash-dotted curve) calculated observables for $p+{}^{40}Ca$ at 300 MeV. The data are taken from Ref. 17.

tial.

Although the target wave functions used here are adequate for comparing the full-folding model with the $t\rho$ approximations, more refined bound-state wave functions are required for a more detailed comparison of the full-folding model results with the data at momentum transfers larger than $\sim 2.5\ \mathrm{fm}^{-1}$.

In summary, we have calculated full-folding momentum-space optical potentials for protons on ^{40}Ca at 200 and 300 MeV incident energy using the Paris potential. Results obtained using these potentials have been compared with those obtained using a conventional on-shell $t\rho$ approximation, a more recent off-shell $t\rho$ approximation, and with measured cross sections and spin observables. Our primary finding is that there are substantial differences between the full-folding model and each of the $t\rho$ approximations considered here. Additionally, the full-folding model provides a significantly better description of the observables at the studied energies, especially in the momentum-transfer range where the target wave functions are most reliable. A systematic understanding of the implications of the full-folding model will require its application to other systems of different sizes over a broader range of energy as well as a study of the sensitivity of the results to the off-shell behavior generated by alternative *NN* potentials.

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