Strong *CP* Violation and the Neutron Electric Dipole Moment

Jiang Liu
Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan 48109

C. Q. Geng and John N. Ng
TRIUMF Theory Group, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3
(Received 22 March 1989)

We derive an identity that relates the fermion electric dipole moment from weak *CP*-violation effects to the strong *CP* parameter θ. In the absence of Peccei-Quinn-type symmetries, we find that requiring θ naturally small generally implies that, for a large class of models, the dominant contribution to the neutron electric dipole moment d_n comes from strong *CP* violation rather than directly from weak *CP*-violation effects.

PACS numbers: 11.30.Er, 13.40.Fn

In the twenty-five years since the discovery of *CP* violation[1] little progress has been made in determining where this violation arises in the elementary-particle interactions.[2] The violation of *CP* invariance has been observed in three decays of the K_L meson and, possibly, in the $K^0 \to \pi\pi$ decay,[3] and nowhere else.[4] In order to determine the source of *CP* violation and hence distinguish between different theoretical models, it is necessary to have additional experimental information. Among the various possibilities, the possibility of detecting *CP* violation by measuring the neutron electric dipole moment d_n has long been of special interest.[5]

Model predictions[6] on d_n are essentially arbitrary. This arises because a nonzero value of d_n could always be blamed on the "strong *CP* violation."[7] Hence, for the class of models in which a small strong *CP* parameter θ is realized by means of fine tuning, the neutron electric dipole moment is fundamentally incalculable. Thus, in general, a nonzero value of d_n actually cannot be unambiguously related to weak *CP* violation. The most familiar example is the Kobayashi-Maskawa (KM) model.[8] Although weak-interaction effects appear to give a very small contribution[9] to d_n, one practically has no control[10] on the size of θ. As a result, in the KM model the value of d_n induced by the strong *CP* parameter θ is essentially arbitrary.

Clearly, in order to be able to predict d_n, we must first consider how to solve the strong *CP* problem. The best known solution is found by introducing Peccei-Quinn-type symmetries,[11] which make θ a small but calculable parameter. However, by doing so either the theory has to have an axion, which is still yet to be borne out by experiments,[7] or we have to make at least one of the light quarks massless which, on the other hand, may not be compatible with the present view of chiral symmetry of hadrons.[12] One possibility is to make the axion very light and weakly coupled to elude experimental detection by adding a Higgs-singlet field.[13] Another one is to require that the theory violates *CP* symmetry either spontaneously or through soft breaking terms. We would like to show in this Letter that for the latter class of models,[14–17] the dominant contribution to d_n *generally*[18] comes from the radiatively induced strong *CP* θ parameter rather than directly from weak *CP*-violation effects.

We begin by considering the effective θ parameter which characterizes the strength of strong *CP* violation,

$$\theta = \theta_{\rm QCD} + \operatorname{Arg}\operatorname{Det}(M_u M_d) = \theta_{\rm QCD} + i \ln \frac{\operatorname{Det}(M_u M_d)}{\operatorname{Det}(M_u^\dagger M_d^\dagger)}\,. \tag{1}$$

Here $\theta_{\rm QCD}$ comes from a term $(\theta_{\rm QCD}/64\pi^2)\mathrm{Tr}G\tilde{G}$ in the QCD Lagrangian. $M_{u(d)}$ is the up- (down-) quark mass matrix. Requiring that *CP* be violated only spontaneously or through soft breaking terms amounts to setting $\theta_{\rm QCD}=0$. In this case, it is convenient to represent the last term in Eq. (1) by a parameter $\theta_{\rm tree}$.[19] In doing so, we have explicitly chosen a basis in which the quark mass matrices are diagonal and positive definite,

$$M_q \to D_q, \quad \text{with} \quad (D_q)_{ij} = m_q^i \delta_{ij}, \quad m_q^i > 0\,, \tag{2}$$

where q represents up and down quarks. Clearly, at higher loop levels, D_q will receive radiative corrections, $D_q \to D_q'$, but if all the corrections are real, they will only contribute to the renormalizations of quark mass and mixing but not to $\theta_{\rm tree}$. Thus in the basis in which D_q' remains diagonal and positive definite, θ is unchanged.

Now, suppose at some loop level nonzero complex values of corrections to D_q first show up,

$$D_q \to D_q' + \delta m_q\,. \tag{3}$$

Here δm_q is the radiative correction to the diagonalized quark mass matrices D_q'. In general, δm_q is not diagonal. For the consideration of d_n, only the imaginary part of δm_q will be relevant. We will therefore assume that δm_q is much smaller than the *diagonalized* matrices D_q'. Now, using the identities

$$\operatorname{Det}(D_u' + \delta m_u) = (\operatorname{Det} D_u') \prod_i (1 + D_{u,i}'^{-1} \delta m_{u,ii}) + \cdots\,,$$

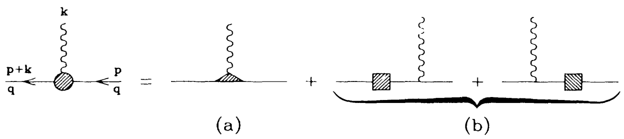

FIG. 1. The effective $qq\gamma$ vertex where (a) and (b) are the one-particle irreducible and reducible parts, respectively.

and

$$\ln(1+D'^{-1}_{u,i}\delta m_{u,ii})=D'^{-1}_{u,i}\delta m_{u,ii}+\cdots,$$

the ellipses representing terms with higher powers of δm_u, we find to the leading order of δm_q

$$\theta=\theta_{\text{tree}}+\sum_i\left[\frac{\text{Im}(\delta m_u)_{ii}}{(D'_u)_i}+\frac{\text{Im}(\delta m_d)_{ii}}{(D'_d)_i}\right]. \tag{4}$$

Hence, we see that to this order only the imaginary part of the diagonal elements of δm_q enters into the expression of θ. This justifies our earlier assumption on the size of δm_q and shows that perturbation expansion in powers of δm_q is indeed valid as far as computing the θ parameter is concerned. When M_q is Hermitian, $\theta_{\text{tree}}=0$ and thus only the last term in Eq. (4) survives. For this special case, our formula reduces explicitly to a form given by Bég and co-workers.[14]

Experimental limits on the size of d_n have yielded[5] $\theta\leq 10^{-9}$–10^{-10}. Now we ask the following: How can we make θ sufficiently small? One possibility is, of course, to introduce arbitrary fine tunings so that the smallness of the θ parameter arises because of a complete cancellation among the different terms in Eq. (4). This possibility, while viable in principle, cannot, however, answer the original question of naturalness which is the center of the strong CP problem. In what follows we will, therefore, not discuss this possibility any further. Alternatively, we may assume that the reason that θ is small is because every term in Eq. (4) is small

$$\theta_{\text{tree}}\leq\theta,\quad \frac{\text{Im}(\delta m_q)_{ii}}{(D'_q)_i}\leq\theta\quad(i=1,2,3,\ldots). \tag{5}$$

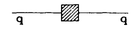

FIG. 2. Contributions to δm_q.

Theoretical models that realize the conditions in Eq. (5) in a natural way are discussed in Refs. 14–17. In practice, if $\theta_{\text{tree}}\neq 0$ one finds that the radiative correction terms are usually much smaller than θ_{tree}. Among the various relations in Eq. (5) of special interest are those that relate the θ parameter to the first-generation quark masses,

$$\frac{\text{Im}(\delta m_u)}{m_u}\leq\theta,\quad\frac{\text{Im}(\delta m_d)}{m_d}\leq\theta, \tag{6}$$

where m_u and m_d represent, respectively, the u- and d-quark masses. Of course relations given by Eqs. (5) and (6) are valid only if there are no Peccei-Quinn-type symmetries. Had we introduced such symmetries, the contribution to θ from Eq. (5) would have been eliminated by a trivial rotation, and it would be unnecessary to require that each term in Eq. (5) be small.

Now we explore the consequences of Eq. (6). First, we realize that in the basis where the quark mass matrices D'_q are diagonal and positive definite, there is a relation between $\text{Im}(\delta m_q)$ and the quark electric dipole moment d_q. Indeed, for every effective $qq\gamma$ coupling graph shown in Fig. 1 that generates a contribution to d_q, there will be a corresponding diagram illustrated in Fig. 2 contributing to $\text{Im}(\delta m_q)$ and hence to θ from Eq. (4). The contributions of the effective $qq\gamma$ vertex can be separated into one-particle irreducible (1PI) and reducible diagrams as depicted in Figs. 1(a) and 1(b), respectively. One finds[20] that only the 1PI vertex of Fig. 1(a) contributes to the quark electric dipole moment d_q term via

$$\bar q(p+k)if_D(k^2)\sigma_{\mu\nu}k^\nu\gamma_5 q(p), \tag{7}$$

where $d_q=f_D(0)$. From the Gordon decomposition relation

$$\bar q(p+k)\sigma_{\mu\nu}k^\nu\gamma_5 q(p)=\bar q(p+k)(2p+k)_\mu\gamma_5 q(p), \tag{8}$$

we have

$$\bar q(p+k)if_D(k^2)\sigma_{\mu\nu}k^\nu\gamma_5 q(p)=\bar q(p+k)if_D(k^2)(2p+k)_\mu\gamma_5 q(p). \tag{9}$$

To relate the right-hand side of Eq. (9) to the parameter θ we use the Ward-Takahashi identity[21]

$$\Gamma^{qq}_\mu(p+k)=eQ_q\frac{\partial}{\partial p^\mu}D_q(p)+O(k), \tag{10}$$

where Γ^{qq}_μ is the effective $qq\gamma$ vertex of Fig. 1(a) and Q_q is the charge of the quark. Putting Eqs. (3), (9), and (10) together one obtains that

$$\bar q(p+k)\left[eQ_q\frac{\partial}{\partial p^\mu}\text{Im}[\delta m_q(p)]\gamma_5+O(k)\right]q(p)=\bar q(p+k)f_D(k^2)(2p+k)_\mu\gamma_5 q(p) \tag{11}$$

which immediately gives an identity

$$f_D(0)\big|_{p^2=m_q^2}=eQ_q\frac{\partial}{\partial p^2}\text{Im}[\delta m_q(p)]\bigg|_{p^2=m_q^2} \tag{12}$$

by taking all the k-dependence terms to zero. As a consequence, for arbitrary m_q, we can relate d_q to $\mathrm{Im}(\delta m_q)$ as follows:[22]

$$d_n^{\mathrm{weak}} \sim d_q \sim eQ_q \frac{\mathrm{Im}(\delta m_q)}{\Lambda^2}, \tag{13}$$

where d_n^{weak} represents the contribution to d_n directly from weak CP-violation effects, and Λ is a parameter with the dimension of mass. Strictly speaking, the first step of Eq. (13) follows only if long-distance contributions to d_q are negligible. Although this is an assumption we will make, for the class of models discussed in Refs. 14–17 this turns out to be the case.[23]

It is important to realize from Eq. (12) that in the limit $\mathrm{Im}(\delta m_q)=0$ for arbitrary m_q,[22] contributions to d_q^{weak} at that level must vanish as well. One interesting implication of this observation can be summarized as follows: Theoretical realistic models which have a contribution to d_n at, say, the nth-loop level must also generate a contribution to θ at the same loop.[23,24] For example, in the standard KM model, since there is no contribution to θ up to two-loop levels for any m_q,[12] one concludes from Eq. (12) that $d_q(\text{two-loop})=0$ which agrees with the explicit calculations of the two-loop diagrams done by Shabalin.[9]

Now, using the constraint on $\mathrm{Im}(\delta m_q)$ from Eq. (6), we have from Eq. (13)

$$d_n^{\mathrm{weak}} \sim \theta \frac{e m_d}{\Lambda^2}. \tag{14}$$

On the other hand, the contribution to d_n through the strong CP parameter θ, denoted by d_n^{strong} hereafter, is of the order[5]

$$d_n^{\mathrm{strong}} \sim 10^{-16}\theta\, e\,\mathrm{cm} \sim \theta \frac{e}{M_W}. \tag{15}$$

Comparing Eq. (14) with Eq. (15) we then see

$$\frac{d_n^{\mathrm{weak}}}{d_n^{\mathrm{strong}}} \sim \theta \left[\frac{m_d}{M_W}\right]\left[\frac{M_W}{\Lambda}\right]^2 \sim 10^{-4}\left[\frac{M_W}{\Lambda}\right]^2. \tag{16}$$

To have a feel on the order-of-magnitude estimate of our result we take $\theta \sim 10^{-9}$ as an example so that d_n^{strong} ($\sim 10^{-25}\, e\,\mathrm{cm}$) saturates the present experimental limits. Assuming

$$\Lambda \gtrsim M_W \tag{17}$$

we find from Eq. (14), $d_n^{\mathrm{weak}} \lesssim 10^{-29}\, e\,\mathrm{cm}$. Thus, if Λ is not too much smaller than M_W, it is actually strong CP violation that provides the dominant contribution to d_n. A detailed calculation shows, in fact, that all the existing theoretical models[14-17] which have solved the strong CP problem without Peccei-Quinn-type symmetries turn out to have this interesting feature.[23]

Evidently, for different models, Λ takes a different form. The graph that contributes to d_q differs from that to $\mathrm{Im}(\delta m_q)$ by an additional external photon coupling and, therefore, contains also one additional internal propagator; we then expect that the order of magnitude of Λ is roughly of the order of the largest energy scale of that loop (or subloop). Therefore, the assumption in Eq. (17) is valid only if the corresponding highest energy scale of these loops or subloops is not much smaller than M_W. Indeed, this turns out to be the case in the class of models discussed in Refs. 14–17.

However, it is conceivable to have such models that have $\Lambda \ll M_W$. This may happen, for instance, if a subloop diagram to which the external photon couples contains only the light quark and/or gluon, photon internal lines. In that case $\Lambda \sim m_q \ll M_W$, and as a result d_n^{weak} could be of the same order of magnitude as or even bigger than d_n^{strong}. Fortunately, models of this sort are difficult to construct. In fact, to have this sort of diagram also violate CP, we may have to consider three or higher loops. Although we are not aware of any such models, the possibility of having $\Lambda \ll M_W$ can be illustrated, for instance, by "calculating" θ and d_n^{weak} in the KM model.[25] It was shown explicitly by Khriplovich[26] that the first nonzero loop contributions to θ and d_n^{weak} arise from three-loop diagrams with $\Lambda \sim m_s$ and as a result $d_n^{\mathrm{weak}}/d_n^{\mathrm{strong}} \gtrsim 1$.

We thank M. Dine for pointing out to us several misleading statements. One of us (J.L.) wishes to thank R. Akhoury and L. Wolfenstein for many interesting discussions, and his work was supported in part by the U.S. Department of Energy. The work of C.Q.G. and J.N.N. was supported in part by the Natural Sciences and Engineering Research Council of Canada.

[1]J. Christenson, J. Cronin, V. Fitch, and R. Turlay, Phys. Rev. Lett. **13**, 138 (1964).

[2]For a recent review on the present status of CP violation, see, for example, L. Wolfenstein, Annu. Rev. Nucl. Part. Sci. **36**, 137 (1986).

[3]NA31 Collaboration, H. Burkhardt *et al.*, Phys. Lett. B **206**, 169 (1988).

[4]Possible detections of the neutron electric dipole moment of the order of $10^{-25}\, e\,\mathrm{cm}$ have recently been reported by two different groups: I. S. Altarev *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 360 (1986) [JETP Lett. **44**, 460 (1986)]; J. M. Pendlebury, in Proceedings of the Ninth Symposium on Grand Unification, Aix les Bains, France, 1988 (to be published). However, it is too early to see these as positive signals.

[5]For a recent review on electric dipole moments, see S. Barr and W. Marciano, in *CP-Violation,* edited by C. Jarlskog (World Scientific, Singapore, 1989).

[6]X. G. He, B. H. J. Mckellar, and S. Pakvasa, Melbourne University Report No. UH-511-666-89, 1989 (unpublished).

[7]For recent reviews on the strong CP problem, see J. E. Kim, Phys. Rep. **150**, 1 (1987); H. Y. Cheng, Phys. Rep. **158**, 1 (1988); R. D. Peccei, in Ref. 5.

[8]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[9]E. P. Shabalin, Yad. Fiz. **28**, 151 (1978) [Sov. J. Nucl.

Phys. **28**, 75 (1978)]; Usp. Fiz. Nauk **141**, 561 (1983) [Sov. Phys. Usp. **26**, 297 (1983)].

[10]J. Ellis and M. K. Gaillard, Nucl. Phys. **B150**, 141 (1979).

[11]R. D. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).

[12]A way of avoiding this difficulty has been suggested by N. V. Krasnikov and V. A. Matveev, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 270 (1982) [JETP Lett. **35**, 338 (1982)]; D. B. Kaplan and A. V. Manohar, Phys. Rev. Lett. **56**, 2004 (1986).

[13]M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981).

[14]M. A. B. Bèg and H.-S. Tsao, Phys. Rev. Lett. **41**, 287 (1978); M. A. B. Bèg, D. J. Maloof, and H.-S. Tsao, Phys. Rev. D **19**, 2221 (1979).

[15]H. Georgi, Hadronic J. **1**, 155 (1978); G. Segrè and H. A. Weldon, Phys. Rev. Lett. **42**, 1191 (1979); S. Barr and P. Langacker, Phys. Rev. Lett. **42**, 1654 (1979); R. N. Mohapatra and G. Senjanović, Phys. Lett. **79B**, 283 (1978); R. N. Mohapatra and D. Wyler, Phys. Lett. **89B**, 181 (1980); A. Masiero, R. N. Mohapatra, and R. D. Peccei, Nucl. Phys. **B192**, 66 (1981).

[16]A. Nelson, Phys. Lett. **136B**, 387 (1984); **134B**, 422 (1984); **143B**, 165 (1984); S. Barr, Phys. Rev. D **30**, 1805 (1984); Phys. Rev. Lett. **53**, 329 (1984).

[17]K. Hagiwara, F. Halzen, and K. Hikasa, Phys. Lett. **92B**, 103 (1980); **94B**, 557(E) (1980); S. Barr and A. Zee, Phys. Rev. Lett. **55**, 2253 (1985).

[18]By general we mean our statement will remain true unless arbitrary fine tunings are allowed when dealing with the θ parameter.

[19]Here we assume that θ comes only from the phase of the determinant of the quark mass matrix. However, this may be an incomplete treatment of θ as indicated by H. Georgi, T. Tomaras, and A. Pais, Phys. Rev. D **23**, 469 (1981).

[20]Compare J. Ellis, M. K. Gaillard, and P. V. Nanopoulos, Nucl. Phys. **B109**, 213 (1976).

[21]This identity was used to calculate d_n explicitly in Refs. 9 and 20.

[22]Here, in the spirit of understanding the strong *CP* problem without invoking accidental vanishing of Green's functions, we do not consider the cases that (i) $\mathrm{Im}[\delta m_q(p)]\,|_{p^2=m_q^2}\neq 0$ and $(\partial/\partial p^2)\mathrm{Im}[\delta m_q(p)]\,|_{p^2=m_q^2}=0$ or (ii) $\mathrm{Im}[\delta m_q(p)]\,|_{p^2=m_q^2}=0$ and $(\partial/\partial p^2)\mathrm{Im}[\delta m_q(p)]\,|_{p^2=m_q^2}\neq 0$ for some special values of m_q. See Ref. 21 for the detailed calculations on various models.

[23]Jiang Liu, C. Q. Geng, and John N. Ng (to be published).

[24]To our knowledge, the only occasion that appears to be in conflict with our result is the recent claim of K. S. Babu and R. N. Mohapatra [Phys. Rev. Lett. **62**, 1079 (1989)] with $d_n \sim d_d \sim 10^{-26}$–$10^{-28}\,e\,\mathrm{cm}$ at the one-loop level, whereas θ is zero at the same level. In their model, instead of θ being zero for the one-loop diagrams, we estimate that $\theta \sim 0.01$–0.001, which is too large for d_n (Ref. 21).

[25]One can make the computation of θ in the KM model meaningful by introducing a cutoff. Since the coefficient of the divergent term of θ (from seven-loop diagrams) is very small, even a cutoff of the order of the Planck scale will not disturb the stability of θ once we choose it small.

[26]I. B. Khriplovich, Phys. Lett. B **173**, 193 (1986).