Comment on "Vortex-Pair Excitation near the Superconducting Transition of $Bi_2Sr_2CaCu_2O_8$ Crystals"

Recently Martin *et al.*[1] found that the in-plane resistivity $\rho_{ab}(T,H)$ of a $Bi_2Sr_2CaCu_2O_8$ superconducting crystal behaves consistently with the Kosterlitz-Thouless (KT) theory, both above and below an experimentally determined KT transition temperature T_k [where $\rho_{ab}(T<T_k)$ was determined via $\partial(\ln\rho)/\partial(\ln H)$]. This complements previous results of Stamp *et al.*[2] on YBaCuO crystals [where $\rho_{ab}(T<T_k)\sim\rho_N(I/I_0)^{n(T)}$ in *zero field*, $H=0$], suggesting that all the quasi-2D high-T_c systems may have KT transitions to superconductivity.

Now Martin *et al.*[1] assume a conventional fluxon-pairing KT transition, as occurs in thin superconducting films of effective thickness $\zeta_\perp \ll \lambda_{ab}$, the in-plane penetration depth. They then argue that the ratio $K_\perp/K$ of the interplane (between adjacent planes) to in-plane coupling between fluxons is very small. They estimate it to be roughly the conductivity ratio ρ_c/ρ_{ab} ($\sim 10^{-5}$ for the Bi system, and $\sim 10^{-2}$ for YBaCuO). Then KT scaling would persist until the mean-field fluxon separation $r_{ij}\sim\tilde{\lambda}_{ab}$ or r_0 [where $\tilde{\lambda}_{ab}=\lambda_{ab}^2/\zeta_\perp$, and $r_0=\xi_{ab}(K/K_\perp)^{1/2}$], whichever is the smallest.

However, this argument is incorrect. In systems with *short-range* interplane interactions, the crossover from KT scaling to 3D behavior occurs[3] for a mean soliton separation $r_{ij}\sim r_0$. But in any layered system with a *long-range interplane coupling* (in particular, where any unscreened gauge field couples to solitons in the planes), $K_\perp(r_{ij})/K(r_{ij})\sim O(1)$. Then KT scaling cannot start —it immediately crosses over to 3D behavior. Suppose, e.g., the solitons are 2D fluxons, moving on a set of $N=2M+1$ coplanar superconducting sheets of effective thickness $\zeta_\perp$, spaced regularly at $z=\nu D$ ($\nu=0,\pm1,\ldots,\pm M$), and with *no direct interaction* between the planes. Fluxons are described by source terms $\mathbf{\Phi}_\nu(\mathbf{r})=\pm(\mathbf{z}\times\mathbf{r})(\phi_0/2\pi r)$ in the νth plane (where ϕ_0 is the flux quantum), generating a supercurrent $\mathbf{j}_\nu(\mathbf{r})=[\mathbf{\Phi}_\nu(\mathbf{r})-\mathbf{A}(\mathbf{r},\nu D)]/\mu_0\tilde{\lambda}_{ab}$ with the fluxons coupling to the electromagnetic gauge potential $\mathbf{A}(\mathbf{r},z)$. We then have[4] approximately that

$$\tilde{\lambda}_{ab}^2\nabla^2\mathbf{A}(\mathbf{r},z)-\sum_{\nu=-M}^{M}\delta(z-\nu D)[\mathbf{A}(\mathbf{r},z)-\mathbf{\Phi}_\nu(\mathbf{r})]=0\,,\quad(1)$$

which can be solved analytically using Fourier transforms. We then find[5] the following: (i) For r_{ij} less than a separation $L(\nu,D/\zeta_\perp,\tilde{\lambda}_{ab}/ND)$,

$$K_\perp(r_{ij})\simeq -q_iq_j(\phi_0^2/8\pi^2\tilde{\lambda}_{ab})\ln|r_{ij}/\xi_{ab}|\quad(2)$$

and $K(r_{ij})=-K_\perp(r_{ij})$ (with unimportant corrections); $q=\pm$ is the fluxon polarity. Loosely speaking, flux from one fluxon can just as easily thread another fluxon in an adjacent plane as one in the same plane. Equation (1) is exact for $D\gg\zeta_\perp$, and $N\zeta_\perp\gg\tilde{\lambda}_{ab}$. (ii) The "upper cutoff" function $L(\nu,D/\zeta_\perp,\tilde{\lambda}_{ab}/ND)$ is complicated, but usually less than $\tilde{\lambda}_{ab}$ {and as $D\to\zeta_\perp$, as in the real system, L falls rapidly; even a small Josephson overlap between planes will then convert the fluxon interaction to the usual 3D form $\sim\exp[-(r_{ij}/\lambda_{ab})^2]$}. For $r_{ij}>L$, the interaction is no longer logarithmic, and KT scaling breaks down. Note, however, that for thin samples ($N\zeta_\perp\lesssim\lambda_{ab}$) the cutoff L crosses over again to the single-plane limit $L\sim\tilde{\lambda}_{ab}$; i.e., for a sample thickness ~0.5 μm in these systems.

So then, how may we explain the results of Refs. 1 and 2? There seem to be two ways: (a) The solitons are not fluxons.[2] For example, a KT transition may occur amongst holes bound to logarithmically interacting magnetic solitons,[2] and detailed models of this can be developed.[6] However, any soliton pairing mechanism satisfying the criteria specified in Ref. 2 will do. (b) Equation (2) implies that fluxons will link between planes to form (rather floppy) vortex lines. For line separations $<L$ and a very thin crystal these may be roughly modeled as a set of logarithmically interacting rods oriented parallel to $\hat{\mathbf{z}}$, which could exhibit KT scaling. Note that the crystal of Martin *et al.*[1] was only 2 μm thick, so this is a possible explanation of their results. In the much thicker sample of Ref. 2 the KT scaling function $n(T)=1+\pi K(T)$ behaved quite differently from that of Ref. 1 thereby excluding this alternative. One might also try arguing for a "flux creep" explanation[7] for Ref. 1; again, such an explanation would not apply to the zero-field experiments of Ref. 2.

Note added.—Yeh and Tsuei[8] have recently published results which further sharpen the paradox described here.

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[1]S. Martin *et al.*, Phys. Rev. Lett. **62**, 677 (1989).
[2]P. C. E. Stamp *et al.*, Phys. Rev. B **38**, 2847 (1988).
[3]S. Hikami and T. Tsuneto, Prog. Theor. Phys. **63**, 387 (1980).
[4]See J. Pearl, Appl. Phys. Lett. **5**, 65 (1964); P. G. de Gennes, *Superconductivity of Metals & Alloys* (Benjamin, New York, 1966); Eq. (1) generalizes this work.
[5]The following results are easily derived, but lengthy to state in detail; see P. C. E. Stamp (to be published).
[6]Stamp, Ref. 5.
[7]See, e.g., Y. Yeshurun and A. P. Malozemoff, Phys. Rev. Lett. **60**, 2202 (1988); M. Tinkham, Phys. Rev. Lett. **61**, 1658 (1988).
[8]N.-C. Yeh and C. C. Tsuei, Phys. Rev. B **39**, 9708 (1989).

