Photon Pairing and the Strong-Coupling Phase of Massive Quantum Electrodynamics

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(Received 6 December 1988)

By using the well-known expression for the vacuum polarization as the low-energy effective Lagrangian, photon pairing and condensation are discussed. Above a certain critical coupling, pairing instability occurs and magnetic-type condensation is realized with $\langle :F_{\mu\nu}^2: \rangle > 0$. A possible confining property of the condensed vacuum is suggested by the use of the dual potential.

PACS numbers: 12.20.Ds

The question of the existence of the strong-coupling phase in quantum electrodynamics (QED) is now an interesting subject, especially in connection with the technicolor theory[1] and with the anomalous GSI e^+e^- events in heavy-ion collisions.[2] The ladder Schwinger-Dyson equation for massless continuum QED[3] and also lattice numerical calculations[4] suggest that, above a critical value of the fine-structure constant α, a new phase is realized where the electron and the positron condense to form a pair and chiral symmetry is spontaneously broken.

The central problem is to clarify the nature of the condensed vacuum and for that purpose we have to investigate the nonperturbative phenomenon in the photon (A_μ) channel, just as in quantum chromodynamics (QCD) where the nonperturbative nature of the gluon channel is crucial for the confinement of quarks.

In this paper we study the photon pairing phenomenon for *massive* QED using the familiar lowest-order formula for the vacuum polarization[5] as the effective photon Lagrangian. It is obtained by integrating out the electron field and by keeping the first term of the expansion in terms of the number of derivatives of the field strength ($F_{\mu\nu}$). The result is[5]

$$\mathcal{L}(x) = -\tfrac{1}{4}F_{\mu\nu}^2(x) + a[F_{\mu\nu}(x)]^2 + b[F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)]^2 , \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \equiv \partial_{[\mu}A_{\nu]}$, $\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$, $a = \frac{7}{16}b = \alpha^2/90m^4$, and $\alpha = e^2/4\pi$. The symbol $\epsilon_{\mu\nu\rho\sigma}$ is the totally antisymmetric unit tensor, m the electron mass, $F_{\mu\nu}$ the renormalized field strength, and α the renormalized fine-structure constant. Since (1) is the low-energy effective Lagrangian, we use (1) for photon momentum smaller than μ. This μ is the phenomenological parameter and our results depend on μ/m. Adler[6] discussed photon pairing by (1) but with the background nonflat metric. Here we study (1) in the flat metric space. We summarize first the conclusions of this paper. They are obtained by some approximations since (1) cannot be solved exactly. But our approximation scheme is gauge invariant.

(1) Within the ladder approximation, the two-body bound-state equation [Bethe-Salpeter (BS) equation] for the photon has a tachyonic solution above a certain critical coupling constant α_c. This is due to the fact that the a term in (1) produces attraction and the b term repulsion between two photons, and the net effect is an attractive force since $a > b$.

(2) Through the use of the mean-field approximation, the magnetic-type condensation of $F_{\mu\nu}^2$ is shown to occur for $\alpha > \alpha_c'$; $\langle :F_{\mu\nu}^2: \rangle > 0$. Here : : denotes the normal ordering. In order to determine the magnitude of $\langle :F_{\mu\nu}^2: \rangle$ we need higher-order terms such as $(F_{\mu\nu}^2)^3$. The fact $\alpha_c \neq \alpha_c'$ (in fact $\alpha_c'^2 = 2\alpha_c^2$) is common to any theory if it is not solved exactly. The numerical value for the parameter connected with the instability (α_c, for example) and that with the condensation (α_c') differ from each other mainly due to the fact that the many-body effects are not included in the above approximation scheme for the study of the instability.

(3) The magnetic condensation leads to an *antishielding* effect for the electric current. For $\alpha > \alpha_c'$, there arises a possibility that the magnetic potential B_μ, defined below, becomes a good coordinate so we make the dual transformation from A_μ to the dual potential C_μ. The condensation $\langle :F_{\mu\nu}^2: \rangle > 0$ then has a *shielding* property for C_μ: the magnetic shielding. By these observations a possible phase is pointed out where the confinement of the electron is realized. Now we discuss these points separately.

Pairing instability.—Consider the photon four-point Green's function $T^{(4)\mu_1\mu_2\mu_3\mu_4}(k_1,k_2,k_3,k_4)$. This is independent of the gauge chosen due to the form of the interaction given in (1), except for the graphs which contain disconnected free photon propagators. But such graphs do not produce the pole in the two-photon channel so that the BS equation is independent of the gauge to any order of the perturbation. Introducing the expected bound state $|B\rangle$, the BS amplitude can be written in Fourier space as

$$C_{\mu\nu}(p,p') \equiv \int d^4p \int d^4p' (2\pi)^{-8} e^{-ip\cdot x} \times e^{-ip'\cdot y} \langle 0 | A_\mu(x) A_\nu(y) | B \rangle .$$

Now the ladder BS equation is given as

$$C_{\mu\nu}(P,q)=\int d^4k\,(2\pi)^{-4}\hat{T}_{\mu,\nu,\mu',\nu'}(P;q,k)D^{-1\mu'\rho}(\tfrac{1}{2}P+k)D^{-1\nu'\sigma}(\tfrac{1}{2}P-k)C_{\rho\sigma}(P,k)\,,$$

where we have set $p=P/2+q$, $p'=P/2-q$, and $\hat{T}$ is the lowest-order contribution to $T^{(4)}$ apart from the four-momentum conservation factor. The bare propagator has been denoted by $D_{\mu\nu}$, and $C_{\mu\nu}$ satisfies $p^\mu C_{\mu\nu}=p'^\nu C_{\mu\nu}=0$, again due to the type of our interaction. Note also that $C_{\mu\nu}(p,p')=C_{\nu\mu}(p',p)$.

We first discuss the bound state of zero total four-momentum, $P_\mu=0$. Then $C_{\mu\nu}$ has the form $C_{\mu\nu}=(q^2g_{\mu\nu}-q_\mu q_\nu)C(q^2)$. After some algebra and the Wick rotation we see that the constant solution, $C(q^2)=C$, exists if the following relation is satisfied:

$$1=\tfrac{3}{2}\xi\int d^4k/(2\pi)^4,\quad \xi=(2^8/3)(a-b)>0\,.$$

Integrating over the region $k^2<\mu^2$, we conclude that the bound state of $P_\mu=0$ is formed if $\alpha=\alpha_c$ where $\alpha_c\mu^2=2\sqrt{10}m^2$. For $\alpha>\alpha_c$, this bound state is expected to become tachyonic and the system will become unstable. This can be discussed by assuming $P_\mu\neq 0$. Now we have two tensor bases for $C_{\mu\nu}$, one of which is $[(p\cdot p')g_{\mu\nu}-p'_\mu p_\nu]D$ while the other involves the fourth power of the momentum. We neglect the latter since it is not important in the low-energy region, which is in accordance with the approximation scheme of this paper. In this approximation the solution $D=\text{const}$ exists if the following eigenvalue equation is satisfied by P^2:

$$1=\xi\int\frac{d^4k}{(2\pi)^4}\left[\frac{(P^2/4-k^2)^2}{(P^2/4+k^2)^2-(P\cdot k)^2}+\frac{1}{2}\right],\qquad(2)$$

where a Wick rotation has been performed. For small spacelike P, (2) becomes $1=(3/64\pi^2)\xi\mu^4(1-P^2/\mu^2+\cdots)$. We therefore conclude that *for $\alpha>\alpha_c$, the tachyonic bound state is formed in the two-photon 0^+ channel* which is the signal of the instability. Here α_c is given by $\alpha_c\mu^2=2\sqrt{10}m^2$. The appearance of the combination $\alpha_c\mu^2$ is natural since for any QED diagram the photon propagator $D_{\mu\nu}(k)$ is multiplied by e^2 so that the effective coupling constant is given by $e^2\int^\mu d^4k\,D_{\mu\nu}(k)\sim e^2\mu^2$. In Fig. 1, the relation between P^2 and α is plotted.

Magnetic condensation; electric antishielding.—The above picture of the instability can also be understood in the following way. We extract the term $\sigma_0\equiv\langle[F_{\mu\nu}(x)]^2\rangle_0$ from the quartic terms of (1) where $\langle\cdots\rangle_0$ denotes the expectation value in the normal vacuum state. For this purpose the formula[7]

$$\langle F_{\mu\nu}(x)F_{\rho\sigma}(x)\rangle_0=\tfrac{1}{12}(g_{\rho\mu}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho})\langle F^2_{\mu\nu}(x)\rangle_0\qquad(3)$$

is used. We throw away the constant term. As for the bilinear term, we get the result $L=-\frac{1}{4}(1-\xi\sigma_0/8)\times(\partial_{[\mu}A_{\nu]})^2$. For $\xi\sigma_0/8>1$, the coefficient of $F^2_{\mu\nu}$ changes sign, which is a signal of instability. In fact, the Hamiltonian corresponding to the above L is given by

$$H=\tfrac{1}{2}(1-\xi\sigma_0/8)[\mathbf{E}^2(x)+\mathbf{H}^2(x)]\,,$$

where $E_1=F_{01}$, $H_1=F_{23}$, etc. For $\xi\sigma_0/8>1$, the photon condensation occurs in the direction $\langle{:}(\mathbf{E}^2+\mathbf{H}^2){:}\rangle>0$. Note here that since the constant term has been discarded the operator in the above Hamiltonian is, in fact, the normally ordered one. Using the relation

$$\langle{:}(\mathbf{E}^2+\mathbf{H}^2){:}\rangle=\tfrac{1}{2}\langle{:}F^2_{\mu\nu}(x){:}\rangle\equiv\tfrac{1}{2}\sigma$$

obtained by the same formula as (3), we conclude that *for $\alpha>\alpha'_c$ magnetic-type condensation of $F^2_{\mu\nu}(x)$, i.e., $\sigma>0$, occurs.* Here α'_c is given by $\alpha'_c\mu^2=4\sqrt{5}m^2$. We have used $\sigma_0=6\int d^4k/(2\pi)^4$ in the Wick-rotated form. There is a discrepancy of a factor of $\sqrt{2}$ between α_c and α'_c. This is due to the difference of the approximation scheme as has been stated.

The important fact is that the vacuum fluctuation of the photon field has an antishielding property since the dielectric constant $\epsilon=1-\xi\sigma_0/8$ is less than unity and it becomes zero at $\alpha=\alpha'_c$: complete antishielding. The central problem is to clarify the nature of the condensed vacuum which is realized above the critical coupling. It is difficult to give a complete answer to this problem but we suggest a possible confining phase by a simple argument.

Dual potential; magnetic shielding and confinement.—Let us rewrite (1) in term of the dual variables which is quite easily done in the Abelian case. We have simply to change the variable from A_μ to $F_{\mu\nu}$ by introducing the δ function $\delta(\partial^\mu\tilde{F}_{\mu\nu})$ corresponding to the Bianchi identity in the functional measure of the path-integral formula:

$$\int[dF_{\mu\nu}]\prod_x\delta[\partial^\mu\tilde{F}_{\mu\nu}(x)]\exp\left[i\int d^4x\,L(x)\right].$$

The field strength can always be decomposed as $F_{\mu\nu}=\partial_{[\mu}A_{\nu]}+\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^{[\rho}B^{\sigma]}$, where $\partial_\mu A^\mu=\partial_\mu B^\mu=0$ and the Bianchi constraint becomes $\delta(\Box B_\mu)$.

The dual potential C_μ is introduced through the parametric representation of $\delta(\Box B_\mu)$ so that the Lagrangian (1) becomes $L(x)+\frac{1}{2}C_\mu\Box B^\mu$ and the functional integra-

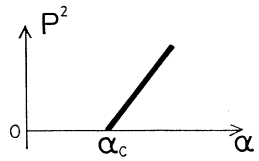

FIG. 1. The eigenvalue P^2 as a function of α. $P^2>0$ is spacelike.

tion is to be done over A_μ, B_μ, and C_μ. The bilinear term is now

$$L_0 = -\tfrac{1}{4}(\partial_{[\mu}A_{\nu]})^2 + \tfrac{1}{4}(\partial_{[\mu}B_{\nu]})^2 + \tfrac{1}{2}C_\mu \Box B^\mu . \qquad (4)$$

The B^2 term here has wrong sign but it does not cause any trouble since after C_μ integration the B_μ field decouples from everything due to the factor $\delta(\Box B^\mu)$. Now we extract the term $\sigma_0 = \langle [\partial_{[\mu}A_{\nu]}(x)]^2 \rangle_0$ from the Lagrangian (1) which is now written in terms of A_μ and B_μ. Then the bilinear term is

$$-\tfrac{1}{4}(1-\xi\sigma_0/8)[(\partial_{[\mu}A_{\nu]})^2 - (\partial_{[\mu}B_{\nu]})^2] + \tfrac{1}{2}C_\mu \Box B^\mu .$$

For $\xi\sigma_0/8 > 1$, the A_μ field becomes tachyonic but at the same time the B_μ field turns into a stable coordinate, suggesting that the strong-coupling phase is described by the magnetic potential.[8]

Therefore, we switch to the dual representation after extracting the condensed value σ from L of (1).[9] This is done by replacing σ_0 by σ in the last expression. The dual Lagrangian L_D is given by integrating over A_μ and B_μ as $L_D = -\frac{1}{4}(1-\xi\sigma/8)^{-1}(\partial_{[\mu}C_{\nu]})^2$. Since $\sigma > 0$ for the strong-coupling phase, the photon condensation has the effect of shielding for the dual potential—magnetic shielding. If we measure the condensation in terms of C_μ, it is seen that $\rho \equiv \langle :[\partial_{[\mu}C_{\nu]}(x)]^2: \rangle$ is opposite in sign to σ):

$$\rho = -(\xi\sigma/8)6\int d^4k/(2\pi)^4 = -\tfrac{32}{3}(a-b)\sigma\sigma_0 .$$

This is natural since A_μ and C_μ are dual to each other.

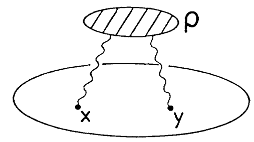

FIG. 2. Graph contributing to the string tension. The loop is the Wilson loop and at x or y we have a factor $\epsilon_{\mu\nu\rho\sigma}\partial^{[\rho}B^{\sigma]}(x)$ or $\epsilon_{\mu'\nu'\rho'\sigma'}\partial^{[\rho'}B^{\sigma']}(y)$.

The condensation of the dual potential has a profound effect. The condensation is represented by the graphs with an infinite number of external C_μ lines and therefore if we calculate the expectation value of any operator the integration over C_μ does not necessarily lead to $\partial_{[\mu}B_{\nu]} = 0$. This can easily be seen in the extreme case where the integration is dominated by the condensed value of $(\partial_{[\mu}C_{\nu]})^2$. The complete dynamics of the C_μ field is governed by the Lagrangian obtained by integrating over A_μ and B_μ (or over $F_{\mu\nu}$) using the Lagrangian $L(x) + \frac{1}{2}C_\mu \Box B_\mu$. We assume, based on the above arguments, the following Lagrangian for the C_μ field in the condensed phase:

$$L_C(x) = -\tfrac{1}{4}[1+(\rho/\sigma_0)]^{-1}(\partial_{[\mu}C_{\nu]})^2, \quad \rho < 0 . \qquad (5)$$

This is the Lagrangian for the vacuum sector of A_μ and B_μ.

A direct consequence is the confining property of this phase. Let us calculate the expectation value of the Wilson loop,

$$\left\langle \exp\left[ie\oint A_\mu(x)dx^\mu\right]\right\rangle \cong 1 - e^2\left\langle \int d\sigma^{\mu\nu}(x)F_{\mu\nu}(x)\int d\sigma^{\mu'\nu'}(y)F_{\mu'\nu'}(y)\right\rangle .$$

Since $F_{\mu\nu}$ now contains a B_μ term we see that nonzero string tension emerges through the graph shown in Fig. 2, where the B_μ-C_μ transition comes from the term $C_\mu \Box B^\mu$ in (4). We note here that the $\partial_{[\mu}A_{\nu]}$ term and the unity in $1+(\rho/\sigma_0)$ do not contribute to the string tension. For a large area S enclosed by the loop, we get

$$\left\langle \exp\left[ie\oint A_\mu(x)dx^\mu\right]\right\rangle \cong 1 - Se^2(-\rho/\sigma_0)\int d^2k_\perp/(2\pi)^2 ,$$

where $\int d^2k_\perp$ indicates integration over the momentum perpendicular to the plane of the loop. We get, therefore, the string tension $\alpha = e^2(4\pi/3)(-\rho/\mu^2)$. Here we have cut off the integration at $|\mathbf{k}_\perp| = \mu$ since $\mathbf{k}_\parallel = 0$ for large S.

The confinement picture we have is not of the flux-tube type since the B_μ or C_μ field remains massless. Instead of flux squeezing we have a linear potential between the electron and positron with the electric flux spreading over the whole space.

We finally discuss several points. They should of course be studied in a more complete way.

(1) The long-range van der Waals force exists in the condensed phase, which will lead to rich phenomenological consequences. These may affect the GSI experiment of anomalous e^+e^- events.

(2) For massless QED, we know that the e^+e^- pair condensation occurs and the critical coupling has been known to exist leading to the spontaneous chiral-symmetry breaking. We are, of course, interested in the relationship between our photon condensation and the electron-positron condensation, but in order to discuss this problem we have to utilize an expression different from (1) since the formula is not applicable to the massless QED.

(3) If the above phenomenological approach is correct, the physical picture of confinement will be different for the strong-coupling phases of QED and of QCD. The differences appear in three ways. (i) For QCD we believe that the critical coupling constant is zero due to the asymptotic freedom, whereas for QED we will have a finite critical coupling constant. The reason for this discrepancy is most easily seen by looking at the instabil-

ity equation for gluon pairing in QCD[10] and for e^+e^- pairing in QED[10] and our Eq. (3) for photon pairing in QED. We see that it is due to the different behavior of the kernel of the instability equation near small momentum. The relationship of our new phase with the zero of the β function in the renormalization-group equation should also be investigated. (ii) As has been stated above, for the condensed phase of QED, the electric flux tube will not be formed which is quite different from what is expected for QCD. The cause of this difference is that for QCD the dual potential is expected to acquire mass, but for QED it will remain massless. (iii) The gluon will be confined in QCD, but the photon will not be. In fact, for Abelian theory there is a formal proof[11] for the existence of the zero-mass pole of the Green's function in the vector channel. The same argument is expected to be applicable to our case and we have a zero-mass pole which prevents the formation of the flux tube. The precise excitation spectrum in the condensed phase has to be investigated by a more reliable method.

The author thanks Professor T. Kugo for discussion on the existence of the massless pole in Abelian gauge theory.

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