Nonrenormalization of the Superstring Tension

Atish Dabholkar[(a)] and Jeffrey A. Harvey[(b)]
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544
(Received 5 June 1989)

It is argued that the superstring tension is not renormalized in perturbation theory for vacua which preserve $N=1$ spacetime supersymmetry. Some implications of this result for macroscopic superstrings are discussed, as well as some analogies between macroscopic superstrings and solitons in supersymmetric theories.

PACS numbers: 11.17.+y, 11.30.Pb

In superstring theory, the only really fundamental constant is the string tension μ. All other constants of nature are, at least in principle, related to the string tension for a given vacuum configuration. It is of interest, therefore, to know how quantum corrections alter the classical value of the string tension because it is the *renormalized* string tension that would be measured at low energies. For example, if cosmic superstrings were to exist, the deficit angle measured from double images of quasars would directly determine the renormalized superstring tension. Many authors have addressed the questions of the renormalization of low-energy couplings and mass renormalization for massive string states, but to our knowledge the renormalization of the string tension itself has not been addressed. As we will see, for spacetime supersymmetric vacua, the superstring tension receives no renormalization in perturbation theory. If the scale of supersymmetry breaking is much below the Planck scale, then we would expect the renormalized string tension to be very close to its classical value.

To evaluate this renormalization we first of all have to define what we mean by the superstring tension. Consider a vacuum configuration of the form $M^3 \times S^1 \times K$. Here K can be any internal $N=2$ superconformal theory that results in $N=1$ spacetime symmetry. M^3 is three-dimensional Minkowski space and S^1 is a large circle of radius R, say in the z direction. A closed string state that wraps around this circle will look like a cosmic superstring in four dimensions. We can *define* the superstring tension as the energy per unit length of such a state with winding number one in the limit that R goes to infinity. To calculate the energy of this winding state we evaluate the mass shift in perturbation theory by evaluating the two-point function of the corresponding vertex operator on genus-g Riemann surfaces.

Before proceeding with the calculation, let us discuss what we expect to find. At large distances a macroscopic superstring should look much like a cosmic axion string.[1] Classically, an axion string receives an ultraviolet-divergent contribution to its self-energy as a result of the surrounding static axion field.[2] This is quite analogous to the quadratically divergent renormalization of the electron mass due to the static Coulomb field in the Dirac-Lorentz theory of classical electrons. Since there is an axion in the massless spectrum of the superstring, with a coupling to a macroscopic superstring which is like that of an axion string, we expect classically that this coupling should renormalize the string tension. Quantum corrections typically soften the classical ultraviolet divergence and superstring theory is in fact completely finite in the ultraviolet. We therefore expect an ultraviolet finite-but-nonzero contribution to the string tension from the interaction of the string with its various modes.

Finiteness of string theory, of course, does not preclude infrared divergences of the amplitudes and such divergences, if present, signify important physics. In our case, there is a very good physical reason to expect the two-point function of a cosmic winding state to diverge in the infrared. A classical string coupled to an axion is much like a vortex line of a spontaneously broken global symmetry. The energy contained in the surrounding static axionic field of such a global string is well known to be infrared divergent and goes as $\mu \ln(\mu R)$, where R is the infrared cutoff, the radius of the string in our case. This classical infrared divergence should show up in the two-point amplitude at the quantum level. Thus, if we take the radius R of the winding state to be sufficiently large, we should be able to extract the string-tension renormalization by looking at the coefficient in front of the leading-log divergence and comparing it with the classical result.

We will show explicitly that the two-point function vanishes on the torus, and then argue that it vanishes to all orders in perturbation theory. We will then discuss how the apparent contradiction between this result and the above discussion can be resolved. Some connections between macroscopic superstrings and solitons in supersymmetric theories will also be presented.

The mass shifts and decay rates of massive states are related to the real and the imaginary parts of the two-point amplitude. For a stable winding state with vertex operator V the mass shift is directly given by the two-point function. The two-point function A at order g is given by integrating the correlation function of vertex operators $\langle VV \rangle$ over the genus-g moduli space with the appropriate measure and summing over spin structures. The right-moving part of the vertex for the lightest winding state in the 0 picture, evaluated in its rest frame, is

$$W=\zeta_\mu\cdot(\partial X^\mu+ik\cdot\psi\psi^\mu)e^{ik\cdot X},$$
$$k=(E,0,0,k^3),\quad \zeta\cdot k=0. \tag{1}$$

The mass-shell conditions are

$$\tfrac{1}{2}k^2=\tfrac{1}{2}\bar{k}^2+\bar{N}-1=0,$$
$$k^3=(M/2R+LR),\quad \bar{k}^3=(M/2R-LR), \tag{2}$$

where $\bar{N}$ is the left-moving number operator and $\bar{k}=(E,0,0,\bar{k}^3)$. It follows from (2) that for unit winding number ($L=1$) we have two solutions: ($M=-1,L=1,\bar{N}=0$) and ($M=0,L=1,\bar{N}=1$) of energy $R^2+1/4R^2-1$ and R^2, respectively. Thus, for a large enough radius, the former has the lowest energy. The left-moving vertex for this state has no oscillators ($\bar{N}=0$) and has only the tachyonic part, $e^{i\bar{k}\cdot\bar{X}}$.

It will be useful to also consider the infinite tower of bosonic states created by adding various left-moving excitations to the right-moving ground state. For a given value of $\bar{N}$ these states have momenta and winding satisfying $\bar{N}=1-ML$ and energy $E=(k^3)^2=(M/2R+LR)^2$. The right-moving part of the vertex operator for all these states is given by (1). Note also that (1) is the right-moving part of the vertex operator for a massless gauge boson in 3+1 dimensions.

Without the correlator $\langle VV\rangle$, the one-loop amplitude A is just the cosmological constant at one-loop level and vanishes by supersymmetry after summing over spin structures. Thus, the nonzero part of the answer can come only from the spin-structure-dependent part of the correlation function. We therefore need to concentrate on the correlation function,

$$\langle ik_1\cdot\psi\zeta_1\cdot\psi e^{ik_1\cdot X}e^{i\bar{k}_1\cdot\bar{X}}(v_1,\bar{v}_1)ik_2\cdot\psi\zeta_2\cdot\psi e^{ik_2\cdot X}e^{i\bar{k}_2\cdot\bar{X}}(v_2,\bar{v}_2)\rangle. \tag{3}$$

In this correlation we have four fermions, i.e., enough to soak up the fermion zero modes in the path integral in order that it does not vanish trivially by supersymmetry. After summing over spin structures the one-loop amplitude is proportional to the kinematic factor,

$$\zeta_1\cdot k_2\zeta_2\cdot k_1-\zeta_1\cdot\zeta_2k_1\cdot k_2,$$

which results from performing the fermion contractions. Since $k_1=-k_2$ by momentum conservation, $\zeta_1\cdot k_2=\zeta_2\cdot k_1=k_1\cdot k_2=0$, and hence this factor vanishes. As a result the mass shift vanishes at one-loop level. As discussed in Ref. 3 there are some theories in which this naive argument fails. In theories with anomalous U(1) factors which lead to supersymmetry-breaking D terms there can be singularities of the form $1/k_1\cdot k_2$ arising from the integration over the relative position of the vertex operators. We have checked by an explicit evaluation of the one-loop amplitude that such singularities do not arise here. At higher loop level there are many subleties in superstring perturbation theory, and it is not clear that nonrenormalization theorems for N-point functions hold order by order in perturbation theory without adjustment of the string tension.[4] However, we would expect that any formulation of superstring perturbation theory in which mass shifts for massless gauge bosons vanish (as would be expected by gauge invariance) should also lead to vanishing mass shifts for the tower of states discussed here since the right-moving part of the vertex operator for massless gauge bosons is the same as for these states. In particular, this would show that the superstring tension is not renormalized in perturbation theory. Note also that the vanishing of the imaginary part of the two-point function for this tower of states implies that they are all stable, since otherwise the two-point function would have to have an imaginary part by unitarity. The stability of these states at tree level is easily checked.

The vanishing of the mass shift at the quantum level seems in contradiction with our original classical expectation. We will now show that a more careful analysis of the infrared behavior of the energy of a winding state leads to a result consistent with the above.

We want to calculate the contribution to the energy per unit length of a winding state from the long-range part of the axion, graviton, and dilaton fields. As in the string calculation, we will work in perturbation theory about flat space. To do this we start from a σ-model action S that describes the coupling of a string to the metric $G_{\mu\nu}$, antisymmetric tensor $B_{\mu\nu}$, and dilaton Φ. Demanding conformal invariance of S leads to the equations of motion for the background fields. Our point of view will be somewhat different. We wish to calculate the equations satisfied by the massless fields in the presence of a string source. We will thus use S to evaluate the source terms for the background fields in the presence of a macroscopic string source. This could also be done by simple vertex operator calculations as in Ref. 1.

Following the treatment in Ref. 5, it is convenient to perform a conformal rescaling $G_{\mu\nu}\to e^{4\phi/(D-2)}G_{\mu\nu}$ in D spacetime dimensions. We can then incorporate the equations of motion which follow from conformal invariance and the classical couplings of the massless fields to the string in the combined action

$$-\frac{\mu}{2}\int d^2\sigma(\sqrt{\gamma}\gamma^{mn}\partial_mX^\mu\partial_nX^\nu G_{\mu\nu}e^{4\Phi/(D-2)}+\epsilon^{mn}\partial_mX^\mu\partial_nX^\nu B_{\mu\nu})$$
$$+\frac{1}{16\pi G}\int d^Dx\sqrt{G}\left[R-\frac{4}{D-2}\partial_\mu\Phi\,\partial^\mu\Phi-\frac{1}{12}e^{-8\Phi/(D-2)}H^{\mu\nu\lambda}H_{\mu\nu\lambda}\right].$$

We now take $D=4$ and consider a macroscopic string running along the z axis and take $z=\sigma$ and $t=\tau$ for the world-sheet coordinates. For this source the linearized equations of motion which follow from the above action are

$$2\nabla^2\Phi=\nabla^2 h_{11}=\nabla^2 h_{22}=\nabla^2 B_{03}=16\pi G\mu\delta(x)\delta(y),$$

which are easily solved in cylindrical coordinates to give

$$2\Phi=h_{11}=h_{22}=B_{03}=8G\mu f(\rho),$$

where $f(\rho)=\ln(\rho/\rho_0)$ with ρ_0 an arbitrary cutoff corresponding to the width of the string core.

The leading contribution to the energy density outside the string is then given by $T_{00}+t_{00}$, where $T_{\mu\nu}$ is the energy-momentum tensor of the antisymmetric tensor and dilaton and $t_{\mu\nu}$ is the gravitational energy-momentum pseudotensor. Using

$$16\pi G T_{\mu\nu}=\tfrac{1}{2}(H_{\mu\nu}^2-\tfrac{1}{6}G_{\mu\nu}H^2)e^{-8\Phi/(D-2)}+\frac{8}{D-2}[\partial_\mu\Phi\partial^\nu\Phi-\tfrac{1}{2}G_{\mu\nu}(\partial_\mu\Phi\partial^\mu\Phi)]$$

and the usual expression for $t_{\mu\nu}$, we find in the linearized approximation with $D=4$,

$$16\pi G T_{00}=(8G\mu)^2[\tfrac{1}{2}(\partial_i f\partial_i f)+\tfrac{1}{2}(\partial_i f\partial_i f)],$$

$$16\pi G t_{00}=-(8G\mu)^2(2f\partial_i\partial_i f+\partial_i f\partial_i f).$$

The $f\partial^2 f$ term in t_{00} is a divergent self-energy part which does not contribute to the logarithmically divergent part of the energy density. The $(\partial f)^2$ terms separately give rise to logarithmically divergent contributions to the energy density, but the dilaton and antisymmetric tensor contributions are precisely canceled by the graviton contribution. Thus we see that the logarithmic divergence vanishes to lowest order in perturbation theory about flat space, in agreement with the superstring calculation. The superstring analysis indicates that this should also be true in higher orders of perturbation theory.

This delicate cancellation occurs purely between bosonic fields, but can probably be understood by embedding the theory in a supersymmetric theory in which the bosonic fields are part of the same supermultiplet. The situation here is in fact reminiscent of magnetic monopoles in the Prasad-Sommerfield limit. The details will be presented elsewhere,[6] but let us sketch some of the connections here.

In SU(2) gauge theory spontaneously broken to U(1) by an adjoint Higgs scalar with vanishing potential (Prasad-Sommerfield limit) there is a universal mass formula for the masses of particle states at the classical level given by

$$(\text{mass})^2=v^2(e^2+g^2),\tag{4}$$

where v is the Higgs expectation value, e is the electric charge of the state, and g is the magnetic charge. This shows that classically a magnetic monopole with, say, charge two, is neutrally stable into decay into two monopoles of charge one. One manifestation of this is the fact that the force between two monopoles of charge one vanishes due to a cancellation between vector repulsion and scalar attraction.[7] This is also connected to the fact that the classical monopole equations can be reduced to first-order equations and possess exact multimonopole solutions.[8] Montonen and Olive[9] have conjectured that there is a dual theory in which the roles of e and g are interchanged, and the magnetic monopoles of one theory become the gauge bosons in the dual picture.

Some support for this conjecture and a deeper understanding of the formula (4) is obtained by embedding the theory into an $N=2$ supersymmetric gauge theory with the gauge fields and the Higgs scalar in the same supermultiplet. The mass formula (4) then becomes exact in perturbation theory and arises as a consequence of the appearance of the magnetic and electric charges in a central extension of the supersymmetry algebra.[10]

Let us compare this to the infinite tower of string states we have discussed. Reinstating the string scale, they obey a classical mass formula

$$(\text{mass})^2=\frac{M^2}{R^2}+\frac{L^2R^2}{\alpha'^2}+\frac{2}{\alpha'}(\bar{N}-1)=\left(\frac{M}{R}-\frac{LR}{\alpha'}\right)^2,\tag{5}$$

where $\alpha'=\frac{1}{2}\pi\mu$, μ is the string tension, and M and L label the momenta and winding, respectively. A string of winding two is thus neutrally stable into decay into two strings of winding one. We would thus expect that the force between two strings of winding one should also vanish due to cancellation between the graviton, antisymmetric tensor, and dilaton. The string calculation indicates that this formula is in fact exact in perturbation theory. Again we expect that this can be understood in terms of a central extension of the supersymmetry algebra for the low-energy effective-field theory. Finally, it is well known that in string theory there is a duality symmetry which interchanges M and L and takes $R\rightarrow\frac{1}{2}R$. This interchanges electric and magnetic world-sheet charges. From a spacetime point of view, there are two U(1) gauge symmetries when we compactify one dimension on a circle, with gauge bosons corresponding to the components $G_{\mu 3}$ and $B_{\mu 3}$ of the metric and antisymmetric tensor field. The duality symmetry interchanges the electric charges of these two U(1) gauge groups.

These considerations also make us suspect that there will exist exact multistring solutions of the equations of motion for the graviton, dilaton, and antisymmetric tensor fields, albeit with explicit source terms for the strings. One might speculate that superstrings themselves might arise as some sort of soliton sector of an underlying theory, but there seems to be little evidence to support this beyond the analogies discussed here.

This work was supported in part by NSF Contract No. PHY 80-19754. J.A.H. was also supported by NSF Grant No. PHY 86-58033 and by the A. P. Sloan Foundation.

[(a)]Bitnet address: asd@pupthy.princeton.edu.

[(b)]Bitnet address: jah@pupthy.princeton.edu or jah@pucc.bitnet.

[1]E. Witten, Phys. Lett. **153B**, 243 (1985).

[2]F. Lund and T. Regge, Phys. Rev. D **14**, 1524 (1976); A. Dabholkar and J. Quashnock, Princeton University Report No. PUPT-1129, 1989 (to be published).

[3]J. Atick, L. Dixon, and A. Sen, Nucl. Phys. **B292**, 109 (1987); M. Dine, I. Ichinose, and N. Seiberg, Nucl. Phys. **B293**, 253 (1987).

[4]See J. Atick, G. Moore, and A. Sen, Nucl. Phys. **B307**, 221 (1987), and references therein; P. Nelson, Phys. Rev. Lett. **62**, 993 (1989); H. La and P. Nelson, *ibid.* **63**, 24 (1989).

[5]C. G. Callan, D. Friedan, E. J. Martinec, and M. J. Perry, Nucl. Phys. **B262**, 593 (1985).

[6]A. Dabholkar and J. Harvey (to be published).

[7]N. Manton, Nucl. Phys. **B126**, 525 (1977).

[8]E. B. Bogomolny, Yad. Fiz. **24**, 861 (1976) [Sov. J. Nucl. Phys. **24**, 449 (1976)]; S. Coleman *et al.*, Phys. Rev. D **15**, 544 (1977).

[9]C. Montonen and D. Olive, Phys. Lett. **72B**, 117 (1977).

[10]E. Witten and D. Olive, Phys. Lett. **78B**, 97 (1978).