Quenching of the Hall Resistance in Ballistic Microstructures: A Collimation Effect

Harold U. Baranger
AT&T Bell Laboratories, 4G-314, Holmdel, New Jersey 07733

A. Douglas Stone
Section of Applied Physics, Yale University, Box 2157, New Haven, Connecticut 06520
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We present both calculations and a physical interpretation of the observed suppression (quenching) of the low-field Hall resistance in quasi-one-dimensional ballistic microstructures. We find that quenching is due to a property of the contact geometry and is not intrinsic to the quasi-one-dimensional limit. Generic quenching, as observed experimentally, is found only when the width of the wires is gradually increased near the junction to the Hall probes. The resulting collimation of the electrons in the forward direction reduces the sensitivity to a magnetic field.

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Several recent transport experiments have demonstrated novel quantum-mechanical effects in small pure samples where the transport is ballistic. Examples include the nonlocal "bend resistance,"[1] electron-focusing fine structure,[2] and anomalous quantization of the Hall resistance[3] in four-probe measurements and the quantized point-contact resistance in two-probe measurements,[4] all of which have received adequate theoretical explanation.[1,3,5,6] However, one of the earliest of these discoveries, the disappearance or "quenching" of the low-magnetic-field Hall resistance,[7] has resisted a complete theoretical explanation.

Roukes *et al.* observed[7] that the Hall resistance, R_H, of very narrow high-mobility wires (width ≈ 100 nm) measured in the standard cross geometry typically is suppressed below the expected two-dimensional value near zero field. Later work confirmed the effect and clearly indicated that it was *generic* in the sense that it occurred for differently fabricated ballistic microstructures over a wide range of carrier densities.[1,8-11] Typically, R_H versus magnetic field, although not completely structureless, has an average slope near zero when B is less than a characteristic value which varies with the width of the wire. Earlier theoretical work[12] argued that the effect should always occur in the quasi-one-dimensional limit since there is a crossover length scale, $3\lambda_{\text{th}} = 3(\hbar/2k_F eB)^{1/3}$, at which the electronic states at the Fermi energy change from being extended across the wire to being localized by the Lorentz force at one edge. It was proposed but not demonstrated that a wire with width less than this length scale should always show quenching. Subsequent microscopic calculations assuming idealized weakly coupled Hall probes[13,14] failed to show quenching. Our calculations below show that quenching is not an intrinsic property of the Hall resistance of narrow wires even with strongly coupled probes.

We find that generic quenching only arises due to a property of the contact geometry, namely, the widening of the wires near the junction region. This widening, which is typically present in quasi-one-dimensional microstructures but need not be, acts like an acoustic horn and sets up a nonequilibrium distribution in which modes with high longitudinal momentum are preferentially populated. The resulting collimation of the electrons injected into the junction suppresses R_H since, as shown below, electrons in these modes contribute little to R_H.

The first step in calculating the resistance of a particular phase-coherent structure in a magnetic field is to find a correct expression for the four-probe resistance, which is valid before spatial or impurity averaging, and is able to account for a specific measuring geometry. Other work[12,13,15,16] has relied on the Landauer-type formulation proposed by Büttiker[17] which, although physically appealing on a number of grounds, had not been connected to standard linear-response theory. The Büttiker approach considers the linear response of a multiprobe system to fixed potentials V_n applied to leads n which cause current to flow in leads m according to $I_m = \sum g_{mn} V_n$. His fundamental result is that $g_{mm} = T_{mn}$ (at $T=0$ for $m \neq n$), where T_{mn} is the sum of all the transmission coefficients from lead n to m between the modes at the Fermi surface. Since in the presence of a magnetic field there are always circulating currents arising from states below the Fermi surface, it was not obvious that this expression was correct in arbitrary magnetic field.

We have recently shown that Büttiker's result[17] can be derived rigorously from linear-response theory in an arbitrary magnetic field; details will be presented elsewhere.[18] Our calculation leads to expressions for the nonlocal conductivity tensor, $\sigma(x,x')$, and for the g_{mn} in terms of the exact eigenstates or Green's functions of the noninteracting problem. We find that although $\sigma(x,x')$ (which describes both diamagnetic and transport currents) does depend on all filled states, the g_{mn} (and hence R_H) are determined solely by wave functions at the Fermi surface, and can be transformed to yield precisely Büttiker's result. For a symmetric cross structure in which each lead supports $N_{\rm chan}$ transverse channels at the Fermi energy, the Hall resistance is

$$R_H = \frac{h}{e^2}\,\frac{T_R - T_L}{2T_F(T_F+T_R+T_L)+T_R^2+T_L^2}\,, \qquad (1)$$

where T_L, T_R, and T_F are the transmission intensities to turn left, turn right, and go forward, respectively, at the Fermi energy. We find the transmission intensities needed in Eq. (1) by applying the recursive Green's-function method to a nearest-neighbor tight-binding Hamiltonian for a square lattice of the desired geometry.[19]

The first structure that we study is a cross formed of perfect straight wires of width W with infinite hard walls (Fig. 1).[20] Very recently, other groups[15,16] have presented results for similar structures; we present our independent results here since the comparison of the straight wire case and the graded-width wire case is essential for a clear understanding of quenching.

The Hall-resistance traces in Fig. 1(a) show that a variety of behavior occurs in the perfect cross, including quenching, a slope greater than the two-dimensional value, and at higher fields the quantum Hall effect. (Quantum Hall plateaus for the multichannel cases shown are off scale.) To present the behavior in the low-field regime more systematically, Fig. 1(b) shows the slope of the $R_H(B)$ curve near $B=0$ normalized to the two-dimensional value. There is a great deal of structure in this curve including cusps at the thresholds of the subbands;[15] the energy range shown is from threshold up to $N_{\rm chan}=6$. It is apparent that quenching of R_H occurs only in very restricted ranges of energy for this structure and only for $N_{\rm chan}\le 2$. For higher channel fillings, R_H essentially oscillates around its classical value and reasonable averages over energy to include the effect of temperature give approximately the classical value. The same qualitative features are found for junctions with harmonic instead of hard-wall confinement.[16,21] Thus the behavior of these structures, and hence the mechanism discussed in Ref. 16, does not explain the generic quenching observed experimentally[7-11] which in almost all cases corresponds to the range $N_{\rm chan}\gtrsim 3$. Furthermore, Fig. 1(b) demonstrates that the slope found using a very small magnetic field ($3\lambda_{\rm th} > 100W$, solid) agrees with that fit over a broad range of fields (up to $3\lambda_{\rm th}\approx W/2$, dotted). Since the broad field range includes values well above the quenching threshold of Ref. 12 while the small field is far below this threshold, this demonstrates that the length-scale arguments proposed previously[12] are not relevant to the occurrence of quenching.

To obtain generic quenching it is crucial to include the widening of the wires near the junction. While the electrostatic potential profile for a junction of two such wires has not been calculated or measured, it is certain that there is considerable rounding of the corners at the junction. We include this in our structures by slowly increasing the width of each lead as the junction is approached

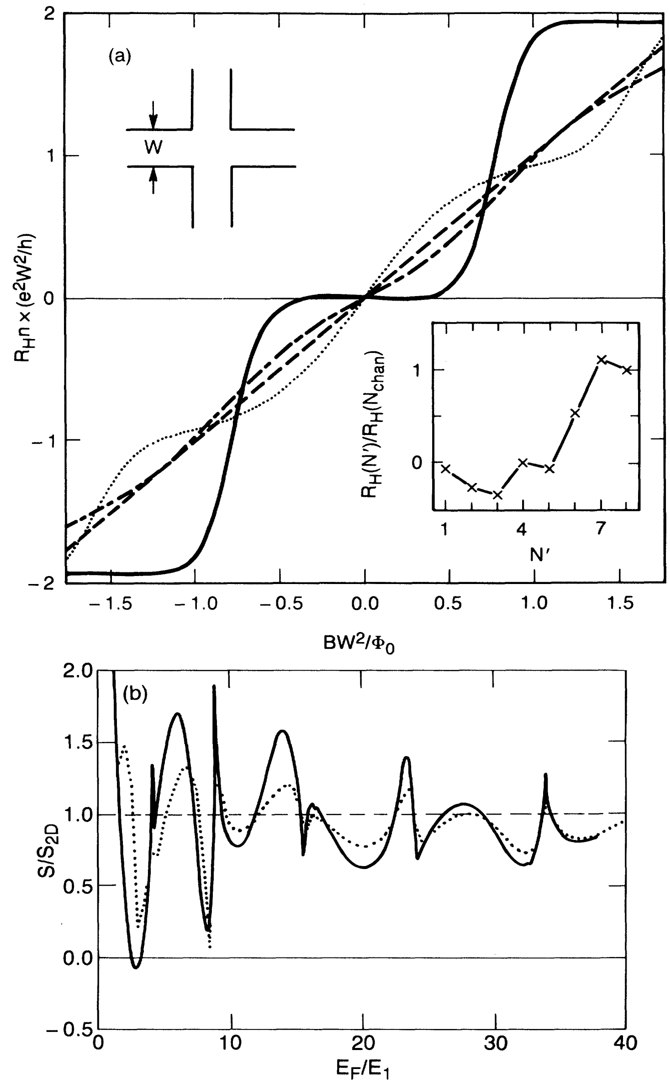

FIG. 1. (a) The product of the Hall resistance and the density as a function of the magnetic field for the perfect hard-wall cross structure shown. At different energies, R_H quenches before rising to the $n=1$ quantum Hall plateau ($E_F=2.7E_1$, $N_{\rm chan}=1$, solid line) is greater than the two-dimensional value ($E_F=14E_1$, $N_{\rm chan}=3$, dotted line) or is close to the 2D value ($E_F=20E_1$, $N_{\rm chan}=4$, dash-dotted line). The 2D value is dashed and E_1 is the threshold energy of the lowest transverse subband. Using T_{mn} summed over the lowest N' channels, the inset shows $R_H(N')$ for a $N_{\rm chan}=8$ case ($E_F=20E_1$ with twice the width). (b) The slope of the Hall resistance S normalized to its 2D value as a function of Fermi energy. Quenching occurs only in narrow regions. The slope at low field (solid line, $BW^2/\Phi_0=4\times10^{-7}$) agrees with that obtained by fitting $R_H(B)$ over a broad range of fields $[-B_1,B_1]$ (dotted line, $B_1W^2/\Phi_0=0.6$ for $E_F<14E_1$, $B_1W^2/\Phi_0=0.8$ for $E_F>14E_1$).

[Fig. 2(a)].[20]

The Hall-resistance trace in Fig. 2(a) shows that strong quenching is possible in the graded-width structure with $N_{\rm chan}=4$. The $T=0$ trace shows a great deal

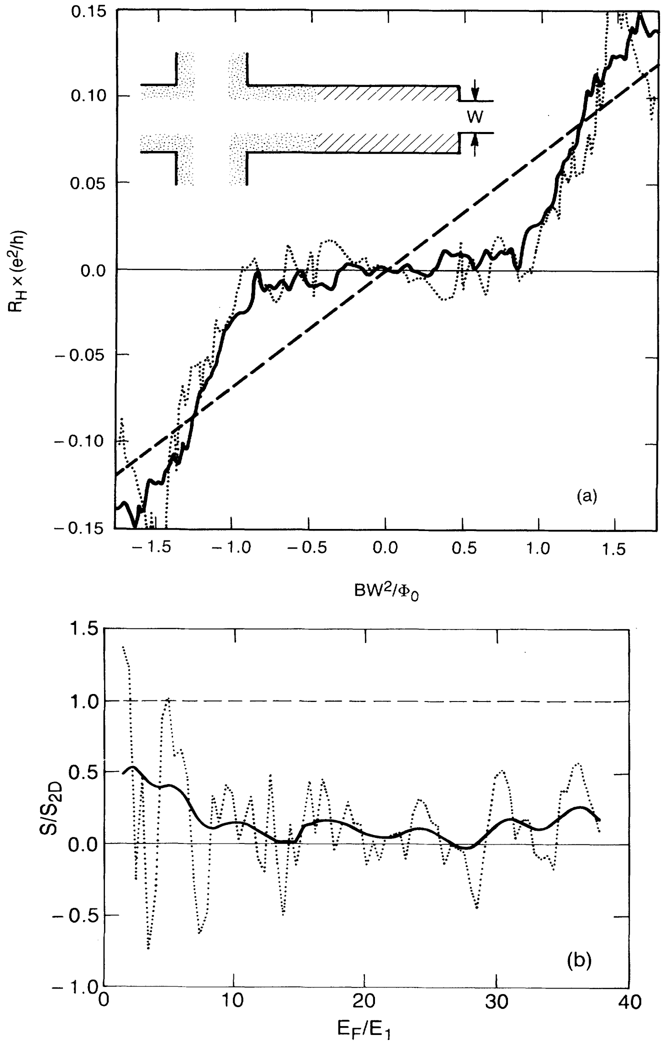

FIG. 2. (a) The Hall resistance as a function of magnetic field for a graded-width structure at $E_F=20E_1$. R_H shows strong quenching ($T=0$, dotted line; $T=3E_1$, solid line; two-dimensional value, dashed line). The structure consists of a hard-wall narrow wire which gradually widens (hatched) to a wire of twice the original width with some softness in the side walls in the junction region (shaded). (b) The slope of the Hall resistance normalized to its two-dimensional value as a function of Fermi energy. Quenching occurs over a broad range of energies ($T=0$, dotted line; $T=E_1$, solid line). Slopes are obtained by fitting $R_H(B)$ as in Fig. 1(b).

of fine structure as a function of magnetic field even though there is no disorder. Such fine structure is observed experimentally as the temperature is lowered and had previously been ascribed to impurity effects.[18] Our calculation indicates that such structure may be intrinsic and distinct from diffusive conductance fluctuations.

The crucial result, that quenching is a *generic* feature of the graded-width structure, is evident from the normalized slopes in Fig. 2(b). While there is still substantial modulation of the slope with Fermi energy at $T=0$, the temperature average is close to zero and certainly far below the two-dimensional value. We have studied many different graded-width structures, e.g., both hard-wall and harmonic transverse confinement, with or without a softening of the potential in the junction region, and with different types of grading as a function of length along the wire. All show generic quenching, in some cases even if the smooth grading is only over a distance of order of the junction size (a realistic scale for experimental systems). However, some region of gradual widening is essential in producing this quenching behavior; an abrupt change from a narrow to a wide wire produces a normalized slope which modulates about 1.

These results suggest a natural physical explanation for the quenching of R_H. A gradual widening of the wires causes injected electrons to travel adiabatically from the narrow to the wide region so that their transverse quantum state (channel number) is conserved. This leads to a transfer of transverse momentum $k_\perp$ into longitudinal momentum $k_\parallel$. Thus the current that was injected into the narrow wire equally distributed across all channels (i.e., consistent with a fixed chemical potential in the reservoir) reaches the wide-junction region distributed across only the low-lying channels; these are the channels with large $k_\parallel/k_\perp$. Ultimately the widening of the equipotential lines at the junction must become nonadiabatic, since they must turn through 90°. From this point on, one expects $k_\perp$ to be conserved instead of the mode number, as demonstrated for an abruptly terminated constriction,[22] yielding a *collimated* beam of electrons in the forward direction, as discussed recently for a constriction.[23]

What is the effect of this collimation on R_H? It is useful to rewrite Eq. (1) as $R_H=\alpha T_{RL}/D$, where $\alpha=(T_R-T_L)/(T_R+T_L)$ is the fractional right-left transmission asymmetry, $T_{RL}=T_R+T_L$ is the total sideways transmission, and

$$D=(e^2/h)[2T_F(T_F+T_{RL})+T_{RL}^2(1+\alpha^2)/2]$$

is relatively insensitive to whether the electrons are transmitted forwards or sideways. It is then clear that R_H can be suppressed either by (1) reducing the fractional asymmetry α due to the field, or simply by (2) increasing the forward transmission at the expense of sideways transmission (since $R_H \to 0$ as $T_{RL} \to 0$ at fixed α). Collimation will obviously produce the latter effect, at least up to a threshold field at which the collimated beam is bent into a voltage probe, and we find numerically that grading also reduces α. To analyze the effect of grading on R_H, we define two quantities which measure the importance of each mechanism separately: $R_1 \equiv \alpha_G(T_{RL}/D)_S$ is the asymmetry from the graded-wire calculation multiplied by the magnitude factor from the straight wire case, while $R_2 \equiv \alpha_S(T_{RL}/D)_G$ incorporates the effect of grading on the magnitudes but uses α for straight wires. Calculating the normalized slopes of these approximate Hall resistances exactly as for the true R_H and averaging over E_F for which there are 3–5 modes present, we find that $\langle S_1/S_{2D}\rangle = 0.45 \pm 0.16$ and $\langle S_2/S_{2D}\rangle = 0.29 \pm 0.02$, while for the true R_H, $\langle S_{graded}/S_{2D}\rangle = 0.18 \pm 0.05$ and $\langle S_{straight}/S_{2D}\rangle = 0.92 \pm 0.02$. Thus we see that grading causes quenching through both mechanisms noted above.

In fact, if we analyze in more detail the transmission intensities for the uniform cross (which does not show generic quenching), we can make a plausibility argument that an adiabatically graded cross of the same final width should show generic quenching (as in Fig. 2). If the uniform cross had been graded from an initial width with only N' propagating channels, then, roughly speaking, only the lowest N' channels would be occupied at the junction, and to estimate the change in the Hall resistance which would result we use the same transmission intensities as for the uniform case, but only sum over the lowest N' incoming and outgoing channels. In Fig. 1(a) we plot R_H as a function of N' ($N_{chan} = 8$); if we are trying to estimate R_H for a graded cross which doubles in width (as in Fig. 2), $N' = 4$ is the appropriate value to consider, and for this value R_H is indeed strongly quenched. The fact that $R_H(N')$ for the uniform cross jumps up sharply at $N' \geq 6$ directly verifies that the high transverse channels make the dominant contribution to R_H.

In conclusion, generic quenching is present in structures when there is an approximately adiabatic grading from a narrow wire to a larger junction region in order to collimate the electrons. An obvious experimental implication of our explanation is that the geometry of the junction should be crucial in determining whether quenching is present or not. Very recently, the dependence of quenching on the geometry of the junction has been studied experimentally by Chang and Chang[9] and Ford *et al.*[10] Change and Chang showed that a uniform narrow wire is not necessary to produce generic quenching; instead a constriction on each lead of a wide cross structure is a necessary and sufficient condition for strong quenching, a result consistent with our model since the constriction introduces the necessary narrow region before the wide junction. In one geometry studied by Ford *et al.*, a deflector is placed in the center of the junction. This sample showed no quenching at all, as one would expect from our model since the deflector prevents forward transmission of the collimated beam. In another geometry in which the lithographic junction is diamond shaped, they found the sign of R_H inverted; our calculations for this geometry indicate that inversion only occurs if there is collimation. Together all these experimental and theoretical results indicate conclusively that junction geometry, and specifically the collimation effect, is the basic mechanism for the observed generic quenching of the Hall resistance. The creation of a nonequilibrium momentum distribution by the sample geometry appealed to in our explanation of the quenching appears to be a concept which unifies many of the novel quantum phenomena[1-6] observed in ballistic transport.

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