Evolution of Superfluid Vortex Line Density behind a Heat (Second-Sound) Pulse in Helium II

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A new probe for superfluid vortex line density with a response time of 400 μs and a spatial resolution of 1 mm was developed and used to investigate the growth and decay of superfluid vortex lines during the passage of a heat pulse. The evolution process turns out to be in accordance with existing theories.

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One of the most striking features of liquid-helium II is its high thermal conductivity, resulting from an internal convection process that can be described in terms of Tisza's two-fluid model as formulated by Landau.[1] According to this model the fluid consists of two components, each having its own density and velocity field. An important case of flow is that generated by injecting a heat flux into the fluid. Heat is transported solely by the normal fluid component, whereas the superfluid moves so as to provide no net mass flux of the whole liquid: counterflow. For small heat fluxes the only dissipation is caused by the viscosity of the normal fluid component. Early experiments of Gorter and Mellink[2] on temperature and pressure gradients in flows in tubes showed an extra dissipation that could be described by an additional friction force in the equations of motion, and is related to the appearance of superfluid vortex lines, as was later shown by Hall and Vinen.[3,4]

This feature has often been investigated in steady-state pipe flows, which are now quite well understood as far as the dependence of the total amount of vortex line density (VLD) (i.e., length of vortex line per unit volume) on temperature and heat flux is concerned,[5] whereas substantial uncertainties remain regarding the growth of the vortex tangle and its motion.[6,7] Therefore we used heat pulses to get a closer insight into the resulting temporal evolution of the vortex line density.

Our experimental apparatus is a square tube of 2.6 $\times$2.6-cm^2 cross section and 8-cm height made of Lucite located vertically in a Dewar. The second-sound shock waves are generated by means of a chromium thin-film heater at the bottom of the tube. The VLD, L, is deduced by measuring its effect on the damping of second sound. The additional attenuation due to the vortex lines is given by[8] $\alpha_S' = (B\kappa/6c_2)L$, where B is a mutual friction coefficient, $\kappa = 10^{-3}$ cm^2/s is the quantum of circulation, and c_2 is the speed of second sound.

The VLD probe consists of two plane parallel glass plates ($12\times12\times0.1$ mm^3) fixed at a distance of 1 mm from one another forming a resonator (Fig. 1). On one plate a thin-film heater (1×2 mm^2) consisting of vapor-deposited chromium acts as a second-sound pulse emitter. Facing it on the other plate, the bolometer consisting of a thin evaporated layer of gold and tin[9] (1×0.02 mm^2) is located. At the beginning of a measurement, an electric pulse is applied to the heater located at the bottom of the channel, producing a primary heat pulse of duration t_H and heat flux Q. After a certain delay time taking into account the time of flight to the probe, a short probe pulse ($t_{HP} = 10$ μs, $Q_P = 3$ W/cm^2) is released from the emitter of the VLD probe. Its temperature signal superimposed on the primary pulse and the following echoes resulting from the reflections of the pulse on both emitter and receiver are detected by the bolometer, amplified, and stored by means of a storage oscilloscope [Fig. 2(a)]. To separate the primary pulse from the probe pulse and its echoes, a reference primary pulse signal is generated without releasing the probe pulse [Fig. 2(b)]. The difference of the two signals gives the consecutive echoes [Fig. 2(c)]. The values of the first five maxima are fitted with a minimum square error to an exponential decay curve. The damping coefficient is evaluated from the slope of the curve divided by the velocity of the second sound determined from the distance between the peaks, and averaged over five measurement cycles. By subtracting the coefficient α_{S0} obtained for the nonperturbed helium, i.e., without releasing a primary pulse, the coefficient α_S' due to the VLD can be estimated. Using the spline fits of Swanson *et al.*[10] for the values of B and c_2, the VLD averaged over a time interval of 400 μs is obtained from the relation above. To ensure that for the measurements the initial conditions were always the same when the test parameters were modified, a set of ten primary heat pulses was generated before each recorded experimental cycle.

In the first series of experiments, the evolution of VLD during the passage of a heat pulse was investigated. To

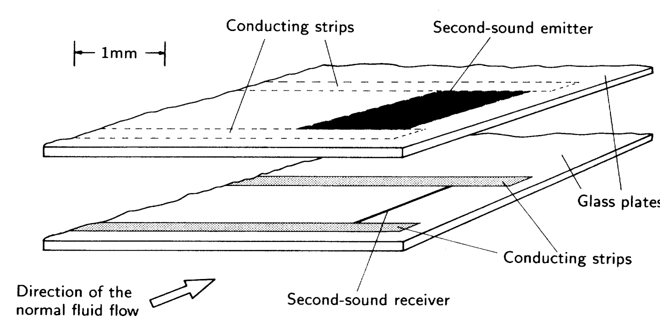

FIG. 1. Schematic drawing of the main part of the vortex-line-density probe.

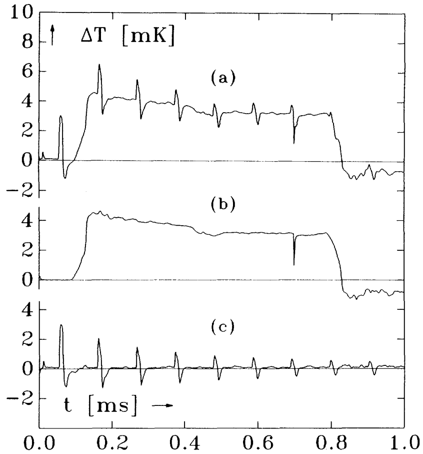

FIG. 2. Sample of damping measurement. (a) The primary pulse, with superimposed testing pulse and several subsequent echoes, as registered by the sensor strip. (b) The primary pulse solely. (c) The testing signal, obtained from the difference between (a) and (b). To the maxima an exponential decay curve can be fitted for evaluation. The peak of 0.7 ms is due to electrical crosstalk when the heater is switched off.

obtain a satisfactory time resolution, a 10-ms primary pulse duration has been chosen. Figure 3 shows results obtained at 1.65 K for different distances $d=5$, 20, and 40 mm between the primary heat pulse source and the probe. The heat flux was 2 W/cm^2, the repetition time 5 s. t_D denotes the time interval between the arrival of the heat pulse at the probe and the beginning of the measurement. For $d=5$ mm, a nonlinear increase of VLD with time is observed up to $t_D=7$ ms. From this point on, the increase slows down until the vortex line density reaches a maximum of $12\times10^6/\mathrm{cm}^2$ at $t_D=9$ ms, which roughly represents the end of the primary heat pulse.

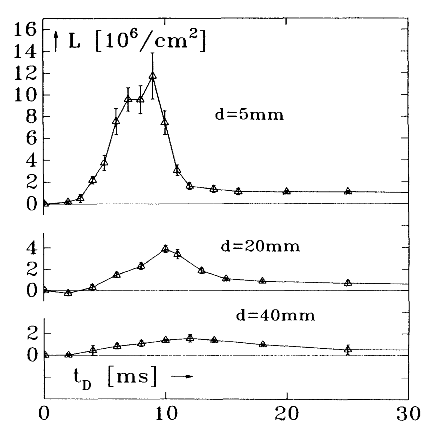

FIG. 3. The temporal evolution of the vortex line density during the passage of a second-sound pulse for different distances d from the heater. The primary pulse duration was 10 ms, the temperature 1.65 K.

Afterwards, the superfluid vorticity decreases at first rapidly—within times of the order of a few ms—to about one-fifth of the maximum value, followed by a much slower decrease. Finally, far beyond the time shown on the figure the VLD tends asymptotically to zero, but up to 40 ms a small amount of remaining VLD could still be detected. Measurements at larger distances show a similar behavior, but the maximum amount of vorticity generated (at $t_D=10$ to 12 ms) is monotonically decreasing with increasing distance. The evolution in time of the VLD generated by the heat pulse can be compared with the prediction based on the following approximate phenomenological relation deduced from experiments on flow in tubes, shown by Hall and Vinen:[3]

$$dL/dt=\alpha wL^{3/2}-\beta L^2\,, \tag{1}$$

where α and β are Hall and Vinen's constants and w is the counterflow velocity. Schwarz[11] obtained a theoretical relation of the same form from more fundamental considerations of vortex motion. This equation is said, however, to be valid only close to equilibrium.

Figure 4 shows a fit of the solution of (1) to our experimental results at 1.85 K for $d=5$ mm and heat flux $Q=3$ W/cm^2. The magnitude of the counterflow velocity has been taken equal to its equilibrium value $w=Q/\rho_s sT$ for $0<t_D<t_H$, and 0 elsewhere, where ρ_s, s, and T are the superfluid density, entropy, and temperature, respectively. The deviation of the measured points from the theoretical curve are within the experimental error. The values for α and β obtained from the best fits are listed in Table I. They can be compared to the values obtained by Awschalom, Milliken, and Schwarz[12] using the quotient $\gamma\equiv\alpha/\beta$, related to L_0, the equilibrium VLD, in steady-state counterflow where $L_0=\gamma^2w^2$. It should be noticed that the VLD generated in our experiments is several orders of magnitude larger than that of steady-state experiments performed so far,[6,12] but the coefficient γ differs only by at most 30% between the two cases and

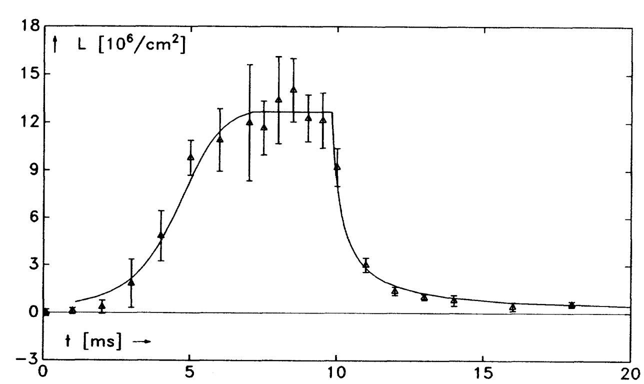

FIG. 4. The temporal evolution of vortex line density and a matched solution of Eq. (1) with $\alpha=29.5\times10^{-3}$, $\beta=0.23\times10^{-3}$ cm^2/s, and $w=38.1$ cm/s.

TABLE I. Variations of the coefficients α, β, and γ with temperature.

T (K)	$10^3\alpha$	β (10^{-3} cm^2/s)	$\gamma=\alpha/\beta$ (s/cm^2)	γ^a (s/cm^2)
1.40	8.3	0.08	104	99
1.65	20.7	0.19	109	142
1.85	29.5	0.23	128	172

[a]Data taken from Ref. 12.

shows a similar variation with temperature. The results turn out to be fairly independent of the resonator's width, as follows from corresponding experiments with a 2-mm resonator.

In a second series of experiments, the average vortex line density produced during the passage of the primary heat pulse was measured for various heat fluxes. For that purpose the testing pulse was released a few μs after the arrival of the trailing edge of the heat pulse at the probe located at a distance of 5 mm. Figure 5 shows the results at $T=1.65$ K and a primary pulse duration $t_H=0.5$ ms for two different primary pulse repetition times $t_R=0.5$ and 5 s. For $t_R=5$ s, no superfluid vorticity at all could be measured for heat fluxes less than 6 W/cm^2. Beyond this limit, the production of vorticity sets in strongly. Increasing the repetition rate ($t_R=0.5$ s) leads to a decrease of the critical heat flux, below which no vorticity could be observed, down to values of 4.5 W/cm^2. For higher heat fluxes, the vortex line density shows a stronger dependence on the heat flux than for $t_R=5$ s.

A similar effect is caused by the pulse duration t_H. Figure 6 shows measurements at 1.40 K and $t_R=5$ s for pulse durations $t_H=1$ and 0.1 ms. The increase in t_H leads to a decrease of the critical heat flux Q_c from 7 W/cm^2 ($t_H=0.1$ ms) to 2.5 W/cm^2 ($t_H=1$ ms). Similar results are obtained at other bath temperatures. It turns out that the strength of the critical heat flux depends on the temperature and reaches a maximum near 1.85 K.

These observations can be explained as follows: The few vortex lines that are always present in helium II (Ref. 13) grow during the passage of the heat pulse, and decay afterwards, until, after the repetition time t_R, another pulse is produced, releasing the same mechanism, but starting then from a higher level of vortex line density. After some cycles, the starting level of vorticity remains constant, and the dynamical steady state is reached. The total amount of superfluid vorticity produced during the passage of the heat pulse depends, hence, not only on temperature, heat flux, and pulse duration, but also in a crucial manner on the starting level of vorticity and hence on the repetition time t_R.

The occurrence of a critical heat flux, below which the measured VLD is smaller than the experimental scatter, is due to the nonlinear character of Eq. (1). From the solid line in Fig. 4 it can be seen that at a heating time of 6 ms or more, 90% of the maximum VLD is produced. A slight decrease of heating time leads to a strong decrease in VLD, until at $t_D=2.5$ ms only 10% are produced. It becomes apparent that for sufficiently small heating times only a negligible amount of VLD is produced. The same effect is caused by a small initial VLD, caused by a large repetition time, and small heat flux, i.e., counterflow velocity.

It seemed, therefore, of some importance to look into the possibility of finding a relation between the critical parameters at which superfluid turbulence appears. This is indicated by a modification of the shape of the propagating temperature pulse, caused by the decrease of the counterflow velocity, i.e., heat flux, due to the vortex lines formed, and hence an accumulation of heat close to the heater. At small distances from the heater, the presence of VLD manifests itself in an increase of the pulse amplitude with time, followed by a slower decay to zero at the end of the pulse.[14] At larger distances, an asymptotic decay of temperature is observed close behind the shock front. Then the pulse shape depends only slightly

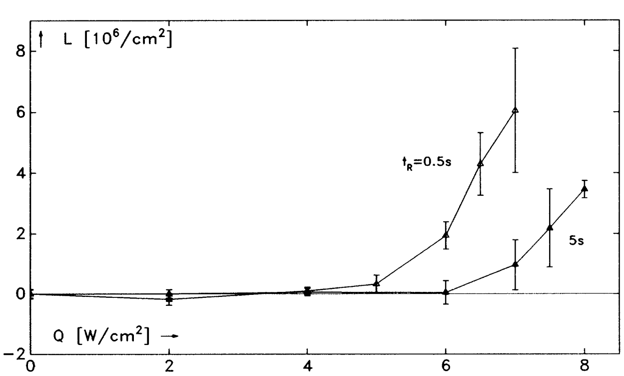

FIG. 5. The vortex line density depending on heat flux for repetition times $t_R=0.5$ and 5 s, respectively. The temperature was 1.65 K, the primary pulse duration 0.5 ms.

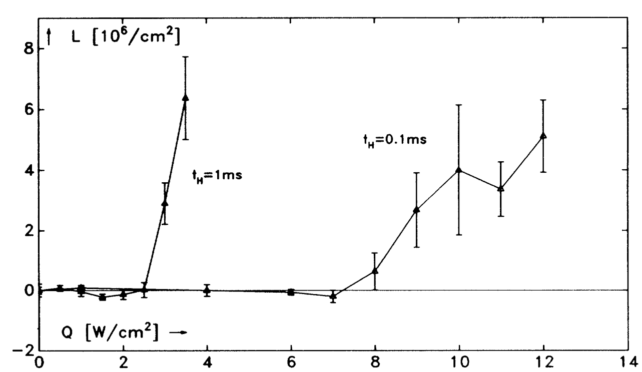

FIG. 6. The vortex line density depending on heat flux for primary pulse durations $t_H=0.1$ and 1 ms, respectively. The temperature was 1.40 K, the repetition time 5 s.

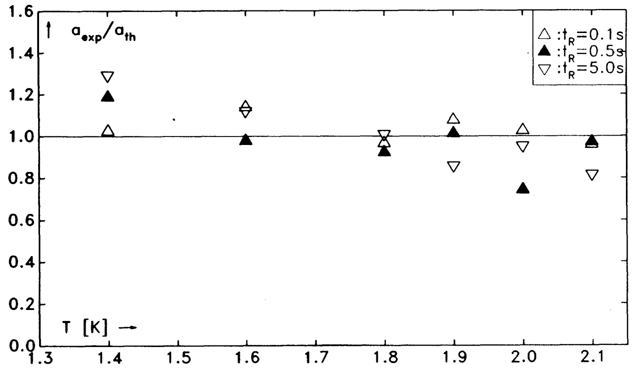

FIG. 7. The variation of the ratio of the experimental to the theoretical coefficient, $a_{\mathrm{expt}}/a_{\mathrm{th}}$, with temperature.

on distance, because due to the decrease of counterflow velocity with distance, the amount of VLD produced also decreases (see Fig. 3). From a remote analogy with Vinen's[15] critical heating time, related to a change of the flow character, we introduced a phenomenological relation between the critical heating time t_{Hc} and the critical heat input Q_c. This relation was derived using the characteristic time $t_L=(\kappa L_0)^{-1}$ with $L_0=\gamma^2 w^2$, where $w=Q/\rho_s sT$, and introducing the coefficient $b(t_R)$ depending on the heat-pulse repetition time:[16]

$$t_{Hc}=b(t_R)\frac{1}{\kappa}\left(\frac{\rho_s sT}{\gamma}\right)^2 Q_c^{-2}=a_{\mathrm{th}}Q_c^{-2}. \qquad (2)$$

To check the validity of this relation a set of temperature measurements was made at t_{Hc} between 100 and 2000 μs, $t_R=0.1$, 0.5, and 5 s, and bath temperatures between 1.4 and 2.1 K. As can be seen from Fig. 7, taking $b(t_R)=(t_R/t_0)^{1/3}$ and $t_0=1.79$ μs$=$const, the measured values of $a_{\mathrm{expt}}/a_{\mathrm{th}}$, where a_{expt} are the experimentally obtained values of t_{Hc}/Q_c^{-2}, collapse fairly well around 1 ± 0.2.

It should be noted here that our observations are based on experiments made of unsteady flows generated by shock waves following fairly large heat inputs. It is very difficult to maintain a constant bath temperature necessary for steady-state measurements under these conditions, and a comparison made with steady-state flows in tubes remains outstanding.

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[1]L. D. Landau, J. Phys. (U.S.S.R.) **5**, 71 (1941).

[2]C. J. Gorter and J. H. Mellink, Physica **15**, 285 (1949).

[3]H. E. Hall and W. F. Vinen, Proc. Roy. Soc. London A **238**, 204 (1956); **238**, 215 (1956).

[4]W. F. Vinen, Proc. Roy. Soc. London A **240**, 114 (1957); **240**, 128 (1957); **242**, 493 (1957).

[5]R. J. Donnelly and C. E. Swanson, J. Fluid Mech. **173**, 387 (1986).

[6]R. A. Ashton and J. A. Northby, Phys. Rev. Lett. **35**, 1714 (1975).

[7]R. T. Wang, C. E. Swanson, and R. J. Donnelly, Phys. Rev. B **36**, 5240 (1987).

[8]J. T. Tough, in *Progress in Low Temperature Physics,* edited by D. F. Brewer (North-Holland, Amsterdam, 1982), Vol. 8, Chap. 3, p. 133.

[9]H. Borner, T. Schmeling, and D. W. Schmidt, J. Low Temp. Phys. **50**, 405 (1983).

[10]C. E. Swanson, C. F. Barenghi, W. T. Wagner, and R. J. Donnelly, J. Low Temp. Phys. **66**, 263 (1987).

[11]K. W. Schwarz, Phys. Rev. B **38**, 2398 (1988).

[12]D. D. Awschalom, F. P. Milliken, and K. W. Schwarz, Phys. Rev. Lett. **53**, 1372 (1984).

[13]D. D. Awschalom and K. W. Schwarz, Phys. Rev. Lett. **52**, 49 (1984).

[14]W. Fiszdon and M.v. Schwerdtner, J. Low Temp. Phys. **75**, 253 (1989).

[15]W. F. Vinen, Inst. Int. Froid., Int. Inst. Refrig., Saclay, Refrig. Sci. Technol. **1981**, 43.

[16]M. v. Schwerdtner, Ph.D. thesis, Max-Planck-Institute für Strömungsforschung 90, 1988 (unpublished).