Observation of the Topological Aharonov-Casher Phase Shift by Neutron Interferometry

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The phase shift predicted by Aharonov and Casher for a magnetic dipole diffracting around a charged electrode has been observed for the case of thermal neutrons, using a neutron interferometer containing a 30-kV/mm vacuum electrode system. The judicious use of the Earth's gravitational field introduces a spin-independent phase shift which enables unpolarized neutrons to be used. A supplementary magnetic bias field of the correct magnitude allows first-order sensitivity to be achieved; even so, the theoretically predicted phase shift is only 1.50 mrad for the geometry and conditions of the experiment. We observe a phase shift of 2.19 ± 0.52 mrad.

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The well-known topological interference effect of Aharonov and Bohm (AB) concerns a phase shift for electrons diffracting around a tube of magnetic flux.[1] The effect has been observed and measured in a series of investigations culminating in the experiments of Tonomura *et al.*[2] More recently, an extension of the Aharonov-Bohm effect was presented by Aharonov and Casher (AC).[3] They predict that a neutral particle possessing a magnetic dipole moment (e.g., a neutron) should experience an analogous phase shift when diffracted around a line of electric charge. The situation may be visualized by referring to Fig. 1. In Fig. 1(b) the AB flux tube of Fig. 1(a) has been replaced by an equivalent line of magnetic dipoles. Between Figs. 1(b) and 1(c) the role of charge and magnetic dipole have been interchanged, resulting in the AC configuration. In this sense the AC effect is an electrodynamic and quantum-mechanical dual of the AB effect. The differences between the two effects were recently discussed by Goldhaber who also comments on new theoretical aspects of the AC effect.[4] The purpose of this paper is to describe a neutron interferometry experiment in which we have detected the AC phase shift for the first time.[5]

In this experiment, carried out at the University of Missouri Research Reactor (MURR), the AC configuration was realized using a Bonse-Hart single-crystal neutron interferometer[6] schematically shown in Fig. 2. Dynamical Bragg diffraction in the perfect silicon crystal splits the neutron beam in the interferometer plate S, reflects each of the resulting beams in plate M, and recombines them in plate A. As pointed out by Aharonov and Casher, their phase shift depends on the lineal charge density Λ enclosed by the beam paths, but not on any details of the geometry of the beam paths relative to the line charge. In this sense the effect is topological. Consequently, a series of line charges can be used, as long as they are enclosed by the beam paths, thereby amplifying the expected phase shift. Thus, instead of a line charge, a charged prism-shaped electrode system was placed between the splitter (S) and mirror (M) plates of the interferometer. The charged central metallic electrode is identical to a continuous series of line charges residing on its surfaces, perpendicular to the plane of the interferometer beam paths shown in Fig. 2.

The canonical momentum for a neutron having magnetic moment $\boldsymbol{\mu}$, mass m, and velocity $\mathbf{v}$, in an electric field $\mathbf{E}$ is

$$\mathbf{p}=m\mathbf{v}+\frac{\boldsymbol{\mu}}{c}\times\mathbf{E}\,. \tag{1}$$

For a neutron diffracting around a line charge, one obtains the AC phase shift (in cgs units) by evaluating the line integral of $\mathbf{p}$, namely

$$\Delta\Phi_{\rm AC}=\frac{1}{\hbar}\oint\mathbf{p}\cdot d\mathbf{r}=\sigma\frac{4\pi\mu\Lambda}{\hbar c}\,, \tag{2}$$

where $\sigma=\pm 1$ depending on whether the neutron spin is up or down with respect to the plane of the neutron motion. For an electrode, Gauss's law allows us to replace the lineal charge density Λ by $2VL/4\pi D$, where V is the potential difference between the electrodes, D is their separation, and L is the effective path length shown in Fig. 2. In terms of these quantities, for $V=45$ kV ($=150$ statvolts), $D=0.154$ cm and $L=2.53$ cm (corrected for end effects, an 8% effect), we find

$$\Delta\Phi_{\rm AC}=1.50\sigma \text{ mrad}\,. \tag{3}$$

The neutron counting rates in detectors C_2 and C_3 are given by

$$C_2=a_2-b_2\cos\Delta\Phi\,, \tag{4}$$

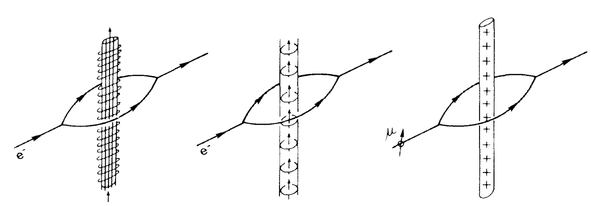

FIG. 1. Duality between the Aharonov-Bohm topology and the Aharonov-Casher topology.

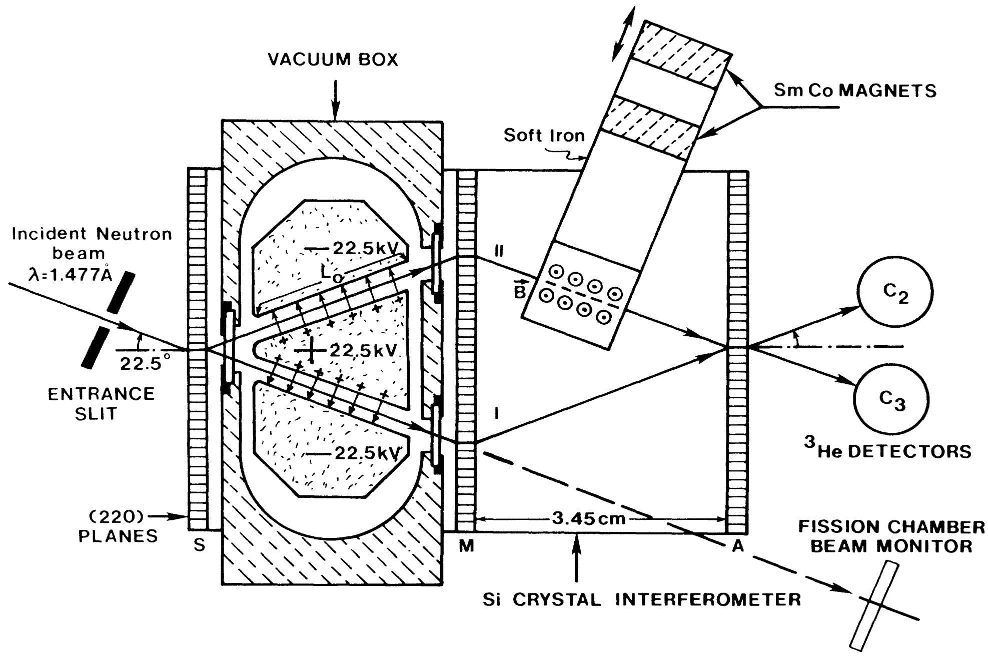

FIG. 2. Schematic diagram of the Bonse-Hart perfect-Si-crystal LLL interferometer. An unpolarized neutron beam of wavelength $\lambda = 1.477$ Å is used. The neutron wave on path II passes through a region of electric field **E** and then through a vertical magnetic bias field **B**. (For the null geometry the magnetic bias field is horizontal.) The neutron wave on path I passes on the opposite side of the center electrode, whose polarity was reversed periodically as described in the text. The interferometer, along with the entrance slit, the vacuum electrostatic cell, and the bias magnet could be tilted about the incident beam direction to adjust the spin-independent gravitational phase shift $\Delta\Phi_G$. The ^{3}He proportional detectors C_2 and C_3 count the exit beams, and the fission chamber monitors the incident beam intensity.

and

$$C_3 = a_3 + b_3 \cos\Delta\Phi\,, \tag{5}$$

where $\Delta\Phi$ is the phase shift for the neutron wave on path II relative to path I. The constants a_2, a_3, b_2, and b_3 characterize the actual interferometer setup, and $a_2/a_3 \approx 3$, while $b_2 = b_3$, such that $C_2 + C_3$ is independent of $\Delta\Phi$.

In the derivation of Eq. (2), it is assumed that the neutrons are polarized along an axis parallel to the line charge. However, as will be shown below, it is not necessary to use polarized neutrons if an additional spin-independent phase shift is judiciously introduced and adjusted. It is common practice to obtain a spin-independent phase shift by transmission through an aluminum plate inserted between two of the crystal slabs of the interferometer. However, due to space limitations within our interferometer, this was not feasible. In this experiment a gravitationally induced quantum phase shift was introduced instead.[7] This was obtained by tilting the interferometer about the incident beam direction. Neutrons traveling along the two paths I and II, being at different heights, experience different gravitational potentials, resulting in a relative phase shift $\Delta\Phi_G$. The introduction of a further spin-dependent phase shift $\Delta\Phi_M$, by means of a magnetic bias field, enabled the maximum sensitivity to the AC effect to be realized. This magnetic phase shift is due to spin precession.[8] We take the z axis of spin quantization to be perpendicular to the plane of the neutron's motion in the interferometer (i.e., parallel to the line charge). As the neutron passes through the electrostatic cell, it precesses about an effective magnetic field $-\mathbf{v}/c \times \mathbf{E}$ directed along the positive z axis on path I and parallel to the negative z axis on path II. If the magnetic bias field **B** is also along the z axis (i.e., vertical), the total spin-dependent phase shift is simply $\sigma(\mp |\Delta\Phi_{AC}| + |\Delta\Phi_M|)$, where the + sign is for negative polarity and the − sign is for positive polarity of the center electrode. Thus, the count rate in detector C_3 for spin-up neutrons is

$$C_3^{\uparrow}(\pm) = \tfrac{1}{2} a_3 + \tfrac{1}{2} b_3 \cos[\Delta\Phi_0 + \Delta\Phi_G + (\mp |\Delta\Phi_{AC}| + |\Delta\Phi_M|)]\,, \tag{6}$$

while for spin-down neutrons we have

$$C_3^{\downarrow}(\pm) = \tfrac{1}{2} a_3 + \tfrac{1}{2} b_3 \cos[\Delta\Phi_0 + \Delta\Phi_G - (\mp |\Delta\Phi_{AC}| + |\Delta\Phi_M|)]\,, \tag{7}$$

where $\Delta\Phi_0$ is an experimental offset phase.

We have assumed that the incident beam is unpolarized; thus the total count rate in detector C_3 is

$$C_3(\pm)=C_3^{\uparrow}(\pm)+C_3^{\downarrow}(\pm)=a_3+b_3\cos[\Delta\Phi_0+\Delta\Phi_G]\cos[|\Delta\Phi_M|\mp|\Delta\Phi_{AC}|]. \tag{8}$$

Similarly, the count rate in detector C_2 is given by

$$C_2(\pm)=a_2-b_2\cos[\Delta\Phi_0+\Delta\Phi_G]\cos[|\Delta\Phi_M|\mp|\Delta\Phi_{AC}|]. \tag{9}$$

The gravitational phase shift $\Delta\Phi_G$ is adjusted by tilting the crystal so that $\Delta\Phi_0+\Delta\Phi_G=0$ (mod2π), giving maximum sensitivity to $\Delta\Phi_{AC}$. Furthermore, adjusting the magnetic phase $\Delta\Phi_M$ to be $\pi/2$, we have

$$C_3(\pm)=a_3+b_3\sin(\pm|\Delta\Phi_{AC}|)\approx a_3\pm b_3|\Delta\Phi_{AC}|, \tag{10}$$

and

$$C_2(\pm)=a_2-b_2\sin(\pm|\Delta\Phi_{AC}|)\approx a_2\mp b_2|\Delta\Phi_{AC}|. \tag{11}$$

Thus, the count rates C_3 and C_2 are linearly proportional to the small AC phase shift. It is important to note that dimensional stability of the interferometer affects only the spin-independent phases, and thereby enters the results only in second order. Had polarized neutrons been used, this would not have been the case, and typical drifts in $\Delta\Phi_0$ of order 50 mrad/day would have precluded obtaining the sensitivity required.

For a horizontally directed bias magnetic field, the analysis is a bit more subtle, since the neutron first precesses about the z axis in the electrostatic cell, and then about the x axis (horizontal) in the magnet. In this case, the leading term is quadratic in $\Delta\Phi_{AC}$, and clearly unobservable, since $\Delta\Phi_{AC}$ is only of order 1 mrad. Nevertheless, we have also carried out an extensive run under these conditions, which we shall refer to as the "null experiment," as a check on the stability of, and potential spurious contributions to the experiment.

The AC effect was expected to change the count rates by an order of 1 part in 1000. To detect such a small effect it was necessary to accumulate an order of 10^7 neutron counts (which, with available intensities, took several months) and to use a synchronous detection method in which the polarity of the electrodes was periodically reversed. A monochromatic neutron beam ($\lambda=1.477$ Å) corresponding to a Bragg angle of 22.5° in the interferometer crystal was employed. The beam was restricted in size by a cadmium slit in front of the interferometer, and gadolinium paint was suitably applied to the entrance quartz windows of the vacuum box enclosing the electrodes, thus ensuring that neutrons did not strike the electrodes. The electrodes were made of highly polished stainless steel and supported on an insulator block. The voltage of 45 kV between the electrodes, which was periodically reversed in polarity, was the highest potential difference that could be maintained during the months-long experiment. The voltage cycle was divided into three parts: positive, negative, and zero. The signal for the voltage cycle was provided by the monitor counter reaching a preset value (35000), taking about 10 min, for a total of 30 min per complete cycle.

The magnetic bias field, which was provided by SmCo magnets, was varied by changing the reluctance of a magnetic circuit (see Ref. 8). To find the correct operating point for $\Delta\Phi_G$ and $\Delta\Phi_M$, the following procedure was followed: With the magnetic field B set to zero, a gravity scan was carried out by tilting the interferometer in steps about the incident beam direction. The first maximum near zero tilt angle sets $\Delta\Phi_G+\Delta\Phi_0=0$ (mod2π). Leaving the tilt fixed at this maximizing angle, operating points of negative slope ($\Delta\Phi_M=\pi/2$) and positive slope ($\Delta\Phi_M=3\pi/2$) were selected by scanning the magnetic field as shown in Fig. 3. This procedure was repeated every week, since the spin-independent phase drifts 50 to 100 mrad over this time period. The following additional procedure was also carried out periodically to fix the operating point more accurately: The magnetic field was changed in small increments around the value giving $\Delta\Phi_M\approx\pi/2$. Gravity scans were executed for each value of the magnetic field and the optimum was chosen as that giving minimum oscillation.

A summary of the difference counts $C_2(+)-C_2(-)$ and $C_3(+)-C_3(-)$ is given in Table I. There were four experimental operating conditions: null experiment, positive slope, negative slope, and zero slope. For each, the data were accumulated over approximately 1700

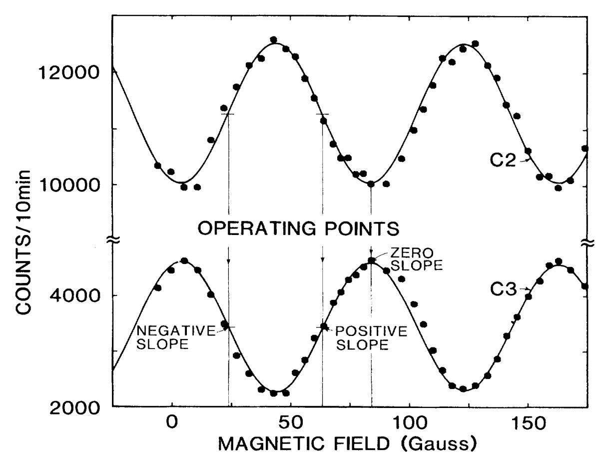

FIG. 3. Magnetic scan taken at an interferometer tilt angle of $-1.10°$. The bias-magnetic-field operating points of negative, positive, and zero slopes of the C_3 interferogram are indicated. The fact that $\Delta\Phi_M=0$ does not occur precisely at $B=0$ is due to fringing fields from the magnet.

TABLE I. This table summarizes the difference counts $C_2(+)-C_2(-)$ and $C_3(+)-C_3(-)$ for the four operating conditions of the bias magnetic field. There were approximately 1700 voltage cycles for each operating condition, taking approximately 35 days each.

Experimental condition	$C_2(+)-C_2(-)$	$C_3(+)-C_3(-)$
Null	-5136 ± 6428	$+643\pm3469$
Positive slope	$+13462\pm6223$	-8019 ± 3494
Negative slope	-9986 ± 6097	$+8243\pm3413$
Zero slope	$+1718\pm6020$	-3631 ± 3979

voltage cycles, taking about 35 days of counting time each. The total accumulation of counts was approximately 6000000 in detector C_3 and 19000000 in detector C_2 for each operating condition. Statistically combining the results for positive and negative slope and normalizing each count to $C_2+C_3=25000000$ ($=1700$ cycles), we find

$$\langle\Delta C\rangle\equiv w_2|C_2(+)-C_2(-)|+w_3|C_3(+)-C_3(-)|$$
$$=9010\pm2130\ \text{counts}/(1700\ \text{cycles}), \qquad (12)$$

where w_2 and w_3 are statistical weighting factors (inversely proportional to the square of the standard deviations).

A fit to the magnetic scan of Fig. 3 gives

$$b\equiv b_2=b_3$$
$$=2057000\pm57000\ \text{counts}/(1700\ \text{cycles}). \qquad (13)$$

Thus, according to Eqs. (10) and (11) we have for the measured AC phase shift

$$\Delta\Phi_{\mathrm{AC}}=\frac{\langle\Delta C\rangle}{2b}=2.19\pm0.52\ \text{mrad}. \qquad (14)$$

We note that the expected sign reversals of ΔC_2 and ΔC_3 for positive- and negative-slope operating points were observed. Furthermore, the difference counts ΔC_2 and ΔC_3 were zero to within statistical error for both the null experiment and the zero-slope operating points, as expected. The experimental result given by Eq. (14) is to be compared with the theoretical prediction given by Eq. (3) resulting in the ratio

$$(\Delta\Phi_{\mathrm{AC}})_{\mathrm{expt}}/(\Delta\Phi_{\mathrm{AC}})_{\mathrm{theor}}=1.46\pm0.35. \qquad (15)$$

The accuracy of this experiment is limited by available neutron intensity, running time, and long-term apparatus stability. The error bars quoted in Eq. (15) are strictly statistical, leaving systematic sources of error to future identification and assessment. Significant improvement will await the advent of much larger neutron interferometers or atomic beam interferometers, for which the magnetic moment is ~2000 times larger than for the neutron. It has recently been pointed out to us that Anandan[9] first calculated the phase shift for neutrons passing through a region of electric field; however, the topological aspect of the AC effect was not envisioned. We have avoided comment on the current theoretical controversy regarding the existence or absence of a force on the neutron in the AC configuration.[10,11] We will address this and other theoretical issues in a full-length paper on our experiment.

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