PHYSICAL REVIEW LETTERS

VOLUME 63 | 24 JULY 1989 | NUMBER 4

Gravitational Field of a Global Monopole

Manuel Barriola and Alexander Vilenkin

Physics Department, Tufts University, Medford, Massachusetts 02155

(Received 26 April 1989)

We present an approximate solution of the Einstein equations for the metric outside a monopole resulting from the breaking of a global O(3) symmetry. The monopole exerts practically no gravitational force on nonrelativistic matter, but the space around it has a deficit solid angle, and all light rays are deflected by the same angle, independent of the impact parameter.

PACS numbers: 04.20.Jb, 11.30.Qc, 14.80.Hv, 98.80.Cq

Phase transitions in the early Universe can give rise to topological defects of various kinds.[1,2] The type of defects depends on the topology of the vacuum manifold μ. In particular, monopoles are formed when μ contains surfaces which cannot be continuously shrunk to a point, that is, when $\pi_2(\mu)\neq I$. Monopoles formed as a result of a gauge-symmetry breaking are similar to elementary particles. Most of their energy is concentrated in a small region near the monopole core. Grand unified theories predicting such monopoles usually predict too many of them,[3] and one has to involve inflation to resolve the problem.

In this paper we would like to consider global monopoles, resulting from a global symmetry breaking. Such monopoles have Goldstone fields with energy density decreasing with the distance only as r^{-2}, so that the total energy is linearly divergent at large distances. The large energy in the Goldstone field surrounding global monopoles suggests that they can produce strong gravitational fields. By analogy with global strings[4,5] one could even expect the corresponding metric to be singular. The main purpose of this paper is to derive the solution of the Einstein equations for the metric outside a global monopole. We shall then examine light propagation in that metric. Finally, we shall briefly discuss nongravitational properties of global monopoles and their cosmological evolution.

The simplest model that gives rise to global monopoles is described by the Lagrangian

$$L=\tfrac{1}{2}\,\partial_\mu\phi^a\partial^\mu\phi^a-\tfrac{1}{4}\lambda(\phi^a\phi^a-\eta^2)^2\,, \qquad (1)$$

where ϕ^a is a triplet of scalar fields, $a=1,2,3$. The model has a global O(3) symmetry, which is spontaneously broken to U(1). The field configuration describing a monopole is

$$\phi^a=\eta f(r)x^a/r\,, \qquad (2)$$

where $x^a x^a=r^2$. The most general static metric with spherical symmetry can be written as

$$ds^2=B(r)dt^2-A(r)dr^2-r^2(d\theta^2+\sin^2\theta\, d\phi^2) \qquad (3)$$

with the usual relation between the spherical coordinates, r,θ,ϕ, and the "Cartesian" coordinates x^a. The field equations for ϕ^a in the metric (3) reduce to a single equation for $f(r)$:

$$\frac{1}{A}f''+\left[\frac{2}{Ar}+\frac{1}{2B}\left(\frac{B}{A}\right)'\right]f'-\frac{2f}{r^2}-\lambda\eta^2 f(f^2-1)=0\,. \qquad (4)$$

The energy-momentum tensor of the monopole is given by

$$T_t^t=\frac{\eta^2 f'^2}{2A}+\frac{\eta^2 f^2}{r^2}+\tfrac{1}{4}\lambda\eta^4(f^2-1)^2\,,$$
$$T_r^r=-\frac{\eta^2 f'^2}{2A}+\frac{\eta^2 f^2}{r^2}+\tfrac{1}{4}\lambda\eta^4(f^2-1)^2\,, \qquad (5)$$
$$T_\theta^\theta=T_\phi^\phi=\frac{\eta^2 f'^2}{2A}+\tfrac{1}{4}\lambda\eta^4(f^2-1)^2\,.$$

In flat space the monopole core has size $\delta\sim\lambda^{-1/2}\eta^{-1}$.

Its mass is

$$M_{\rm core} \sim \lambda \eta^4 \delta^3 \sim \lambda^{-1/2} \eta . \qquad (6)$$

For $\eta \ll m_P$, where m_P is the Planck mass, we expect that gravity does not substantially change the structure of the monopole at small distances, so that the flat-space estimates of δ and $M_{\rm core}$ still apply. Outside the core[6] $f(r) \approx 1$ and the energy-momentum tensor can be approximated as

$$T_t^t \approx T_r^r \approx \eta^2/r^2, \quad T_\theta^\theta = T_\phi^\phi \approx 0 . \qquad (7)$$

The general solution of the Einstein equations with this T_μ^ν is

$$B = A^{-1} = 1 - 8\pi G \eta^2 - 2GM/r , \qquad (8)$$

where M is a constant of integration.

To estimate the value of M, we use the general relation[7]

$$A^{-1} = 1 - \frac{8\pi G}{r} \int_0^r T_t^t r^2 dr = 1 - 8\pi G \eta^2 - \frac{8\pi G \eta^2}{r} \int_0^r \left[\frac{f'^2}{2A} + \frac{f^2-1}{r^2} + \tfrac{1}{4}\lambda \eta^2 (f^2-1)^2 \right] r^2 dr . \qquad (9)$$

Comparing this with Eq. (8) we find

$$M = 4\pi \eta^2 \int_0^\infty \left[\frac{f'^2}{2A} + \frac{f^2-1}{r^2} + \tfrac{1}{4}\lambda \eta^2 (f^2-1)^2 \right] r^2 dr . \qquad (10)$$

The integral in Eq. (10) is of the order of $\delta \sim \lambda^{-1/2} \eta^{-1}$, and thus $M \sim M_{\rm core}$. For reasonable values of η and λ this mass is totally negligible on the astrophysical scale. [We note in passing that the metric (8) with a large value of M, $M \gg \delta/G$, describes a black hole of mass M carrying a global monopole charge. Such a black hole can be formed if a global monopole is swallowed by an ordinary black hole.] Neglecting the mass term and rescaling the r and t variables, we can rewrite the monopole metric as

$$ds^2 = dt^2 - dr^2 - (1 - 8\pi G \eta^2) r^2 (d\theta^2 + \sin^2\theta \, d\phi^2) . \qquad (11)$$

The metric (11) describes a space with a deficit solid angle. The area of a sphere of radius r is not $4\pi r^2$, but $4\pi(1 - 8\pi G\eta^2) r^2$. The surface $\theta = \pi/2$ has the geometry of a cone with a deficit angle

$$\Delta = 8\pi^2 G \eta^2 . \qquad (12)$$

By symmetry, all surfaces passing through the origin and cutting space into two symmetric parts have the same geometry. A striking feature of the metric (11) is that the monopole exerts no gravitational force on the matter around it (apart from the tiny gravitational effect of the core). This can be understood qualitatively if we note that the Newtonian potential $\Phi = GM(r)/r =$const, since $M(r) \propto r$. Some properties of the monopole metric are similar to those of the metric for a gauge cosmic string,[8]

$$ds^2 = dt^2 - dz^2 - dr^2 - (1 - 8G\mu) r^2 d\phi^2 , \qquad (13)$$

in which all surfaces $z =$const are cones with a deficit angle $\Delta = 8\pi G\mu$, μ being the mass per unit length of string. An important difference is that the monopole metric is not locally flat.

We now turn to light propagation in the gravitational field of a global monopole. Consider a light signal propagating from source S to an observer O. Without loss of generality we can assume that both S and O lie on the surface $\theta = \pi/2$. Then it is clear from the symmetry that the whole trajectory lies on that surface. Moreover, it is easily verified that the geodesic equation describing light propagation on the surface $\theta = \pi/2$ of the metric (11) is identical to that on the surface $z =$const of the cosmic string metric (13) with the same deficit angle. The problem is thus reduced to the problem of light propagation in the metric of a cosmic string.[9,10] If S, O, and the monopole (M) are perfectly aligned, then the image has the form of a ring of angular diameter

$$\delta\phi_0 = 8\pi^2 G \eta^2 l (d+l)^{-1} , \qquad (14)$$

where d and l are the distances from the monopole to the observer and to the source, respectively. If SM and OM are misaligned by a small angle $\alpha < \Delta$, then the observer will see two point images separated by an angle $\delta\phi_0$ given by Eq. (14). Note that $\delta\phi_0$ is independent of α. It is easily shown that the brightness ratio of the two images is given by the ratio of the impact parameters of the corresponding light rays. This is an important difference from the case of cosmic strings, in which the two images have equal brightness.

The subscript of $\delta\phi_0$ refers to the fact that the monopole was assumed to be at rest with respect to the observer. As we shall explain shortly, cosmological monopoles are expected to move at relativistic speeds, and so Eq. (14) has to be generalized to the case of a moving monopole. The same argument as in Ref. 10 gives

$$\delta\phi = \gamma^{-1} (1 - \mathbf{n}\cdot\mathbf{v})^{-1} \delta\phi_0 , \qquad (15)$$

where $\mathbf{n}$ is a unit vector along the line of sight, $\mathbf{v}$ is the monopole velocity, and $\gamma = (1 - v^2)^{-1/2}$.

For a typical grand unification scale $\eta \sim 10^{16}$ GeV, the angular separation (14) is of the order of 10 arcsec and is certainly within the observable range. The crucial question now is what is the expected density of global monopoles. In the rest of this paper we would like to make some comments on the cosmological evolution of global monopoles, which may help to answer this question.

Let us first consider a monopole-antimonopole ($M\bar{M}$) pair. The energy of the pair is $E \sim \eta^2 R$, where R is the

$M\bar{M}$ distance. The attractive force acting on M and $\bar{M}$ is $F = \partial E/\partial R \sim \eta^2$ and is independent of the distance. Under the action of this force, M and $\bar{M}$ oscillate and lose their energy by emitting Goldstone bosons. If the $M\bar{M}$ separation is greater than the core radius, $R > \delta$, then the motion of the pair is relativistic. The energy-loss rate $\dot{E}$ can be roughly estimated[11] if we note that the energy flux is $T_i^0 \sim \partial_t\phi^a\partial_i\phi^a$. At a distance $r \sim R$ from the pair, $\partial_t\phi^a \sim \partial_i\phi^a \sim \eta/R$ and $T_i^0 \sim \eta^2/R^2$. Hence, $\dot{E} \sim T_i R^2 \sim \eta^2$, and the lifetime of the pair is $\tau \sim R$. We see that the Goldstone-boson radiation is a very efficient energy-loss mechanism. As the pair loses its energy the $M\bar{M}$ separation decreases, and when it becomes $\sim\delta$, the pair annihilates.

The large attractive force between global monopoles and antimonopoles suggests that $M\bar{M}$ annihilation is very efficient and that the monopole overproduction problem does not exist. In fact, it can be so efficient that not a single monopole will be left within the observable Universe. One can draw the analogy between global monopoles and quarks, which also interact with a force independent of the distance. We do not expect any free quarks to be left after the quark-hadron phase transition, so why should we expect any global monopoles? An important difference here is that even if two quarks get separated by a large distance, the gluon "string" connecting them can easily break up producing quark-antiquark pairs. The probability of producing an $M\bar{M}$ pair by the Goldstone field is proportional to $\exp(-kM_{\rm core}^2/F) \sim \exp(-k'/\lambda)$, where k and k' are numerical coefficients. For small λ this probability is negligibly small. One can also draw the analogy between global monopoles and monopoles connected by strings, for which it is known[12] that large $M\bar{M}$ separations are exponentially suppressed, and so no $M\bar{M}$ pairs are expected to survive in the observable Universe. But again there is an important difference. Unlike monopoles connected by strings, a global monopole is not paired with any particular antimonopole, and it is not clear how efficiently it can find a partner. A convincing analysis of global monopole evolution will probably require a numerical simulation.

This research was partly supported by the National Science Foundation and by contributions from General Electric Co. and Dennision Manufacturing Co.

[1]T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).

[2]For a review see A. Vilenkin Phys. Rep. **121**, 263 (1985).

[3]J. P. Preskill, Phys. Rev. Lett. **43**, 1365 (1979).

[4]A. G. Cohen and D. B. Kaplan, Phys. Lett. B **215**, 67 (1988).

[5]R. Gregory, Phys. Lett. B **215**, 663 (1988).

[6]In flat space $f(r) = 1 - (\lambda\eta^2 r^2)^{-1}$.

[7]L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon, Oxford, 1971).

[8]A. Vilenkin, Phys. Rev. D **23**, 852 (1981).

[9]A. Vilenkin, Astrophys. J. Lett. **282**, L51 (1984); J. R. Gott, Astrophys. J. **288**, 422 (1985).

[10]A. Vilenkin, Nature (London) **322**, 613 (1986).

[11]This estimate is similar to that of the radiation of Goldstone bosons by global strings in R. L. Davis, Phys. Rev. D **32**, 3172 (1985).

[12]A. E. Everett, T. Vachaspati, and A. Vilenkin, Phys. Rev. D **31**, 1925 (1985); E. Copeland *et al.*, Nucl. Phys. **B298**, 445 (1988).