Simple Systematics of the Shape Transitions in Hot Rare-Earth Nuclei

Y. Alhassid and J. M. Manoyan[(a)]

Center for Theoretical Physics, Sloane Laboratory, Yale University, New Haven, Connecticut 06511 and A. W. Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06511

S. Levit

Department of Nuclear Physics, The Weizmann Institute of Science, Rehovot 76100, Israel

(Received 14 October 1988)

The Landau theory of shape transitions in hot rotating nuclei is applied to even-even rare-earth nuclei from cerium to hafnium. The parameters of the Landau expansion are extracted from microscopic calculations based on the Nilsson-Strutinsky procedure, and are shown to follow simple systematics. In particular, the systematics of the critical temperature and the critical angular momentum of the shape transitions as functions of neutron and proton number are investigated. Simple rules are provided for the behavior of these quantities.

PACS numbers: 24.60.Dr, 27.70.+q

Recent experiments are providing new information on the properties of hot nuclei formed in heavy-ion fusion reactions.[1-5] Experimental tools involve measurements of the spectra of γ rays as well as light particles emitted from the compound nucleus during its cooling process. Of special interest is the evolution of the nuclear shape as a function of temperature and angular momentum and, in particular, the predicted prolate-to-oblate shape transitions. Recently a unified framework based on the Landau theory of phase transitions was introduced to describe the universal features of the nuclear shape transitions.[6-8] In particular, a universal phase diagram emerged in terms of a certain "reduced" temperature and a scaled angular velocity. The unfolding of this diagram to the experimentally accessible variables—excitation energy and spin—requires the knowledge of certain parameters which are specific to the nucleus under consideration, are not universal, and should be determined from the experiment or from microscopic models. We have performed microscopic calculations for all known even-even isotopes of the rare-earth elements and extracted their respective Landau parameters. In particular, we have calculated the critical temperature and angular momentum of the shape transitions for these nuclei and found that they follow a rather simple systematic trend. In addition, all free-energy surfaces of these rare-earth nuclei are now available in a compact parametrized form for $T \lesssim 3$ MeV. In this Letter we present the most important features of our systematics. The complete account of our investigation is reported elsewhere.[9,10]

We start with a brief review of the Landau theory of the nuclear shape transitions.[6] The relevant macroscopic variables of a hot rotating nucleus are its excitation energy E^* and spin J. Technically, it is more convenient to work with their intensive partners, the temperature T and angular velocity ω. In analyzing shape transitions the quadrupole deformation parameters $\alpha_{2\mu}$ play the role of the order parameters with respect to which the nuclear free energy F should be minimized. Since $F(T,\boldsymbol{\omega},\alpha_{2\mu})$ is a rotational scalar, it must be built out of invariant combinations of $\alpha_{2\mu}$'s and $\boldsymbol{\omega}$. ω-independent invariants are considered up to fourth order in $\alpha_{2\mu}$ and ω-dependent ones—up to second order in ω. Instead of $\alpha_{2\mu}$ we can use the parameters β,γ to describe the deformation of the nucleus in the intrinsic principal frame and the Euler angles Ω to describe the orientation of the nucleus with respect to the rotation axis $\boldsymbol{\omega}$. Minimizing F with respect to Ω, we find that the nucleus in equilibrium rotates around a principal axis with the largest moment of inertia. We call this axis z. F then has the general form[6]

$$F(T,\omega,\alpha_{2\mu}) = F_0(T) + A(T)\beta^2 - B(T)\beta^3\cos 3\gamma + C(T)\beta^4 - \tfrac{1}{2} I_{zz}(\beta,\gamma,T)\omega^2 , \quad (1)$$

where the moment of inertia is

$$I_{zz} = I_0(T) - 2R(T)\beta\cos\gamma + 2I_1(T)\beta^2 + 2D(T)\beta^2\sin^2\gamma . \quad (2)$$

The temperature-dependent coefficients A, B, C, I_0, R, I_1, and D are phenomenological parameters which are not determined by the Landau theory. However, the topography of the free-energy surfaces (1) depends only on certain combinations of these parameters which have simple behavior. Thus in the $D=0$ case (rigid body moment of inertia) the relevant dimensionless combinations are the reduced temperature $\tau = AC/B^2$ and ω/ω_c, where $\omega_c = \frac{9}{16}(B/C)(B/R)^{1/2}$ is a critical angular velocity. In these reduced variables a universal phase diagram emerges as described in Ref. 6. It shows rapid changes of nuclear shapes near the so-called tricritical point at $\tau_c = \frac{63}{128}$, $\omega = \omega_c$. The values of the temperature and angular momentum which give the critical values of τ and ω depend on the functions $A(T), B(T), \ldots$ and thus on the nucleus under consideration. We have investigated the systematics of these functions for the even-even rare-earth isotopes between $Z=58$ and 72. For that purpose we have calculated their free-energy surface at $\omega=0$ by using a Nilsson-Strutinsky procedure. The standard Nilsson single-particle Hamiltonian,

$$h=-\frac{\hbar^2}{2m}\Delta+\frac{1}{2}m\sum\omega_k^2x_k^2 -\kappa\hbar\omega_0[2\mathbf{l}\cdot\mathbf{s}+\mu(\mathbf{l}^2-\langle\mathbf{l}^2\rangle_N)]\,, \quad (3)$$

was used where the frequencies ω_k were parametrized according to Hill and Wheeler,[11]

$$\omega_k=\omega_0\exp\left[-\left(\frac{5}{4\pi}\right)^{1/2}\beta\cos\left(\gamma-\frac{2\pi}{3}k\right)\right], \quad (4)$$

and $\langle\mathbf{l}^2\rangle_N$ is the expectation value of $\mathbf{l}^2$ in a major oscillator shell N. For all rare-earth nuclei we used[12]

$$\begin{aligned}&\kappa=0.0637,\ \mu=0.420\,,\\ &\hbar\omega_0=41A^{-1/3}\left(1+\frac{1}{3}\frac{N-Z}{A}\right),\ \text{for neutrons}\,,\\ &\kappa=0.0637,\ \mu=0.60,\\ &\hbar\omega_0=41A^{-1/3}\left(1-\frac{1}{3}\frac{N-Z}{A}\right),\ \text{for protons}\,,\end{aligned} \quad (5)$$

where $\hbar\omega_0$ is given in MeV. We used a BCS model with a monopole pairing force to calculate the oscillating part of the pairing energy. The pairing matrix element was taken to be isospin dependent and proportional to the nuclear surface area (for deformed shapes) as in Ref. 13. The effect of pairing was found to be negligible for $T\gtrsim 0.5$–0.8 MeV. The Strutinsky normalization can be generalized[14] to finite T but in the temperature range of our interest it is sufficient to use the cold-nucleus approximation in the calculation of the average energy $\tilde{E}$. In the liquid-drop energy we have allowed for large deformations by taking the exact Coulomb and surface energies expressed as elliptic integrals.

Except for few surfaces at very low temperatures, all surfaces were found to have the topography predicted by the universal Landau expansion (1) with $\omega=0$, the case we consider at the moment. We have mapped the calculated free energies on a simple form (1) using the uniform mapping technique explained in Refs. 6 and 9. This technique permits reliable extraction of a set of the parameters functions $F_0(T)$, $A(T)$, $B(T)$, and $C(T)$. They turn out to have simple systematics as functions of N and Z of the nucleus.

Figure 1 shows, e.g., $A(T)$ for different erbium isotopes in the temperature range $T\gtrsim 0.8$ MeV. A is the most crucial parameter in the Landau theory and controls the curvature of F at $\beta=0$. The transition of a prolate-to-spherical shape at $\omega=0$ occurs near the temperature T_c where A changes sign. The values of A are very sensitive to shell effects. For mid-shell nuclei, which are strongly deformed in their ground state, A starts large in magnitude and negative and increases monotonically with T towards positive values. Such nuclei undergo prolate-to-spherical shape transition in the absence of rotations. Rotations change this into triaxial-to-oblate transition as discussed in Ref. 6. For nuclei near the closed shell, A starts positive and decreases monotonically but never becomes negative. Such nuclei start and stay spherical but become softer with increasing T. Above $T\sim 3$ MeV all nuclei have approximately the same A (~ 30 MeV) due to the disappearance of shell effects. A complete set of tables and diagrams of the parameters F_0, A, B, and C is presented in Refs. 9 and 10.

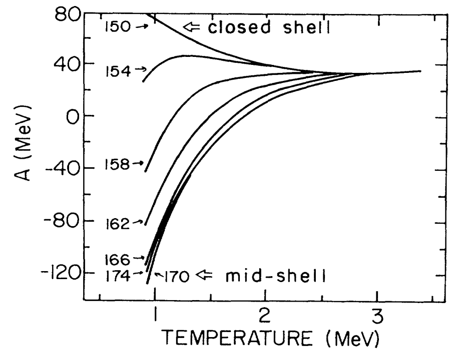

FIG. 1. The Landau parameter $A(T)$ for various erbium isotopes as calculated microscopically (see text). All curves converge to a common value ≈ 30 MeV for $T\gtrsim 3$ MeV.

Figure 2 shows the critical temperature T_c versus neutron number for the even-even rare-earth nuclei. T_c is very close to the temperature at which $\tau=\tau_c$. On the phase diagram of deformed nuclei[6] the value of T_c determines the position of the line of triaxial-to-oblate shape transitions near $J\approx 0$. Qualitatively, rapid transition from almost prolate-to-oblate shape is expected in the vicinity of this line. The systematic of T_c is strikingly simple. For each family of isotopes between two closed neutron shells the values of T_c fall on an inverted parabola-like curve whose maximum is at the mid shell. The curve drops rapidly towards shell closure. This is of course consistent with the A systematics of Fig. 1. For various families the parabolalike curves are arranged like onion shells where the most inside shells correspond to isotopes near proton shell closure at $Z=50$ and 82 and the outside shells are in the region of proton mid shell, $_{66}$Dy. The largest critical temperature of $T_c\approx 1.85$ MeV is found in mid-neutron-shell isotopes of Gd, Dy, and Er.

The respective critical excitation energies E_c^* which correspond to the above critical temperatures are dis-

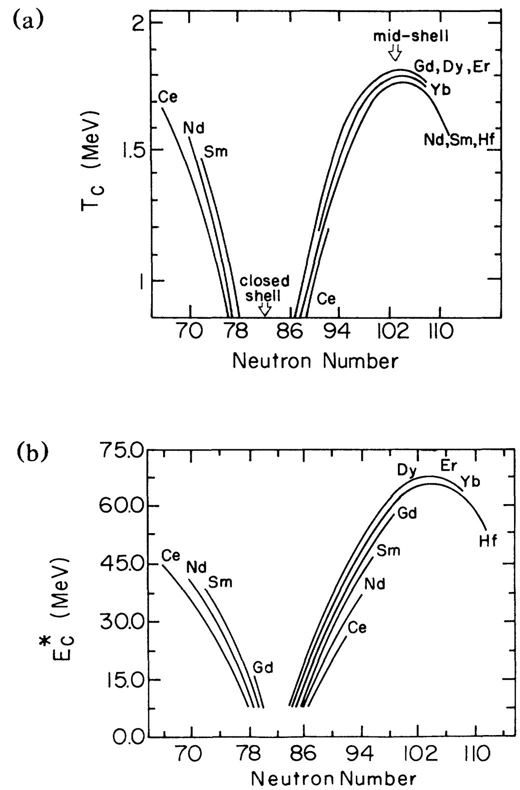

FIG. 2. (a) The critical temperature T_c and (b) the corresponding critical excitation energy E_c^* as functions of neutron number for even-even rare-earth nuclei. The maximal value of T_c occurs near the neutron mid shell ($N=104$) for a given isotope family and near the proton mid shell ($Z=66$) among various families. Shape fluctuations are largest in the vicinity of (E_c^*, J_c).

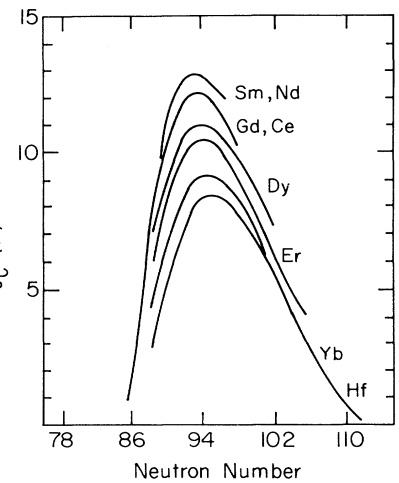

FIG. 3. The angular momentum J_c at the tricritical point vs neutron number for rare-earth nuclei. J_c is maximal around $\frac{1}{3}$ of the shell ($N=96$) and drops to zero around $\frac{2}{3}$ of the shell ($N=112$). Shape fluctuations are largest in the vicinity of (E_c^*, J_c).

played in Fig. 2(b). We emphasize that the calculation of F_0, A, B, and C and T_c and E_c^* above requires only the knowledge of the $\omega=0$ (no rotation) surfaces.

Finally, we consider the angular momentum J_c which corresponds to $\omega=\omega_c$, i.e., $J_c=I_{zz}\omega_c$. Large shape fluctuations are expected in the vicinity of (E_c^*, J_c) on the phase diagram. J_c is shown in Fig. 3 for various rare-earth isotope families. The moment of inertia requires the knowledge of the parameters I_0, I_1, R, and D in Eq. (2). We have calculated them using the rigid body expressions to first order in deformation, i.e., $I_0=\frac{2}{5}mR_0^2$, $R=(5/16\pi)^{1/2}I_0$, and $D=I_1=0$. We note that since $\omega_c \sim B^{3/2}/C$ we expect the J_c systematics to be dominated by the B systematics. B, on the other hand, governs the prolate-oblate asymmetry of the free energy [at $A=0$, the prolate-oblate free-energy difference is $\Delta F=\frac{24}{256}(B^4/C^3)$]. One can apply the Hill-Wheeler "thirds of the shell" rule[11] according to which prolate ground-state deformation dominates in the first two-thirds of the shell while oblate deformation is typical for the last third. It is thus expected that $B\approx 0$ around two-thirds of a filled shell and is expected to be maximal around one-third of a filled shell. The behavior of J_c in Fig. 3 tends to support the foregoing analysis with J_c rising rapidly from neutron shell closure at 82 to a maximum in the neighborhood of the end of the first third of the shell ($N=96$) and then proceeds to fall towards zero at the end of the second third of the shell ($N=112$). An approximate proton thirds-of-a-shell rule is also observed by inspecting the various curves in Fig. 3. The proton shell closures are 50 and 82 and so the "most outside" curves correspond to nuclei with 60–62 protons (Nd and Sm) while the "most inside" curves to 70–72 protons (Yb and Hf).

In general, the values of J_c are relatively small $[\lesssim(10\text{–}15)\hbar]$ indicating that the transitions are very close to being second order (cf. Ref. 6).

The approximation of using rigid body moment of inertia in the calculations of J_c is good if the critical temperature $T_c \gtrsim 0.5$–0.8 MeV so that pairing is negligible. For some transitional nuclei where T_c is lower the effect of pairing on the moment of inertia is important and thus pairing should be included in the $\omega\neq 0$ microscopic surface calculations. Pairing can also be incorporated in the Landau theory by adding a pairing gap order parameter.[15]

This work was partly supported by the Department of Energy Contract No. DE-AC02-76-ER-03074, and by the U.S.-Israel Binational Science Foundation Grant No. 86-00336. Y.A. acknowledges support from the A. P. Sloan Foundation.

[a]Present address: IBM Research Division, T. J. Watson Research Center, P.O. Box 218, Yorktown Heights, NY

10598.

[1]C. A. Gossett, K. A. Snover, J. A. Behr, G. Feldman, and J. L. Osborne, Phys. Rev. Lett. **54**, 1486 (1985); K. A. Snover, Annu. Rev. Nucl. Part. Sci. **36**, 545 (1986).

[2]J. J. Gaardhoje, C. Ellegaard, B. Herskind, and S. G. Steadman, Phys. Rev. Lett. **53**, 148 (1984).

[3]D. R. Chakrabarty, M. Thoennessen, S. Sen, P. Paul, R. Butsch, and M. G. Herman, Phys. Rev. C **37**, 1437 (1988).

[4]R. Holtzmann *et al.*, Phys. Lett. B **195**, 321 (1987).

[5]Z. Majka *et al.*, Phys. Rev. Lett. **58**, 322 (1987).

[6]Y. Alhassid, S. Levit, and J. Zingman, Phys. Rev. Lett. **57**, 539 (1986); Nucl. Phys. **A464**, 205 (1987).

[7]Y. Alhassid, B. Bush, and S. Levit, Nucl. Phys. **A482**, 57c (1988); Phys. Rev. Lett. **61**, 1926 (1988); Y. Alhassid and B. Bush, Yale University Report No. YCTP-N16-88 (to be published).

[8]S. Levit and Y. Alhassid, Nucl. Phys. **A413**, 439 (1984).

[9]J. M. Manoyan, Ph.D. thesis, Yale University, 1987 (unpublished).

[10]Y. Alhassid, J. Manoyan, and S. Levit (to be published).

[11]D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1182 (1953).

[12]C. Gustafson, I. L. Lamm, B. Nilsson, and S. G. Nilsson, Ark. Fys. **36**, 613 (1967).

[13]S. G. Nilsson *et al.*, Nucl. Phys. **A131**, 1 (1969).

[14]M. Brack and P. Quentin, Nucl. Phys. **A361**, 35 (1981).

[15]A. L. Goodman, Nucl. Phys. **A406**, 94 (1983).