# *CP*-Violating Correlations in Electron-Positron Annihilation into $\tau$ Leptons

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An analysis of *CP*-violating effects in $e^+e^- \to \tau^+\tau^-$ is outlined. Some tensor observables sensitive to *CP*-odd couplings arising from possible electric or weak dipole moments $d_\tau, \tilde{d}_\tau$ of the $\tau$ are presented. For the case where both $\tau$'s decay via $\tau \to \pi\nu_\tau$ we estimate the the accuracies with which $\tilde{d}_\tau$ and $d_\tau$ can be measured at the $Z$ resonance and at a $\tau$-charm or $B$ factory, respectively.

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One of the most intriguing phenomena of particle physics is *CP* violation, observed so far only in the neutral-kaon system. Great effort has been made to find evidence for *CP* and/or time-reversal ($T$) violation in other places. Among those efforts are the searches for electric dipole moments (EDM's) of particles. (An EDM of a particle signals breakdown of $T$ invariance which, if the *CPT* theorem applies, also implies *CP* violation.) Stringent upper limits on the EDM of the neutron and the electron have been obtained.[1] Not much is known however, about EDM's of heavy fermions, i.e., of the $\tau$ lepton and of baryons containing $c$ or $b$ quarks.[2] As EDM's of particles are very sensitive to new *CP*-violating interactions[3] (that is, forces not described by the Kobayashi-Maskawa mechanism[4]) it is of great interest to obtain direct experimental information also on EDM's of heavy flavors.

In this Letter we show that the reaction $e^+e^- \to \tau^+\tau^-$ is well suited to study two *CP*-violating parameters of the $\tau$ lepton: to wit, its electric and weak dipole form factors $d_\tau(s), \tilde{d}_\tau(s)$ at c.m. energy $\sqrt{s}$ which may be present in the $\tau\bar{\tau}$ photon[5] and $\tau\bar{\tau}Z$ boson vertices, respectively. [The EDM of the $\tau$ is given by $d_\tau(s=0)$.] We shall also briefly discuss models where these form factors can be sizable.

In the following we consider the reaction

$$e^+(p_+) + e^-(p_-) \to \tau^+(k_+) + \tau^-(k_-) \qquad (1)$$

in the c.m. system. We assume unpolarized $e^+$ and $e^-$ beams. The initial state is then described by a *CP*-invariant density matrix. *CP*-odd observables for the final state can be constructed from the unit vector $\hat{\mathbf{k}}_+ = \mathbf{k}_+/|\mathbf{k}_+|$ and the $\tau^+$ and $\tau^-$ spin operators denoted by $\boldsymbol{\sigma}\otimes 1$ and $1\otimes\boldsymbol{\sigma}$. They act in the product space of the $\tau^+$ and $\tau^-$ spin spaces. Under a *CP* transformation we have

$$\hat{\mathbf{k}}_+ \to \hat{\mathbf{k}}_+, \quad \boldsymbol{\sigma}\otimes 1 \leftrightarrow 1\otimes\boldsymbol{\sigma}. \qquad (2)$$

A basis for all spin-momentum correlation observables is given by irreducible tensors of rank $n$ ($n=0,1,2,\ldots$) constructed from the three vectors above. The *CP* transformation (2) allows the classification of these observables as *CP*-even and -odd ones. Restricting ourselves to tensors of rank $n \le 2$ we find fourteen linearly independent *CP*-odd observables.[6] Here we only give two rank-2 tensor observables which we consider particularly promising for experimental investigations:

$$A_{ij} = \hat{k}_{+i}[\hat{\mathbf{k}}_+ \times (\boldsymbol{\sigma}\otimes 1 - 1\otimes\boldsymbol{\sigma})]_j + (i \leftrightarrow j), \qquad (3)$$

$$B_{ij} = \hat{k}_{+i}[(\boldsymbol{\sigma}\otimes 1)\times(1\otimes\boldsymbol{\sigma})]_j + (i\leftrightarrow j) - \tfrac{2}{3}\delta_{ij}(\text{trace}), \qquad (4)$$

where $1 \le i,j \le 3$ are the Cartesian vector indices. Clearly, $A, B$ are odd under *CP*. Since they are *CPT*-even no final-state interactions are required to render their expectation values nonzero. Thus, they can get contributions from *CP*-violating interactions at the tree-diagram level.

Let us turn now to *CP*-violating interactions. The standard model clearly predicts no observable *CP* violation in the leptonic reaction (1). For classifying the effects of new *CP*-violating interactions on (1) we assume that these interactions are characterized by a scale $\Lambda_{CP} \gg \sqrt{s}$. An effective Lagrangian analysis can then be made where the *CP*-violating forces are represented by local operators of dimension $d$ with couplings proportional to $\Lambda_{CP}^{4-d}$. We neglect the electron mass and keep only contact terms corresponding to operators with $d \le 6$. Then we find[6,7] that for unpolarized $e^+e^-$ beams we are left with the $d=5$ electric and weak dipole interactions,

$$L_{CP} = -\tfrac{1}{2} i\bar{\tau}\sigma^{\mu\nu}\gamma_5\tau(d_\tau F_{\mu\nu} + \tilde{d}_\tau Z_{\mu\nu}), \qquad (5)$$

affecting the $\tau\bar{\tau}\gamma$ and $\tau\bar{\tau}Z$ vertices in Figs. 1(b) and 1(c), respectively. In (5) $F_{\mu\nu}$ is the electromagnetic field tensor, $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$, $d_\tau$ is the EDM of the $\tau$, and $\tilde{d}_\tau$ its neutral-current analog.

Alternatively, a form-factor decomposition of the vertices represented by Fig. 1 can be made. In this approach $d_\tau, \tilde{d}_\tau$ are to be identified with the *CP*-odd dipole form factors $d_\tau(s), \tilde{d}_\tau(s)$ in the $\tau\bar{\tau}\gamma$ and $\tau\bar{\tau}Z$ vertices, respectively. These form factors may have abortive parts which we shall neglect, however, in the following.

In many extensions of the standard model *CP* violation in the lepton sector[3] occurs quite naturally and nonzero form factors $d_\tau(s), \tilde{d}_\tau(s)$ are generated. (*A priori* $d_\tau$ and $\tilde{d}_\tau$ are not related but one expects them to be of the same order of magnitude in such models.) Probably the most sizable potential source of *CP* violation, generating "large" dipole moments for heavy fermions, is Higgs-boson models of *CP* violation. In particular, there are models[8-11] containing neutral spin-0 bosons $\phi$, some of which may couple solely to leptons $l$ through lepton-flavor-conserving scalar *and* pseudoscalar couplings.[9] These interactions then generate $d_l, \tilde{d}_l$ which grow like[6,9-11] $m_l^3$, unless $m_l^2 \gg m_\phi^2$. That is, in such models one gets

$$d_\tau \approx \tilde{d}_\tau \approx \left(\frac{m_\tau}{m_\mu}\right)^3 d_\mu \approx \left(\frac{m_\tau}{m_e}\right)^3 d_e \,. \qquad (6)$$

Using the present experimental limits[1] on $d_e$ and $d_\mu$, this implies that sizable values of $d_\tau, \tilde{d}_\tau$, say of the order $10^{-17}\, e\,\text{cm}$, are conceivable.

We have calculated the expectation values of (3) and (4) arising from the *CP*-odd amplitude parametrized by $d_\tau, \tilde{d}_\tau$ interfering with the *CP*-even standard-model amplitude (of which only the leading order is taken into account). On the $Z$ resonance the $\gamma$ exchange diagram Fig. 1(b) enters only as a radiative correction of order $\alpha$ and we neglect it there. Keeping thus only $\tilde{d}_\tau$ for

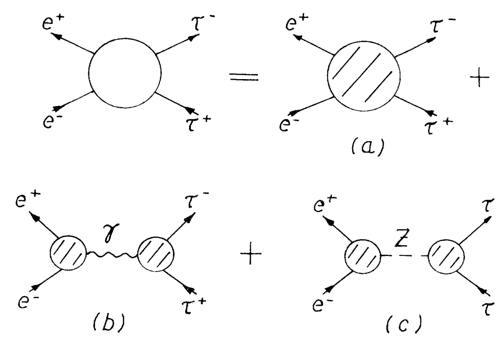


FIG. 1. Decomposition of the amplitude of (1) into one-particle irreducible parts (with respect to standard-model particles) (a)-(c). Higgs-boson exchange is neglected.

$\sqrt{s} = m_Z$ we get

$$\langle A_{ij} \rangle = \frac{12\sqrt{3}}{5} \left(\frac{\tilde{d}_\tau m_Z}{e}\right) s_{ij} \qquad (7)$$

and $\langle B_{ij} \rangle = 0$, where we put the weak angle at $\sin^2\theta_W = \frac{1}{4}$ and neglected terms of order $m_\tau/m_Z$. Here $e > 0$ is the positron charge and

$$s_{ij} = \tfrac{1}{2}\left(\hat{p}_{+i}\hat{p}_{+j} - \tfrac{1}{3}\delta_{ij}\right) \qquad (8)$$

with $\hat{\mathbf{p}}_+ = \mathbf{p}_+/|\mathbf{p}_+|$ the tensor polarization of the $Z$.

On the other hand, we consider the reaction (1) for $\sqrt{s} \le 10$ GeV, an energy region which is already quite well explored experimentally and which will be even better covered if the planned $\tau$-charm and/or $B$ factories are realized. At these energies we can neglect the $Z$ exchange [Fig. 1(c)], being suppressed with respect to $\gamma$ exchange by $s/m_Z^2$. Keeping the EDM $d_\tau$ as the only *CP*-violating coupling we find

$$\langle B_{ij} \rangle = -\frac{12}{5}\left(1 - \frac{4m_\tau^2}{s}\right)^{1/2} \left(1 + \frac{4m_\tau}{3\sqrt{s}}\right) \left(1 + \frac{2m_\tau^2}{s}\right)^{-1} \frac{d_\tau}{e}\sqrt{s}\, s_{ij} \,. \qquad (9)$$

Here $\langle A_{ij} \rangle = 0$ because $A_{ij}$ is $C$-odd, whereas the $\gamma$-exchange diagrams are $C$-even.

The measurement of the correlations $A, B$ requires knowledge of the momentum directions of the $\tau$'s. In principle this can be obtained with a vertex chamber. The polarization of the $\tau$'s can be obtained through their decays, e.g., from $\tau \to \pi\nu$ assuming the standard $V-A$ interaction.

We will now discuss some *CP*-odd observables for the case where both $\tau$'s decay into $\pi\nu$ requiring only knowledge of the momenta $\mathbf{q}_\pm$ or unit momenta $\hat{\mathbf{q}}_\pm = \mathbf{q}_\pm/|\mathbf{q}_\pm|$ of $\pi^\pm$ in the overall c.m. system. Again we find tensor observables to be most useful:

$$T_{ij} = (\mathbf{q}_+ - \mathbf{q}_-)_i (\mathbf{q}_+ \times \mathbf{q}_-)_j + (i \leftrightarrow j) \,, \qquad (10)$$

$$\hat{T}_{ij} = (\hat{\mathbf{q}}_+ - \hat{\mathbf{q}}_-)_i \frac{(\hat{\mathbf{q}}_+ \times \hat{\mathbf{q}}_-)_j}{|\hat{\mathbf{q}}_+ \times \hat{\mathbf{q}}_-|} + (i \leftrightarrow j) \,. \qquad (11)$$

Calculating the expectation values of (10) and (11) we keep as above at the $Z$ resonance only $\tilde{d}_\tau$, and for $\sqrt{s} \ll m_Z$ only $d_\tau$. We then get

$$\langle T_{ij} \rangle = x c s_{ij}, \quad \langle \hat{T}_{ij} \rangle = x \hat{c} s_{ij} \,, \qquad (12)$$

where $x = m_Z \tilde{d}_\tau / e$ for $\sqrt{s} = m_Z$ and $x = \sqrt{s} d_\tau / e$ for $\sqrt{s} \ll m_Z$. Table I contains for some c.m. energies the values of $c, \hat{c}$ and the averages of $T_{33}^2$ and $\hat{T}_{33}^2$ which can be used to assess the attainable accuracy on $d_\tau$ and $\tilde{d}_\tau$ from such correlation measurements. Using the standard branching ratios we conclude that with $10^7$ $Z$'s one can obtain an accuracy $\delta(\tilde{d}_\tau) = 6 \times 10^{-18}\, e\,\text{cm}$. At a $\tau$-charm factory with $\sqrt{s} = 4.6$ GeV, one can reach $\delta(d_\tau) = 2 \times 10^{-16}\, e\,\text{cm}$ given $10^7$ $\tau$ pairs. The same accuracy is attainable at a $B$ factory given $10^6$ $\tau$ pairs at $\sqrt{s} = 10$ GeV.

The observables (10) and (11) may also be used for channels which have higher event rates than $\tau \to \pi\nu$.

TABLE I. Values for $c$ and $\hat{c}$ defined in (12) and standard-model averages of $T_{33}^2$ and $\hat{T}_{33}^2$ for various $\sqrt{s}$. We used $m_Z = 92$ GeV and $\sin^2\theta_W = 0.23$.

| $\sqrt{s}$ (GeV) | $c$ (GeV$^3$) | $\hat{c}$ | $(\langle T_{33}^2\rangle)^{1/2}$ (GeV$^3$) | $(\langle \hat{T}_{33}^2\rangle)^{1/2}$ |
|---|---|---|---|---|
| 4.0 | 0.16 | 0.13 | 0.82 | 0.95 |
| 4.6 | 0.40 | 0.25 | 1.38 | 1.0 |
| 10.0 | 3.95 | 0.44 | 9.74 | 1.42 |
| 92.0 | $-1.42\times10^3$ | $-1.56$ | $8.9\times10^2$ | 0.86 |

For instance, one may sum up the leptonic modes $\tau^+\tau^- \to l^+\nu_l\bar{\nu}_\tau l^-\bar{\nu}_l\nu_\tau$ or one may use all decays of $\tau^\pm$ into one charged prong identifying $\mathbf{q}_+$ ($\mathbf{q}_-$) in (10) and (11) with the positively (negatively) charged-particle momentum. In this way the accuracy of measuring $d_\tau$ and $\tilde{d}_\tau$ can be increased substantially. The relevant formulas will be given elsewhere.[6]

The $CP$-odd observable $O = \mathbf{p}_+ \cdot (\mathbf{q}_+ \times \mathbf{q}_-)$ was suggested in Ref. 5 to search for a nonzero $d_\tau$ in $e^+e^- \to \tau^+\tau^- \to \pi^+\bar{\nu}_\tau\pi^-\nu_\tau$ around the $Z$ resonance. The contribution of $\tilde{d}_\tau$ to $\langle O\rangle$ was not considered. Then $\langle O\rangle$ vanishes in Born approximation at $\sqrt{s} = m_Z$. Assuming the collection of an integrated luminosity of 170 $\mathrm{pb}^{-1}$ in the vicinity of $\sqrt{s} = m_Z$ (which if spent at $\sqrt{s} = m_Z$, would produce about $10^7$ $Z$'s) the 1-standard-deviation accuracy with which $d_\tau$ could be measured was estimated to be $\delta(d_\tau) \simeq 3\times10^{-16}\, e\,\mathrm{cm}$.

To summarize, we have proposed simple $CP$-odd observables (3), (4), (10), and (11) for the reaction $e^+e^- \to \tau^+\tau^-$ which are sensitive to the electric dipole moment of the $\tau$ (if $\sqrt{s} \ll m_Z$) and its weak dipole moment (if $\sqrt{s} = m_Z$). In this respect experimental investigations at low and high energy are complementary. The observables (10) and (11) only require measurement of the $\pi^\pm$ momenta from the $\tau \to \pi\nu$ decays. Analogous observables can be applied to other decay channels. We emphasize that the above observables are unflawed indicators of $CP$ violation also if $CP$-odd interactions in $\tau$ decays are present. Finally, we would like to note that the correlations (3) and (4) can be used to look for $CP$-violating effects in any reaction $e^+e^- \to$ fermion $+$ antifermion and $p\bar{p} \to$ fermion $+$ antifermion. Interesting cases might be $e^+e^- \to \Lambda\bar{\Lambda}$, $e^+e^- \to \Lambda_c\bar{\Lambda}_c$, $e^+e^- \to \Lambda_b\bar{\Lambda}_b$, to be investigated at $\tau$-charm and $B$ factories, and $p\bar{p} \to \Lambda\bar{\Lambda}$ which can be studied at low-energy antiproton facilities. If $10^7$ $\tau^+\tau^-$ pairs are produced at a $\tau$-charm factory, $\sigma$ can be measured with a statistical accuracy of $3\times10^{-4}$. But systematic effects in absolute cross-section measurements are well known to be a difficult problem. Assuming nevertheless optimistically the relative deviation of the cross section from its standard-model value to be measured at $\sqrt{s} = 4$ and 6 GeV as $\delta\sigma/\sigma \leqq 3\times10^{-4}$, we would obtain $|d_\tau| \leqq 3 \times10^{-16}\, e\,\mathrm{cm}$. This bound would be 50% larger than the direct one obtainable from measuring (10) and (11) for the $\tau \to \pi\nu$ mode alone. Moreover, the latter type of investigation allows further increase in sensitivity by taking into account also other decay channels.

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*Note added.*— At a low-energy $\tau$ facility an upper bound on the EDM $d_\tau$ could also be obtained in an analogous fashion as the one deduced by Barr and Marciano at DESY PETRA energies.[2,3] Neglecting radiative corrections the cross section for $e^+e^- \to \tau^+\tau^-$ is

$$\sigma = \frac{4\pi\alpha^2}{3s}\left(1 - \frac{4m_\tau^2}{s}\right)^{1/2}\left(1 + \frac{2m_\tau^2}{s}\right)\left[1 + \frac{s}{2}\left(\frac{d\tau}{e}\right)^2\left(1 - \frac{4m_\tau^2}{s}\right)\left(1 + \frac{2m_\tau^2}{s}\right)^{-1}\right].$$

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[1] For a compilation, see Particle Data Group, G. P. Yost *et al.*, Phys. Lett. B **204**, 1 (1988).

[2] Of course, a sizable EDM $d_\tau$ of the $\tau$ would lead to a considerable deviation ($\sim d_\tau^2$) of the cross section for $e^+e^- \to \tau^+\tau^-$ from its standard-model value, which has not been observed. In this way Barr and Marciano [BNL Report No. BNL-41939, 1988 (to be published)] deduce $|d_\tau| < 10^{-16}$ $e\,$cm. But this argument is an indirect one and cancellations of the EDM contribution with other possible new-physics terms in the cross section cannot be excluded. Nevertheless, this number indicates the orders of magnitude in accuracy which direct experiments should try to surpass.

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