Cosmic-String Evolution

David P. Bennett
Physics Department, Princeton University, Princeton, New Jersey 08544

François R. Bouchet
Institut d'Astrophysique de Paris, 98 bis Boulevard Arago, 75014 Paris, France
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We summarize the results of high-resolution simulations of cosmic-string evolution in an expanding universe. The string network is found to relax to a scaling solution in accordance with Kibble's "one-scale model." However, the long strings are found to have significant structure on scales much smaller than the scale length of the long-string network, and the size of the stable loops produced by the network is determined by this small-scale structure. This implies that the long strings rather than the loops will probably play a dominant role in the observational effects of cosmic strings.

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Cosmic strings are of particular interest among astrophysicists due to the realization by Zel'dovich[1] and Vilenkin[2] that they could serve as seeds for galaxy and large-scale structure formation. They also have direct observable consequences through gravitational lensing,[3,4] microwave-background anisotropies,[5,6] and a stochastic gravitational-wave background.[7,8]

Shortly after they are formed, the motion of cosmic strings is described very accurately by the Nambu equation of motion in an expanding background.[9] The evolution of a string network can be studied with the "one-scale model" invented by Kibble.[10-13] The basic parameter of this model is the long-string scale L which is defined by $\rho_{\rm LS}=\mu/L^2$, so that a volume L^3 contains a total (proper) length of string L. $\rho_{\rm LS}$ is the energy density of the long strings (those longer than the horizon). The simplest form of the one-scale model holds that

$$\dot{\rho}_{\rm LS}=-2\frac{\dot{a}}{a}(1+\langle v^2\rangle)\rho_{\rm LS}-\frac{C}{L}\rho_{\rm LS}\,, \tag{1}$$

where $\langle v^2\rangle$ is a spatial average of v^2. The first term on the right-hand side of Eq. (1) gives the variation of $\rho_{\rm LS}$ with the expansion of the Universe, and the second term gives the energy loss from the long strings due to loop production. The essential "one-scale" assumption is that the time scale for loop production is just the scale length L of the infinite strings (i.e., C is a constant). Equation (1) has a stable "scaling solution" in which $L\propto t$. A string network must obey such a scaling solution for the cosmic-string galaxy formation to be viable, and there is now increasing numerical evidence that it does.[13-16] If we set $L=\gamma H$, where $H\equiv a\int dt/a$ is the horizon size, then γ is the scale length of the long-string network (in horizon-sized units) which is a constant in a scaling solution.[17] If we eliminate C in Eq. (1) in favor of γ_r (the radiation-era scaling value) and we take $\langle v^2\rangle$ to be a constant [this is correct to ± 0.02 for our radiation-era runs and consistent with the model of Albrecht and Turok[13] for $\langle v^2\rangle$], the solution of (1) is

$$\frac{\gamma-\gamma_r}{\gamma_0-\gamma_r}=\left(\frac{a}{a_0}\right)^{\langle v^2\rangle-1}, \tag{2}$$

where a_0 and γ_0 are initial values. Figure 1 shows the evolution of γ vs $(a/a_0)^{\langle v^2\rangle-1}$ for several radiation-era runs. The dashed curve is from our highest-resolution run with our original code (code I), and the solid curves are from three different runs of our second, higher-resolution code (code II) with different initial γ's. Clearly, the results from our two codes coincide quite closely. When we include our estimated errors, our value for $1/\gamma_r^2=\rho_{\rm LS}H^2/\mu$ is 52 ± 10 which compares to our previous value of 80 ± 40 obtained from code I. The difference between the central values of these two measure-

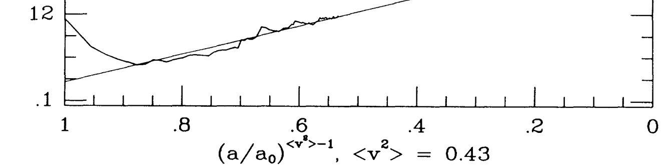

FIG. 1. The evolution of the scale length of the long strings γ vs $(a/a_0)^{\langle v^2\rangle-1}$ for the three code-II runs (solid curves) and one high-resolution code-I run (dashed curve). The straight lines are least-squares fits to the curves. $\langle v^2\rangle$ has been set to its scaling-solution value $\langle v^2\rangle=0.43$.

ments is due to a reduction in our estimated systematic errors rather than an actual difference in the results of the two codes. For comparison, Allen and Shellard[16] (hereafter AS) obtain $1/\gamma_r^2=64\pm16$ while Albrecht and Turok[13] (hereafter AT) obtain $1/\gamma_r^2=210$, which they estimate could be as much as a factor of 4 too large due to systematic errors which they have not thoroughly investigated.

The simplified one-scale model [Eq. (2)] predicts that these curves should be linear and intersect at $(a/a_0)^{\langle v^2\rangle-1}=0$ ($a=\infty$) at the scaling-solution value $\gamma_r\simeq0.14$. Except for an initial transient due to the peculiarities of the initial conditions, these curves fit straight lines that nearly converge to $\gamma=0.14$ at $a=\infty$ very well, so Kibble's model appears to accurately describe the relaxation to scaling in the radiation era. Our matter-era value for $1/\gamma_m^2$ is somewhat smaller: 31 ± 7. AT's matter-era value is $1/\gamma_m^2\approx64$. The Kibble-model prediction for the relaxation to the matter-era scaling solution is worse than in the radiation era, and the matter-era value for C is 40% smaller than the radiation-era value. Nevertheless, Kibble's one-scale model does seem to give a reasonably good description of the behavior of the long strings.

There is, however, a crucial piece of physics that is not included in the Kibble model. Both our simulations and the AS simulations show a significant amount of structure at scales much smaller than L. This structure is due to the presence of discontinuities known as "kinks" in the velocity of and tangent to the string. Four kinks are produced whenever a pair of string segments cross each other and intercommute (break and reconnect the other way). In a flat space-time, the shape of these kinks is preserved by the equations of motion, while in an expanding universe their amplitude decays with the expansion as $a^{2v^2-1}\sim a^{-0.14}$ for $v^2=0.43\pm0.02$, which is our mean radiation-era value.[15] One might expect that the kink density would be reduced by the tendency for the more wiggly pieces of string to chop off the network, but we have found that this can only compensate for the production of kinks when the kink density is quite high. Gravitational-radiation back reaction might also tend to smooth out the small-scale structure on the long strings, but this will only operate on scales that are smaller than our current resolution.

The preponderance of kinks on the long strings makes it very difficult to evolve the strings numerically. Standard finite-differencing schemes will progressively smooth the kinks especially when numerical diffusion is invoked to prevent the growth of short-wavelength instabilities. In code I,[14] we attempted to minimize smoothing by invoking numerical diffusion only when an instability started to develop. AS (Ref. 16) have used a sophisticated algorithm originally developed to handle shock waves which apparently halts kink smoothing after the kink has spread over three or four grid points. Our new code[15] (code II) maintains accurate resolution down to the scale of the interparticle separation by evolving right- and left-moving waves on separate grids that move with respect to each other. Smoothing is only invoked when the separation of two kinks becomes smaller than the mean interparticle separation. The algorithm becomes exact in the flat-space limit (except for the merging of kinks that is sometimes required upon intercommutation). It is quite reassuring that these three codes all seem to give quite similar results. The AT code has a significant amount of numerical smoothing, and we believe that this is responsible for much of the difference between their results and those of the three higher-resolution codes.

In Fig. 2, we have plotted $\log_{10}\mathcal{L}/R$ vs $\log_{10}R/H$ for a radiation-era, transition-era, and a matter-era run. The fractal dimension of the string d is given by $\mathcal{L}\propto R^d$, so the slope of the curves in Fig. 2 is just $d-1$. On large scales ($R>L$), the strings have a fractal dimension very close to 2, just like random walks, but on smaller scales the fractal dimension decreases to a value somewhat larger than 1: from ~1.12 in the radiation era to ~1.05 in the matter era. The matter-era fractal dimension is the smallest because there are fewer string crossings and there is more damping from the universal expansion in the matter era. The fractal dimension at small scales does not remain constant during our simulation runs. This is because our resolution is fixed in physical units so that our small-scale resolution range increases as $L\propto H$. Thus, if the fractal dimension remained fixed, then the total amount of energy in small-scale wiggles would increase with time. Instead, we find that the total amount of energy in small-scale wiggles remains very nearly constant in time. In our radiation-era runs, this is about 45% of the total energy, while in the matter era it is about 28%. There is also a slight variation of the fractal

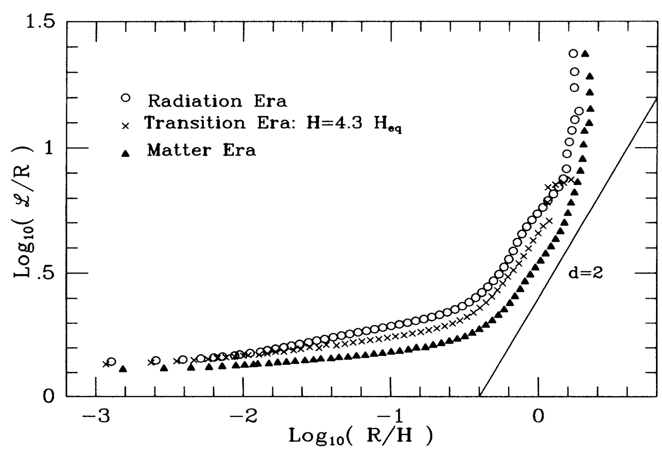

FIG. 2. $\log_{10}\mathcal{L}/R$ vs $\log_{10}R/H$ for a radiation-era run (open circles), a transition-era run (crosses), and a matter-era run (filled triangles). R is the distance between two points on a long string and $\mathcal{L}$ is the proper length of the segment of string which connects those two points.

dimension (i.e., slope of the curves) with size. The fractal dimension is smaller at very small scales (especially in the matter era). This suggests that in the physical limit, d might be 1 at very small scales, and gradually increase to $d \approx 1.1$ ($d \approx 1.05$ for the matter era) for R slightly smaller than L. The effect of the back reaction of the strings' own gravitational radiation might be to smooth the strings on scales of order $G\mu H$ (or somewhat larger).

The presence of this evolving structure on very small scales suggests that our results might be sensitive to numerical parameters that affect the strings on the smallest scales. We have checked very carefully for any evidence of this and found none. We have done two runs with identical initial conditions differing only in that one had 2.5 times as many sampling points as the other. We find that the values of ρ_{LS} differ by less than 4% throughout the two runs. The loop distributions are also very similar, in contrast to some of the lower-resolution runs discussed in Ref. 14. Our high-resolution runs have confirmed the tentative conclusion of Ref. 14 that when the minimum cutoff on the loop size is sufficiently small, it has no effect on the long-string evolution. Another test we have performed involved adding moderate and large amounts of small-scale power (in the form of random velocities at every point) to the initial conditions. This made a noticeable change in the evolution of the loop distribution, but almost no change in the predicted long-string scaling density. Thus, it clear that we have achieved high enough resolution so that the details of the small-scale structure on the strings do not have any effect on the large-scale behavior of the long strings, and this implies that our results for the large-scale behavior of the long strings are independent of the evolution on small scales.

The most important consequence of this small-scale structure on the long strings is that the stable non-self-intersecting loops that are produced by the string network are very much smaller than the scale length of the string network L. Previous work[10,13,18,19] had expected that "parent" loops of size $\sim L$ would break off the network and then fragment into smaller stable loops that would not be very much smaller. We find quite a different picture: First, the size of the stable loops produced is governed by the small-scale structure on the strings and *not* by L. In runs with smooth initial conditions, the size of the stable loops seems to be proportional to the separation between kinks. In these runs, both the mean separation between kinks on the long strings and the average stable-loop size actually *decrease* with time in *physical* units. When we run from initial conditions with excess short-wavelength power, we find that the stable loops are produced at a constant physical size which is smaller than in the smooth case. In all cases, the strings are always growing smoother at fixed physical scales, so the mean kink amplitude must be decreasing. This suggests that the physical size of the stable loops should eventually begin to increase, but we have seen no sign of this even when the typical stable loop has a proper length of about $10^{-3}H$. This implies that the strings are not yet smooth enough to produce larger stable loops.

We also find that more than two-thirds of the energy lost by the network goes directly into very tiny loops. Only 25% to 30% of the energy is chopped off the long strings in the form of large ($\sim L$) parent loops. This might seem puzzling in view of the success of Kibble's one-scale model. Figure 3(a) shows the configuration of the long-string network at a late time in one of our smaller radiation-era runs. The parts of the string that will chop off the network during the next factor of 1.2 in expansion are shown as larger dots. Figure 3(b) shows

(a)

(b)

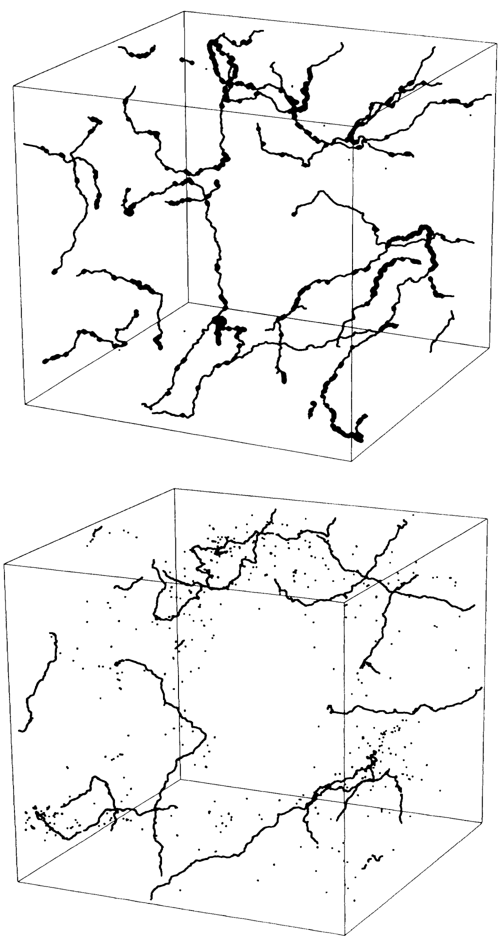

FIG. 3. (a) The configuration of all the long strings in one of our smaller runs when the horizon is 1.57 times the box size. The thick curves show the pieces of the long strings that will be chopped off into loops after an expansion factor of 1.2, and the thin curves show pieces that will remain on the long strings. (b) The configuration of these same string points after expansion by a factor of 1.2. Note that much of the long string that will be chopped into loops is made up of moderately long segments, while the loops produced are essentially microscopic.

the same string points an expansion factor of 1.2 later. Most of the loop production comes from sections of the string that are not very much shorter than L ($L \approx 22\%$ of the box length). The stable loops that are produced, however, are all so small that they just appear as points in Fig. 3(b). Thus, the energy that goes into loop production comes from "parent regions" with length of order L as Kibble's model would suggest. These parent regions tend to be more highly curved and have higher velocities than the rest of the long-string network. The preponderance of small-scale structure on the long strings causes these parent regions to fragment into very tiny loops either before or after the parent region chops off of the long string.

In summary, our high-resolution simulations of cosmic-string evolution have revealed a qualitatively new picture for cosmic-string evolution. Kibble's one-scale model gives a reasonably good description of the large-scale behavior of the string network, but it does not account for the small-scale structure on the long strings which prevents the formation of a significant number of stable loops with proper length larger than $10^{-3}H$ in the radiation era. The very small size of stable loops reduces the expected gravity-wave amplitude enough so that the best limit[20] on $G\mu$ from measurements of millisecond pulsar timing residuals[7] is $G\mu < 4\times10^{-6}$. Lower-resolution simulations had suggested an upper bound[13] that was a factor of 100 smaller. Another important implication of our new results is that the dominant gravitational perturbations from strings probably come from the long strings as Zel'dovich originally suggested[1] and not from the loops. This means that the pattern of galaxy and large-scale structure formation due to cosmic strings is still largely unknown, and it suggests that there may be little difference between structure formation from gauge strings and from global strings which have long-range interactions and do not radiate much gravitationally. Work is currently under way to include the gravitational accretion of matter in our string-evolution code in order to calculate the details of galaxy and large-scale structure formation to be expected in the string scenario.

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