

Fain Replies: Wódkiewicz¹ does not question the validity of the solutions obtained in my paper.² However, conclusions (a) and (b) drawn by him from my paper, which Wódkiewicz considers to be “exact and correct,” are inconsistent.

(a) It is easy to see that the Markovian approximation (master equation) in the case of spontaneous emission with $\omega_0 \rightarrow 0$ leads to the equations

$$\dot{P}_{1/2} = 0, \quad \dot{P}_{-1/2} = 0, \quad (1)$$

where $\frac{1}{2}$, $-\frac{1}{2}$ designate upper and lower levels, respectively, for $\omega_0 \neq 0$. Indeed, solution (22) of my paper describes time evolution different from the constant $P_{1/2}$ and $P_{-1/2}$. These solutions together with (19)–(21) describe the creation of bosons from an initial vacuum state. Thus, Eqs. (21) determine the numbers of bosons in various modes k at every moment of time, while at $t=0$ all boson numbers were equal to zero.

Considering the function $A(t)$ in the limit $t \rightarrow \infty$, Wódkiewicz makes a transition to the Markovian (and Weisskopf-Wigner) approximation. Therefore his statement “that in this limit there is no radiative level shift and no radiative decay” means that there is no spontaneous emission in the *Markovian* approximation [see Eq. (1)]. My paper provides a non-Markovian solution of the problem.

One of the premises of the Weisskopf-Wigner approximation is that at any moment of time only one-boson states are taken into account. On the other hand, the wave function (19)–(21) obtained in my paper takes into account all many-boson states. These many-boson states can be neglected only at high enough frequency ω_0 , when the rotating-wave approximation may be employed. Therefore, the statement of the Comment that the Weisskopf-Wigner approximation becomes exact is inconsistent. The approximation in which all many-boson processes are neglected cannot take them into account in an exact way.

(b) The statement that “For arbitrary small $\omega_0 \dots$ the spontaneous emission is completely different from the one predicted by Fain” is “proved” by mentioning that in this case the spontaneous emission is subject to certain integro-differential equations. These integro-differential

equations have not been analyzed for the case of small nonvanishing ω_0 . On the other hand, as I mentioned in the paper, “all the implications of this exact solution are also valid for nonvanishing but small energy differences $\hbar\omega_0$. This can be verified by comparison with corresponding solutions⁵ in the region of their overlap.” [Reference 5 of the Letter is B. Fain, Phys. Rev. A **37**, 547 (1988).] It has been shown in this reference that for frequencies ω_0 satisfying the condition

$$\omega_0 < \omega_c = \sum_k B_k^2 / \hbar^2 \omega_k, \quad (2)$$

a nondissipative negative energy level appears in the system,

$$E_0 = \hbar\omega_0 - \sum_k B_k^2 / \hbar \omega_k, \quad (3)$$

and the probability $P_{1/2}(\infty)$ to remain in the upper state is

$$P_{1/2} = 1 - 2 \sum_k B_k^2 / \hbar^2 \omega_k^2 \quad (4)$$

[unlike the Markovian approximation, where $P_{1/2}(\infty) = 0$]. These relations are valid for rather small $\sum_k B_k^2 / \hbar^2 \omega_k^2 \ll 1$. Solutions (2)–(4) tend to those obtained in my paper, when $\omega_0 \rightarrow 0$. (Indeed, only for this analysis did I take B_k to be small. The solution obtained in my paper is valid for the arbitrary $B_k, \omega_0 \rightarrow 0$.) It means that, for “arbitrary small ω_0 , the solution is” *not* *not* “dramatically different.” Thus none of the conclusions of the Comment are “exact and correct.”

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¹K. Wódkiewicz, preceding Comment, Phys. Rev. Lett. **63**, 2693 (1989).

²Benjamin Fain, Phys. Rev. Lett. **61**, 2197 (1988).