Comment on "Spontaneous and Induced Emission of Soft Bosons: Exact Non-Markovian Solution"

In a recent Letter,[1] Fain has considered an exact dynamical solution for a degenerate two-level system (with the atomic frequency $\omega_0=0$) coupled to a harmonic boson field.

The conclusions of his work as presented in the Letter are the following. (a) The solution behaves in an explicitly non-Markovian way, i.e., spontaneous and induced emissions do not decay exponentially. This behavior contrasts with the standard Weisskopf-Wigner solutions. (b) The decay depends in an explicit way on the initial condition. This is again a striking difference between the obtained results and the conventional behavior based on the Weisskopf-Wigner theory. (c) All the applications of this exact solution are also valid for nonvanishing but small energy differences $\hbar\omega_0$.

It is the purpose of this Comment to show that all of these conclusions are incorrect. Using the published literature, it is easy to show that, in contrast to the results claimed by Fain, the following statements are exact and correct. (a) In the limit of $\omega_0=0$, the theory is fully Markovian, because independently of the atomic initial state there is no radiative decay at all. In this limit the Weisskopf-Wigner approximation becomes exact. (b) For arbitrary small ω_0, the non-Markovian character of spontaneous emission is completely different from the one predicted by Fain. The non-Markovian spontaneous-emission decay rate for long time behaves as t^{-2}.

These results are well known and have been presented many times in the literature, in order to elucidate my point I will present the exact solution derived by Fain, using the Heisenberg equations-of-motion approach. In this approach I solve first the equation of motion for the boson field and then insert this solution into the atomic equations using the normal ordering of the field. This standard approach has been described and discussed in detail in Refs. 2–4. As a result of this exact procedure I obtain the following equations for the atomic variables described by Pauli spin operators:

$$d\hat{\sigma}_2/dt=0\,, \tag{1}$$

$$d\hat{\sigma}_1/dt=2\lambda(\hat{\sigma}_3\hat{E}_v^{(+)}+\hat{E}_v^{(-)}\hat{\sigma}_3)+2i\lambda^2\int d_3k\,f^2(\omega_k)\int_0^t ds[\exp(i\omega_k s)\hat{\sigma}_2(s)\hat{\sigma}_3(t)-\text{H.c.}]\,, \tag{2}$$

$$d\hat{\sigma}_3/dt=-2\lambda(\hat{\sigma}_1\hat{E}_v^{(+)}+E_v^{(-)}\hat{\sigma}_1)+2i\lambda^2\int d_3k\,f^2(\omega_k)\int_0^t ds[\exp(i\omega_k s)\hat{\sigma}_2(s)\hat{\sigma}_1(t)-\text{H.c.}]\,, \tag{3}$$

where $\hat{E}_v^{(-)}(t)=\int d_3k\,f(\omega_k)\hat{a}_k(0)\exp(-i\omega_k t)$ and $\hat{E}_v^{(-)}(t)=[\hat{E}_v^{(-)}(t)]^\dagger$ denote, respectively, the annihilation and the creation part of the vacuum electric field (boson field), the coupling constant λ involves all the numerical factors related to the atomic dipole moment and polarization, and the function $f(\omega_k)$ for a sharp cutoff is simply equal to $(\omega_k)^{1/2}$. Without the rotating-wave approximation and for $\omega_0=0$, Eq. (1) predicts that $\hat{\sigma}_2$ is a constant of motion. As a result of this, in Eqs. (2) and (3) we have the following exact relations: $\hat{\sigma}_2(s)\hat{\sigma}_3(t)=\hat{\sigma}_2(t)\hat{\sigma}_3(t)=i\hat{\sigma}_1(t)$ and $\hat{\sigma}_2(s)\hat{\sigma}_1(t)=\hat{\sigma}_2(t)\hat{\sigma}_1(t)=-i\hat{\sigma}_3(t)$. In this case the Heisenberg equations (2) and (3) are linear. For spontaneous emission, i.e., when the initial state of the boson field is in the vacuum state, I obtain the following exact equations: $d\langle\hat{\sigma}_2\rangle/dt=0$, $d\langle\hat{\sigma}_1\rangle/dt=-A(t)\langle\hat{\sigma}_1\rangle$, and $d\langle\hat{\sigma}_3\rangle/dt=-A(t)\langle\hat{\sigma}_3\rangle$, where $A(t)=4\lambda^2\int d_3k\,f^2(\omega_k)\int_0^t ds\cos(\omega_k s)$ is a real function of t. Comparing these exact results with the standard theory I conclude that in the limit of $t\to\infty$ this function is exactly equal to the Weisskopf-Wigner result but evaluated at $\omega_0=0$. In this case we have $A(\infty)=0$. This means that in this limit there is no radiative level shift (Lamb shift), and no radiative decay. Moreover, the time evolution of the model is entirely Markovian contrary to the claim made by Fain [see Eqs. (22)–(26) in Ref. 1].

For arbitrary small ω_0, the solution is dramatically different. This follows from the fact that for the free evolution of the atomic variables we have, for example,

$$\hat{\sigma}_1(s)\hat{\sigma}_2(t)=-\sin[\omega_0(t-s)]+i\cos[\omega_0(t-s)]\hat{\sigma}_3(s)\,.$$

This relation, if inserted into the Heisenberg equations of motion (2) and (3), leads to the well-known integro-differential equations of non-Markovian spontaneous emission discussed, for example, in Ref. 4. Stimulated emission can be discussed in the same way. For an initial coherent state of the boson field $|\alpha_k\rangle$, I obtain the following exact equations for the atomic variables: $d\langle\hat{\sigma}_2\rangle/dt=0$, $d\langle\hat{\sigma}_1\rangle/dt=\Omega(t)\langle\hat{\sigma}_3\rangle$, and $d\langle\hat{\sigma}_3\rangle/dt=-\Omega(t)\langle\hat{\sigma}_1\rangle$, where

$$\Omega(t)=2\lambda\int d_3k\,f(\omega_k)[\alpha_k\exp(-i\omega_k t)+\alpha_k^*\exp(i\omega_k t)]$$

is the coherent Rabi frequency. Again this exact result contains no decay, is fully Markovian, and the time evolution is periodic, contrary to the claim made by Fain [see Eqs. (30)–(33) in Ref. 1].

This research has been partially supported by the Research Program CPBO.01.03.

K. Wódkiewicz
Institute of Theoretical Physics
Warsaw University,
Warsaw 00-681, Poland

Received 17 February 1989
PACS numbers: 42.50.Md

[1] B. Fain, Phys. Rev. Lett. **61**, 2197 (1988).

[2] J. R. Ackerhalt, P. L. Knight, and J. H. Eberly, Phys. Rev. Lett. **30**, 456 (1973), and references therein.

[3] L. Allen and J. H. Eberly, *Optical Resonance and the Two-Level Atoms* (Wiley, New York, 1975), Chap. 7, and references therein.

[4] K. Wódkiewicz and J. H. Eberly, Ann. Phys. (N.Y.) **101**, 574 (1976).