High-Energy Heavy-Ion Reactions and Relativistic Hydrodynamics in Three Dimensions

U. Ornik, F. W. Pottag, and R. M. Weiner
Department of Physics, University of Marburg, 3550 Marburg, Federal Republic of Germany
(Received 30 December 1988)

A numerical three-dimensional solution of the equations of relativistic hydrodynamics with Landau-type initial conditions has been obtained. Using an equation of state consistent with lattice-QCD calculations, the rapidity and transverse-momentum distributions are calculated for O+Au collisions at 200 GeV/nucleon. Satisfactory agreement with data is obtained with an initial temperature $T_i = 220$ MeV corresponding to a mixed quark-gluon–hadron phase. A pure hadron phase is within this formalism inconsistent with the data.

PACS numbers: 25.70.Np, 12.38.Mh, 12.40.Ee, 47.75.+f

The possibility that in high-energy hadron-hadron or nucleus-nucleus (AA') collisions a quark-gluon plasma (QGP) is created has resurrected the interest in the hydrodynamical model, because only through hydrodynamics can information about the equation of state (EOS) be obtained. In most hydrodynamical calculations performed so far for AA' collisions, Bjorken scaling[1] was assumed.[2–7] Exceptions are found in Refs. 8 and 9 which consider compressional shock waves or in Ref. 10 in which a bag EOS is assumed and no rapidity and transverse-momentum distributions are calculated. The assumption of Bjorken scaling is equivalent to demanding that the hydrodynamical solutions are invariant under a longitudinal boost. This is based on the fact that for high-energy pp collisions the rapidity distribution $d\bar{n}/dy$ has a plateau in the central rapidity region. This assumption simplifies considerably the mathematics of the problem. However, even for pp collisions the Bjorken scaling is only valid in the central region and for pA and AA' collisions it is certainly unjustified (at least for asymmetric systems at the energies available so far) as suggested by the fact that $d\bar{n}/dy$ has no plateau for pA collisions.[11] The correct procedure is therefore to consider the hydrodynamical flow in its full physical generality. The only acceptable simplification for central collisions is the use of cylindrical (axial) symmetry. In this paper we report on the solution of such a problem with Landau initial conditions;[12] i.e., the hydrodynamics is used after shock waves have disappeared.

At present the handling of shock waves in the ultrarelativistic regime, when (partial) transparency is present, is yet an unsolved problem. Furthermore, even if one knew how to treat compression shocks, this would not necessarily bring more insight, because one would have to assume local equilibrium from the very beginning of the reaction, which is certainly too strong an assumption. At intermediate energies ($<10A$ GeV) the situation may be different.[13] Our hydrodynamical description starts after the formation of an initial fireball with given geometry and thermodynamical properties.

Another point that should be emphasized is that concerning the EOS. In most theoretical calculations done in the past a constant velocity of sound c_0 was assumed leading to an EOS of the form $p = c_0^2\epsilon$, where p is the pressure and ϵ is the energy density of the fluid. Lattice-QCD calculations,[14] as well as general heuristic considerations, on the other hand,[15] suggest the existence of a phase transition so that the velocity of sound becomes a strong function of ϵ. This nonlinear behavior affects the hydrodynamical expansion and has been considered for the first time in Ref. 16.

In the absence of viscosity and thermal conductivity the hydrodynamical flow is governed by the relativistic generalization of the Euler equations

$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\,T_\mu^\nu)}{\partial x^\nu} - \frac{1}{2}\frac{\partial g_{\nu\alpha}}{\partial x^\mu}T^{\alpha\nu} = 0\,, \qquad (1)$$

where

$$T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu + p g_{\mu\nu} \qquad (2)$$

is the energy-momentum tensor, u_μ is the four-velocity of the fluid, and $g_{\mu\nu}$ is the metric tensor. Since we are interested in central collisions we can assume axial symmetry around the beam direction. Equation (1) is then solved for the temperature of a fluid element and its velocity as a function of the space-time variables x,r,t, where x is the coordinate along the longitudinal axis (beam axis) and r is the radial coordinate in the transverse direction. This will be called a $(2+1)$-dimensional solution, to be compared with the $1+1$ case where only x and t are considered (longitudinal expansion).

In order to integrate the hydrodynamical system of partial differential equations (PDE), one eliminates one of the variables through the EOS which can be parametrized as follows:[16,17]

$$c_0^2(\epsilon) = \left[\alpha + \beta\tanh\left(\gamma\ln\frac{\epsilon}{\epsilon_c} + \delta\right)\right]\left(1 - \frac{(1-\xi)\Gamma^2}{\ln^2(\epsilon/\epsilon_c) + \Gamma^2}\right), \qquad (3)$$

where α, β, γ, δ, Γ, ξ, and ϵ_c are constants. These parameters are determined from the results of lattice QCD and from the following asymptotic conditions:[18]

$$\lim_{T\to\infty} c_0^2 = \frac{1}{3} \quad \text{and} \quad \lim_{T\to 0} c_0^2 = \frac{1}{3+a}\,, \qquad (4)$$

TABLE I. List of parameters used in Eq. (3) and obtained from Ref. 14.

α	β	γ	δ	ξ	Γ^2	ϵ_c (GeV/fm^3)
$\frac{5}{21}$	$\frac{2}{21}$	0.24	1.05	0.3	0.73	3.0

where a is a parameter fixed by phenomenological studies of the resonance spectrum,[19-21] with $3 \leq a \leq 4.5$. In this paper we took $a=4$. One possible set of parameters consistent with the above constraints is shown in Table I. Once the EOS is known, one can derive from thermodynamics the following expressions relating the energy density and temperature:

$$\frac{T}{T_0}=\exp\left[\int_{\epsilon_0}^{\epsilon}\frac{c_0^2}{(1+c^2)\epsilon'}d\epsilon'\right],\quad c^2=\frac{1}{\epsilon}\int_{\epsilon_0}^{\epsilon}c_0^2\,d\epsilon'. \tag{5}$$

In Figs. 1(a)–1(d) the temperature dependence and energy-density dependence of the various quantities are shown.

As mentioned previously we are starting our calculation at the beginning of the expansion stage, after the formation of a locally thermalized fireball, with cylindrical symmetry, at rest in the c.m. system. Its spatial extension is determined by the radius of the projectile nucleus R^p and the Lorentz-contracted length of the cylinder cut by the projectile in the target $2R^t/\gamma_{\rm c.m.}$. The initial volume is thus

$$V_i=2\pi(R^p)^2R^t/\gamma_{\rm c.m.}\,. \tag{6}$$

Because of the formation time of the fireball of about 1 fm, the parameters R^p and R^t can fluctuate around the "clean cut geometry" by this distance.

For the calculation of $\gamma_{\rm c.m.}=\cosh(y_{\rm c.m.})$, we assume that the fireball expands symmetrically in the c.m. system, so that the c.m. rapidity $y_{\rm c.m.}$ corresponds to the maximum of the $d\bar{n}/dy$ distribution. From the O+Au data of NA35 and WA80 (Refs. 22 and 23) we get $y_{\rm c.m.}=2.45$. In order to check the consistency of these estimates we calculate $\beta_{\rm c.m.}$ for a system of sixteen projectile nucleons with an incoming energy $E=3200$ GeV and $\bar{n}_p$ participant target nucleons with the mass $\bar{n}_p m_n$, where m_n is the nucleon mass:

$$\beta_{\rm c.m.}=\frac{P_p}{E_p+\bar{n}_p m_n}\,. \tag{7}$$

From Eq. (7) we obtain

$$\bar{n}_p=\frac{1}{m_n}\left[\frac{P_p}{(1-1/\gamma_{\rm c.m.}^2)^{1/2}}-E_p\right]. \tag{8}$$

For $\gamma_{\rm c.m.}=5.5$ we get $\bar{n}_p=57$ which is in good agreement with the estimates obtained from data by using baryon-number conservation.[23] From the experimental information on the number of secondaries produced in the final state $\bar{n}_f$ and using an inelasticity $K\simeq 0.46$,[24] where $K=M/\sqrt{s'}$, with M being the fireball mass and $\sqrt{s'}$ the c.m. energy of all participants ($\simeq 590$ GeV), one can calculate V_i and T_i as follows: We write

$$\bar{n}_f=\alpha S_f\,, \tag{9}$$

where α is a slowly varying function of the temperature and S_f is the entropy in the final state.[12] On the other hand, it follows from Eqs. (1) and (2) that

$$\partial_\mu(su^\mu)=0,\ \text{i.e.,}\ S_i=S_f\,, \tag{10}$$

where S_i is the initial entropy. From Eqs. (9) and (10) and the assumption that the energy is distributed homogeneously in the initial fireball, one gets

$$\epsilon_i V_i=K\sqrt{s'}\,, \tag{11}$$

$$s_i(T_i)=\bar{n}_f\epsilon_i(T_i)/\alpha M\,, \tag{12}$$

where s_i is the initial entropy density.

Equation (1) was solved numerically by using a finite-difference method.[18,25] We define, as usual, the freezeout time t_f in a local fluid element as the time at which particles become free. At this moment the temperature takes the value $T(t_f,x,r)=T_f$, which defines a hypersurface σ, the freezeout isotherm. On σ the parti-

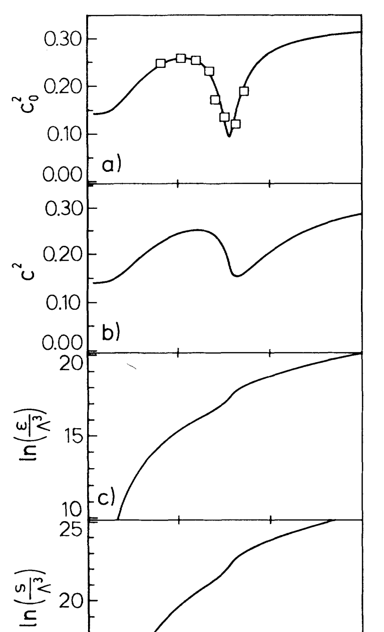

FIG. 1. (a)–(d) Velocity of sound c_0, c, energy density, and entropy density vs T/Λ. The numerical values for $c_0^2(T)$ are taken from Ref. 14. For the parameter Λ we took 1.29×10^{-3} GeV, which means that the phase transition occurs at a temperature of 0.2 GeV.

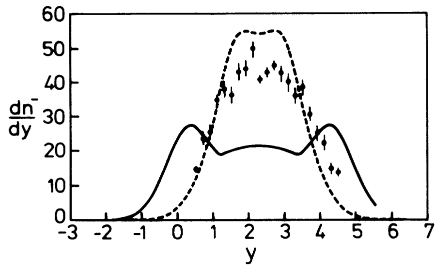

FIG. 2. Comparison of rapidity distribution of the (2+1)-dimensional solution (dashed line) with the (1+1)-dimensional one (solid line) both with constant speed of sound $c_0^2=\frac{1}{3}$. The initial temperature in both cases is 0.38 GeV. For the initial geometry we took $R^p=3.1$ fm and $R^t=1.5$ fm. The data are from Ref. 22. The shoulders in the 1+1 case are due to the simple wave solution which contributes predominantly to large y. In the 2+1 case the transverse expansion fills up the gap between the shoulders.

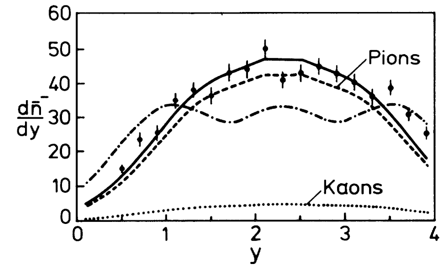

FIG. 3. Rapidity distribution of negative particles (Ref. 22). The contributions from kaons (dotted line), pions (dashed line), and the sum of both (solid line) were calculated with the (2+1)-dimensional hydrodynamical model with an EOS given by Eq. (3). The parameters are listed in Table I. Initial conditions: $T_i=0.22$ GeV, $R^p=3.8$ fm, and $R^t=1.5$ fm. Also represented is the distribution obtained from a (1+1)-dimensional solution with an EOS given by Eq. (3) (dash-dotted lines).

cles are distributed according to a Bose-Einstein or Fermi-Dirac distribution

$$E\frac{d^3N}{dp^3}=\frac{1}{(2\pi)^3}\int_\sigma\int\rho(m)dm\frac{p^\mu d\sigma_\mu}{\exp(p^\mu u_\mu/T_f)\pm 1}, \quad (13)$$

where p_μ is the momentum of the observed particles and $d\sigma^\mu$ is the hypersurface element normal to σ.

Particles produced with momenta p_μ pointing into the interior of the emitting isotherm, i.e., regions of higher density, are assumed to be absorbed and therefore their contribution to the total number $\bar{n}$ is set equal to zero. This procedure neglects the changes of entropy and energy due to absorption. However, the number of absorbed particles is found to be small and therefore, in a first approximation, these changes should not matter. We have checked that the results presented in this paper remained practically the same if the absorption is not considered at all.

The freezeout temperature T_f is a free parameter chosen in the following to be $\simeq 140$ MeV. In applications we consider only pions and kaons, neglect final-state interactions and the decay of resonances, and correct for the fact that in the experiment no distinction between pions and kaons is made. This last circumstance influences the meaured rapidity values[26] and thus the transverse-momentum (p_t) distributions which are given in various rapidity windows.

We found that for an EOS with constant $c_0^2=\frac{1}{3}$ and for the assumption of a pure pionic gas, it is not possible to fit simultaneously the height and the width of the rapidity distribution (Fig. 2). Assuming an EOS with variable speed of sound as described in Eq. (3) and with parameters listed in Table I (lattice-QCD calculations for a pure gluonic system) we can reduce the initial temperature to 220 MeV (Fig. 3). From Fig. 1 it turns out that this means that the fireball is initially in the mixed phase. In Fig. 2 we also compare the rapidity distribution obtained for a 1+1 expansion with a 2+1, both with $c_0^2=\frac{1}{3}$. The initial conditions in both calculations are the same: $T_i=380$ MeV, $R^p=3.2$ fm, $R^t/\gamma_{\text{c.m.}}=1.5$ fm. It is seen that the (1+1)-dimensional solution seriously underestimates $d\bar{n}/dy$. An increase of T_i in the 1+1 solution might increase $d\bar{n}/dy$ but would bring about a much too broad $d\bar{n}/dy$ distribution. It is difficult to conceive that a change of parameters could change this situation, unless very exotic values for these parameters were chosen.

With $T_i=220$ MeV, $R^t/\gamma_{\text{c.m.}}=1.5$ fm, and $R^p=3.8$ fm we obtained a good fit to the rapidity distribution (Fig. 3) and a satisfactory fit to the p_t distribution (Fig. 4) of negative secondaries where we took into account the kaon contribution. Preliminary calculations suggest that an improvement of the agreement with data can be obtained by assuming a phase transition of the first kind $c_0^2(T_c)=0$ and/or by taking into account the contributions of more massive resonances like kaons to the entropy. Both possibilities are under investigation. In Fig. 3 we also present the results for $d\bar{n}/dy$ obtained in a (1+1)-dimensional solution with the EOS given by Eq.

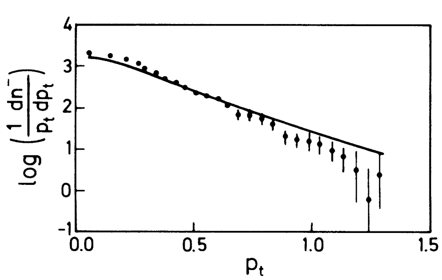

FIG. 4. Transverse-momentum distribution of negative secondaries (Ref. 22). The solid line is a solution of the (2+1)-dimensional hydrodynamical model. Initial conditions: $T_i=0.22$ GeV, $R^p=3.8$ fm, and $R^t=1.5$ fm.

(3). It is seen that it again leads to an unacceptable distribution.

To summarize, the comparison of 1+1 and 2+1 hydrodynamical calculations with data assuming various equations of state strongly suggests the need for a three-dimensional solution with an EOS containing a phase transition from hadronic matter to QGP. The initial temperature of $T_i = 220$ MeV suggested by the data corresponds to a mixed (quark-)gluon–hadron phase and is inconsistent with the existence of a pure hadron phase. This also implies that a pure QGP phase has not been reached yet at present energies and thus clean signals of QGP might be difficult to see.

This work was supported in part by the Deutsche Forschungsgemeinschaft and Gesellschaft für Schwerionenforschung Darmstadt.

[1]J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).

[2]G. Baym *et al.*, Nucl. Phys. **A407**, 541 (1983).

[3]A. Bialas, W. Czyz, and A. Kolawa, Acta Phys. Pol. B **15**, 229 (1984).

[4]J. P. Blaizot and J. Y. Ollitrault, Nucl. Phys. **A458**, 745 (1986).

[5]H. Gersdorff, L. McLerran, M. Kataja, and P. V. Ruuskanen, Phys. Rev. D **34**, 794 (1986).

[6]S. Kagiyama, A. Nakamura, and A. Minaka, Prog. Theor. Phys. **75**, 319 (1986).

[7]X. Wang and R. C. Hwa, Phys. Rev. D **35**, 3409 (1987).

[8]T. Rentzsch *et al.*, Z. Phys. C **38**, 237 (1988).

[9]R. B. Clare and D. Strottman, Phys. Rep. **141**, 177 (1986).

[10]M. Kataja, Z. Phys. C **38**, 419 (1988).

[11]I. Otterlund, S. Garpman, and I. Lund, Z. Phys. C **20**, 281 (1983).

[12]L. D. Landau, Izv. Akad. Nauk SSSR, Ser. Fiz. **17**, 51 (1953), also published in *Collected Papers of L. D. Landau,* edited by D. Ter Haar (Pergamon, New York, 1965).

[13]W. Scheid, H. Müller, and W. Greiner, Phys. Rev. Lett. **32**, 741 (1974).

[14]K. Redlich and H. Satz, Phys. Rev. D **33**, 3747 (1986); R. V. Gavai and A. Gocksch, Phys. Rev. D **33**, 614 (1986); T. Celik, J. Engels, and H. Satz, Phys. Lett. **125B**, 411 (1983); J. Kogut *et al.*, Phys. Rev. Lett. **50**, 393 (1983).

[15]M. Plümer, S. Raha, and R. M. Weiner, Phys. Lett. **139B**, 198 (1984).

[16]K. Wehrberger and R. M. Weiner, Phys. Rev. D **31**, 222 (1985).

[17]U. Ornik and R. M. Weiner, Phys. Rev. D **36**, 1263 (1987).

[18]U. Ornik, F. W. Pottag, and R. M. Weiner, in Proceedings of the Meeting on Hadronic Matter in Collision, Tucson, Arizona, 1988, edited by P. A. Carruthers and J. Rafelski (World Scientific, Sinagpore, to be published).

[19]R. Hagedorn, Nuovo Cimento Suppl. **3**, 147 (1965); J. Rafelski and R. Hagedorn, in *Statistical Mechanics of Quarks and Hadrons,* edited by H. Satz (North-Holland, Amsterdam, 1981), p. 253.

[20]E. V. Shuryak, Sov. J. Nucl. Phys. **16**, 220 (1973).

[21]F. Cooper, G. Frye, and E. Schonberg, Phys. Rev. D **11**, 192 (1975).

[22]H. Ströbele *et al.*, Z. Phys. C **38**, 89 (1988).

[23]T. J. Humanic *et al.*, Z. Phys. C **38**, 79 (1988).

[24]The value K is obtained in the following two ways: (a) by calculating the total energy of secondaries

$$M\gamma_{\text{c.m.}} = \int \frac{d^2\bar{n}}{dy\,dp_t} m_t \cosh y\, dy\, dp_t \simeq 1400 \text{ GeV};$$

(b) by using the theoretical derivation obtained in G. N. Fowler *et al.*, Phys. Rev. C **40**, 1219 (1989); Phys. Lett. B **214**, 657 (1988).

[25]R. W. McCormack, in *Proceedings of the Second International Conference on Numerical Methods in Fluid Dynamics,* edited by Maurice Holt, Lecture Notes in Physics Vol. 8 (Springer-Verlag, New York, 1970).

[26]A. Klar and J. Hüfner, Phys. Rev. D **31**, 491 (1985).