Global Texture as the Origin of Cosmic Structure

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If a unified theory possesses non-Abelian global symmetries which are spontaneously broken, it gives rise to a three-dimensional defect known as texture. In an expanding universe texture obeys a "scaling solution," which produces a constant density perturbation at horizon crossing of $\delta\rho/\rho \approx m_{\rm GUT}^2/m_{\rm Planck}^2$, where $m_{\rm GUT}$ is the scale of symmetry breaking. These (isocurvature) perturbations are broadly of Harrison-Zeldovich form, but are non-Gaussian, having strong "spikes" in the density field. Consequences for microwave background anisotropy are briefly discussed.

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The idea that a symmetry-breaking phase transition in the early Universe produced the density inhomogeneities required to form structure in the Universe is both appealing and plausible. It has inspired a number of recent theories of the origin of cosmic structure—cosmic strings,[1] superconducting cosmic strings,[2] and "soft" domain walls.[3] The inflationary-universe scenario originally shared this motivation, but "working" models of inflation seem to require a singlet scalar field "tacked onto" the unified theory.

In this Letter I propose a new simple mechanism for the generation of large-scale structure via the breaking of a global non-Abelian continuous symmetry, at the grand unification scale.

Despite the existence of continuous global symmetries in the standard model there is some prejudice against their occurring in unified models among particle physicists. This is partly a result of the success of the gauge principle, and partly due to the nonobservation of massless Goldstone bosons. However, *prima facie* there is no reason to believe that a fundamental theory would not have a larger global symmetry group than the standard model, and since the couplings of Goldstone bosons are suppressed by inverse powers of the symmetry-breaking scale, they could be unobservable at low energies. Broken non-Abelian continuous symmetries have been discussed in the context of horizontal symmetries where constraints from low-energy physics imply that the breaking scale η must be greater than 10^{10} GeV.[4] In this paper I will derive a new, approximate constraint (in the absence of inflation) that $\eta < 10^{16}$ GeV from the isotropy of the microwave background.

Broken Abelian [U(1)] global symmetries lead to global cosmic strings, which have been extensively discussed in the literature as a possible mechanism for large-scale structure formation. Non-Abelian global symmetries can lead to global monopoles, as discussed by Linde[5] and by Barriol and Vilenkin.[6] Generically, however, they lead to a third type of defect, global texture.[1,7] It is simple to construct grand-unified-theory (GUT) models where this occurs. An SU(2) global symmetry can be imposed on any model with two identical complex Higgs representations. To do so constrains the masses and couplings, and produces extra "accidental" ungauged degeneracy in the Higgs potential minimum. Conversely any theory which predicts Higgs-boson couplings in a simple way is likely to have such symmetries. Continuous global symmetries can also result from the imposition of discrete symmetries and the restriction to quartic terms in the potential: For example, if one has two complex fields ϕ and χ in a U(1) gauge theory, imposing the symmetries $\phi \to i\phi$, $\chi \to \chi$ and $\phi \to (\phi+\chi)/\sqrt{2}$, $\chi \to (\chi - \phi)/\sqrt{2}$ is enough to guarantee full global SU(2) invariance. The same result can be obtained by imposing a smaller non-Abelian group, $D_3 \times Z_2$. Such symmetries arise in low-energy effective theories derived from superstring models.

Broken non-Abelian global symmetries *always* lead to the formation of texture. To see this, consider the exact sequence of homotopy groups

$$\cdots \to \pi_3(H) \to \pi_3(G) \to \pi_3(G/H) \to \pi_2(H) \to \cdots , \tag{1}$$

where H is the global low-energy symmetry group and G is the original global group. If we assume that the non-Abelian symmetry is completely broken, then $\pi_3(H)$ and $\pi_2(H)$ are trivial. Therefore the vacuum manifold (potential minimum) $M \approx G/H$ has $\pi_3(M) \approx \pi_3(G) \approx Z$, the integers, for *any* non-Abelian compact group. In the simplest case I will discuss below, $G = $SU(2) and $H = 1$, M is a three-sphere. A generalization is to allow the unbroken subgroup H to contain non-Abelian factors; in this case it is possible to have π_3 trivial. An example is SO(5) broken to SO(4) by a vector, for which $M \approx S^4$. This leads to what I shall call "nontopological texture." In most of this paper, however, I will discuss the simpler case, where π_3 is nontrivial.

A texture is easiest to visualize in one dimension where M is a circle. As one travels along in space the field may wind around M in some localized region. Unlike other defects, in a texture the Higgs field can remain in the vacuum manifold throughout. The energy of the texture comes almost entirely from gradient energy in the field. In two or three dimensions one similarly has a localized region (which I shall call a "knot") within which the scalar field winds around M: The field has trivial topology at infinity. Nevertheless to undo the knot requires lifting the Higgs field from M.

If the symmetry is gauged, the texture becomes merely another vacuum configuration, as the gauge field can adjust to "soak up" the gradient in the scalar field everywhere. To see this, write the Higgs-field configuration $\Phi(\mathbf{x})=g(\mathbf{x})\Phi_0$, with Φ_0 any point on M. Now choosing $A_\mu=-(i/e)\partial_\mu g g^{-1}$ is enough to set $D_\mu\Phi=0$ everywhere. Since A_μ is pure gauge, all field strengths and currents vanish everywhere. An example of this occurs in the Weinberg-Salam theory, where different "texture" sectors correspond to different baryon number states in the quantum theory.

Global texture has been largely ignored so far because it is unstable. Davis[7] considered a simple model with global texture, but with only a single knot in the whole universe, stabilized by a large spatial curvature term. However, if one is interested in generating fluctuations, the instability is just what is needed, as I shall explain.

The instability may be understood from Derrick's theorem.[8] The energy of a static configuration is given as the integral of a gradient squared term and a potential term. If one replaces a given configuration $\Phi(\mathbf{x})$ by $\Phi(\lambda\mathbf{x})$ with $\lambda>1$, shrinking it, the gradient term scales as λ^{2-D} while the potential term scales as λ^{-D} in D spatial dimensions. Thus only in $D=1$ is a stable minimum possible—in $D>1$ any localized configuration of scalar fields is unstable to shrinking. Note that the argument fails for global strings or monopoles because the configurations are not localized—the energy of a single object diverges at infinity.

I will consider the simplest texture model, global SU(2) broken by a complex doublet, with potential $\lambda(\Phi^2-\eta^2)^2$. This is equivalent in the pure scalar sector to a theory with a larger symmetry, SO(4), broken by a four-vector to SO(3), as may be seen by writing Φ in real components. As explained above, a texture knot shrinks radially to a point. As long as its size is larger than the symmetry-breaking scale $m^{-1}\sim\lambda^{-1/2}\eta^{-1}$ the field Φ should remain close to M throughout space, but when the knot shrinks to of order m^{-1} the field gradients will be strong enough to pull Φ over the potential barrier and unwind the knot. One is then left with a localized lump of Goldstone-boson field.

As long as the knot is large, it is a good approximation to impose the condition $\Phi^2=\eta^2$ as a constraint on the free-massless-field action. Doing this with a Lagrange multiplier, one finds the equations of motion in a theory with an N-component field Φ^a, $a=1,\ldots,N$,

$$\nabla^\mu\partial_\mu\Phi^a=\frac{\Phi\cdot\nabla^\mu\partial_\mu\Phi}{\eta^2}\Phi^a=-\frac{(\partial_\mu\Phi)^2}{\eta^2}\Phi^a,\quad \Phi^2=\eta^2. \tag{2}$$

This is the equation for the O(N) nonlinear σ model.[9] In the case where $N=2$, $\Phi=\eta(\cos\theta,\sin\theta)$, θ is just a massless Goldstone-boson field. However, for $N>2$ the components of Φ are coupled nonlinearly and the dynamics more interesting. For $N=3$ in $D=2$, (2) has soliton solutions which have the same energy for arbitrary size, as indicated by Derrick's theorem. For $N=3$ in $D=3$ there are global monopoles which probably become connected by "strings" (the $D=2$ solitons) and disappear.[10] However, since M is S_2 in this case, the $\pi_3(S_2)=Z$, as indicated by (1), topological texture also forms. I shall discuss the simpler case $N=4$ below.

Note that while (2) is nonlinear, there is no dimensional scale in the equation apart from the Hubble radius (the background geometry scale). This means that the dynamics of Φ is determined solely by the *geometry* of the background and that of M with no free parameters. The theory should therefore be very predictive.

An interesting point is that texture can actually *survive* a period of inflation. This is because the degree of freedom involved in forming texture is massless, and so the usual quantum fluctuations produced in a de Sitter phase can drive it around the potential minimum. The usual arguments for a massless minimally coupled field in a de Sitter background[11] lead to the estimate $\Delta\Phi\approx H/2\pi$ on the scale H^{-1}. Hence for texture to be produced one requires $H\approx 2\pi\eta$.[12] For $\eta\approx10^{16}$ GeV, this requires a value of H higher than that possible in most simple models, but the addition of an $R\Phi^2$ term to the scalar-field potential can reduce the effective value of η during inflation, and allow copious quantum production of texture.

In a homogeneous hot-big-bang universe I expect Φ to evolve as follows. At high temperatures it is in thermal equilibrium, and the global symmetry is unbroken. As the universe cools, Φ falls to M throughout space, but to different points on M in different correlation volumes. Now as the universe expands and cools, knots in Φ shrink and radiate away into Goldstone bosons. Phase space clearly favors this process—at temperatures below m the formation of new small-scale knots becomes exponentially (Boltzmann) suppressed. The shrinking of knots correlates the Φ field on larger and larger scales. This is analogous to what happens when a ferromagnet cools. Quite quickly the correlation length should grow to of order the horizon scale, and then keep growing with it.

Now as still larger scales come cross the horizon, knots will constantly form anew as Φ points in different directions on M in different horizon volumes. The density in a knot as it forms is just $\approx(\nabla\Phi)^2\approx\eta^2/t^2$. The fractional density perturbation on the background density ρ_b is $\approx\eta^2/\rho_b t^2\approx(20\text{–}30)G\eta^2$, independent of time in the matter or radiation eras. To a first approximation there will be a scale-invariant Harrison-Zeldovich spectrum of density perturbations.[13] This argument should also hold in the case of nontopological texture, except that there would be no localized knots in that case.

It is important that the dynamics of Φ are *not* simply that of a massless field. If they were, different modes would decouple, and the correlation length remain constant in comoving coordinates. The nonlinear dynamics of Φ separate out knots from the smooth background density and lead to a real fluctuation forming.

On small scales the knots produce highly nonlinear perturbations. As they collapse, the density at the center quickly becomes larger than ρ_b. If the scale of the knot is r then the density in the core of the knot is of order η^2/r^2. Because of initial asymmetry, the center of the knot would, in general, be expected to move with a relativistic velocity in the latter stages of collapse. This and the short lifetime of the "spike" would diminish the resulting density perturbation on small scales.

I have evolved (2) numerically in flat spacetime for a spherically symmetric knot,

$$\Phi^a = \eta(\sin\chi\sin\theta\cos\phi, \sin\chi\sin\theta\sin\phi, \sin\chi\cos\theta, \cos\chi)\,, \quad (3)$$

where θ and ϕ are the standard polar angles and $\chi(r,t)$ governs the radial profile of the knot. $\chi(r,t)$ must vanish as r at the origin and, for a localized knot, go to π at infinity. The initial conditions used were $\partial_t\chi(r,0)=0$, $\chi(r,0)=\pi(1-e^{-3r})$. Defining $X(r,t)=r(\chi-\pi)$, vanishing at the origin and infinity, one obtains

$$\ddot{X} - X'' - \sin(2X/r)/r = 0\,, \quad (4)$$

which is, modulo problems at the origin, well solved by a standard leapfrog method. I smoothed χ near the origin by approximating it as linear in r. The graphs shown were obtained using 1000 points in r; increasing the resolution by up to 5 times and varying the smoothing scale similarly produced no noticeable difference in the results.

As can be seen in Fig. 1, the χ configuration collapses at the speed of light towards π everywhere. As explained above, only when the size of the "core" becomes of order m^{-1} (i.e., tiny) does the approximation in (2) break down and χ become free to hop over the potential barrier and thus move away from zero at the origin. I also tried starting configurations where χ went to some other constant value (e.g., $\pi/2$) at infinity (so that the energy density was not localized). For these too, χ collapses towards the chosen constant value everywhere. In the expanding-universe context, one therefore expects "half-knots" to occur just as often, or more often, than full knots. The nonlinearity and resulting density perturbation are, of course, smaller.

Figure 2 shows the energy density ρ in the scalar field times r^2. ρ clearly diverges at the origin as the knot shrinks, as expected.

In an expanding universe (4) is modified by a damping term. The effect of this is that a given mode only starts oscillating on a time scale of order of its wavelength, or when it crosses the Hubble radius. Thus at a time t knots whose size is t should be collapsing, and the correlation length should also be $\approx t$. There should be some fraction f per volume t^3. Triangulating S^3 with five tetrahedra, and considering five neighboring domains (centered at the vertices and center of a tetrahedron, for example), the probability of a full knot existing is just the probability that all domains correspond to different tetrahedra on S^3, from which one finds that $f \approx \frac{1}{25}$.

The source for the density perturbation in dark matter is $2\dot{\Phi}^2$. $r^2\dot{\Phi}^2$ is plotted in Fig. 3 for the latest time—the source clearly diverges at small r, $\rho_{\rm eff} \approx 5\eta^2/r^2$. As well as the spike produced instantaneously as the knot collapses, there is a spherical "blast wave" of outgoing Goldstone radiation. This could well be more important in the resulting accretion pattern, perhaps producing a "bubblelike" overdensity pattern.

With hot dark matter, very little accretion would occur on galaxy scales before decoupling. The gravitational attraction from knots would not counter neutrino diffusion, and the knots would disperse. Knots could accrete baryons after decoupling, and later, dark matter.

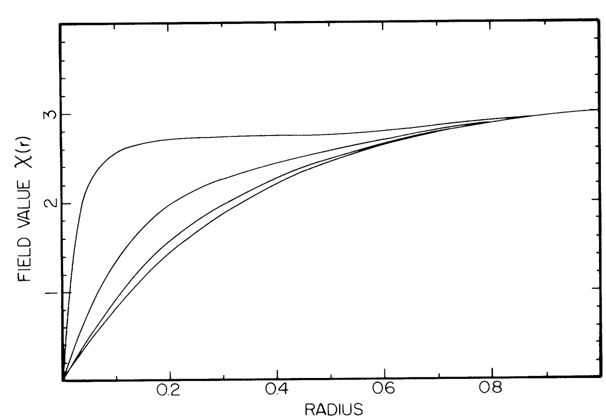

FIG. 1. The scalar-field configuration $\chi(r)$ in a collapsing knot is shown for different times. In units where $\chi(r) = \pi(1-e^{-3r})$ initially, and $c=1$, the times shown are 0, 0.1, 0.2, and 0.31, with the value of $\chi(r)$ increasing with time.

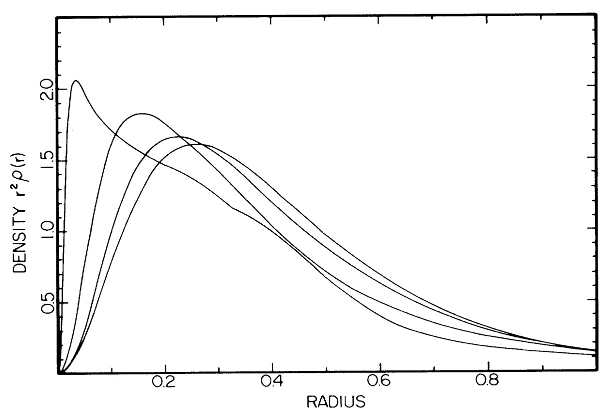

FIG. 2. The energy density in the scalar field at the same times. $r^2\rho(r)$ is plotted in units of η^2. With increasing time, the density collapses towards the origin. The "wiggle" at $r \approx 0.33$ in the last configuration does not seem to be a numerical artifact, but an effect of the boundary condition imposed at the initial time propagating outwards.

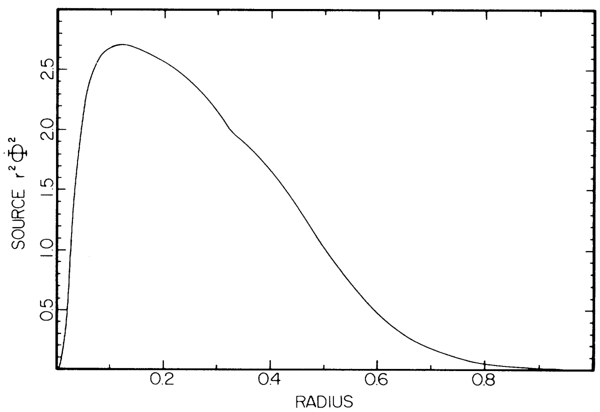

FIG. 3. The source for growth of density perturbations in the surrounding matter, $\dot{\Phi}^2$, shown at the last time in the sequence. $r^2\dot{\Phi}^2$ is plotted in units of η^2.

The theory obviously has more power on small scales than the standard hot-dark-matter model, but probably not enough to form galaxies.

With cold dark matter, on small scales which enter the horizon in the radiation era, the theory should simply reproduce the standard Zel'dovich spectrum. On larger scales, spikes and blast waves should produce noticeable effects. A simple way to normalize the theory is to demand that knots with the mean separation of Abell clusters had enough mass to accrete them. Richness-class-one Abell clusters have a mean separation today of $110h_{50}^{-1}$ Mpc. At equal density there was about one per volume $(2R_H)^3$, close to the density of knots just collapsed at that time. To accrete a region with a mean overdensity today $(\rho/\rho_b)_0$ starting from equal matter-radiation density requires an initial overdensity of $\approx (\rho/\rho_b)_0^{1/3}/(1+Z_{\rm eq})$, where $Z_{\rm eq}$ is $6250h_{50}^{-2}$ with three light neutrinos.

The effective overdensity in Fig. 3 is $M_{<r}/(4\pi/3\rho_b r^3) \approx 100G\eta^2(R_H/r)^2$ at equal density. An Abell cluster has an overdensity today of ≈ 200 in a radius of $3h_{50}^{-1}$ Mpc, i.e., a region with an initial comoving radius of $3\times(200)^{1/3} \approx 18h_{50}^{-1}$ Mpc. Thus $G\eta^2 \approx 10^{-6}$ is needed to produce Abell clusters. Because the perturbations are isothermal (only affecting the photon temperature by secondary gravitational effects), this scenario should produce very small fluctuations in the microwave background temperature. The Sachs-Wolfe effect, for example, is given approximately by $\delta T/T = \phi/3$, where ϕ is the potential on the last scattering surface. Using the numbers above, on the scale of knots formed at the time, this is given by $-4\pi G\eta^2/3 \approx -4\times 10^{-6}$. The potential actually diverges towards the center of a knot, but only logarithmically and this would probably not be observable. The pattern produced in the microwave background by a collapsing knot should, however, be quite characteristic.

Because the probability for high overdensities in the texture model is highly non-Gaussian, events like the great attractor[14] should occur, with a small but significant probability, due to knots entering the horizon late in the matter era.

As mentioned, the spherical waves emitted by collapsing knots could produce overdense shells. If some clusters also form at the vertices of these shells, their observed r^{-2} correlations might be naturally explained.[15] The coincidence between the scale of the observed "bubbles" in the galaxy distribution $[(40\text{–}60)h_{50}^{-1}$ Mpc],[16] the scale of correlations of Abell clusters, and the comoving Hubble radius at equal density might be neatly explained in this theory.

The background of Goldstone-boson radiation, in general, appears quite undetectable. The Goldstone background builds up only logarithmically in the radiation era, and at time t would be $\approx 30G\eta^2\ln(t/t_{\rm GUT})$, too low to present problems for nucleosynthesis. Today the Goldstone radiation would be dominated by the longest wavelengths, with total density $\approx 20G\eta^2(1+Z_{\rm eq})$, a few percent of the microwave background.

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